“This is an Accepted Manuscript of an article published in Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education, CERME11, February 6 – 10, 2019. (pp. 2961-2968), available online: https://hal.archives-ouvertes.fr/hal-02428863v1”

Semiotic analysis of modelling activities in a rich-digital environment

Osama Swidan[1] and Eleonora Faggiano[2]

[1]Ben-Gurion University of the Negev, Be’er-Sheva, Israel; osamas@bgu.ac.il

[2]University of Bari Aldo Moro, Italy; eleonora.faggiano@uniba.it

In this paper, we report on a research study that aims to understand the ways students transit between the simulation of real phenomenon and the mathematical representation, as the modelling activities are introduced by dynamic digital artefacts. The study took place in a high school in Turin, in which 9th grade students were introduced to the quadratic function through a task to be accomplished with the use of a dynamic digital simulation of a rolling ball on an inclined plane - the Galileo experiment. The semiotic mediation approach guided this study to carefully analyze the modelling activity, enhanced by the use of the digital artefact, in terms of meanings that evolve. Our results identify the signs which refer to the simulation of the real phenomenon and those which refer to the mathematical representation and bring to the fore the evolution from the artefact signs to the mathematical signs.

Keywords: Modelling activity, digital artefacts, semiotic mediation, signs, quadratic function.

Introduction

Mathematical modelling (the process of translating between the real world and mathematics in both directions) is one of the topics in mathematics education that has been discussed and propagated most intensely during the last few decades (Kaiser, 2014). Mathematical modelling is now a compulsory part of the mathematics curriculum in many countries worldwide and one of the main skills within international educational standards. Over the last two decades, a considerable variety of models (Borromeo Ferri, 2006) that explain the interplay between the real phenomena and the mathematical representations have emerged and processed (e.g. Blomhøj & Jensen, 2003; Blum & Leiß, 2007; Voskoglou, 2007). Despite the differences between the variety of models, the transition from the real phenomenon to the mathematical world is considered to be a common point among them. For example, to model this transition, Blum and Leiß (2007) distinguished between the mathematical world and the rest of the world. The transition from the real model to the mathematical model happens through a mathematising process, while the transition from the mathematical world to the real results happens through an interpreting process (Figure 1, taken from Blum & Leiß, 2007). These two processes are complex and require careful analysis in order to avoid useless oversimplification (Blum, 2011). This complexity is also valid when the real phenomenon is simulated by digital artefacts which connect the real phenomenon with mathematical representation. Digital artefacts are widely used to design and develop modelling activities, especially, to simulate complex real phenomena. Digital artefacts, indeed, allow linking multiple representations simultaneously. With their use, data concerning the real phenomena can be precisely displayed and mathematically interpreted. In the context of using digital artefacts, modelling activities are designed by using several kind of signs that are embedded in the artefact. Some of these signs are iconic (they have a great similarity with the real object), others are symbolic (they have a meaning in a specific culture). Bridging the gap between the iconic and symbolic signs is a serious challenge and should be taken into account (Arzarello & Sabena, 2011). Therefore, in addition to the traditional complexities of the transition between the real phenomenon and the mathematical one, the affordances and constraints of the digital artefacts, and the design principles of the tasks, should be taken into account when analyzing the learning processes of modelling activities (Naftaliev, 2017).

Herein, we assume that the construction of mathematical meanings throughout a modelling activity can be achieved through the mediation of dynamic digital artefacts. Moreover, we believe that the semiotic mediation approach could provide a suitable framework to carefully analyze the transition between the real world and the mathematical representations, identifying the evolution of personal meanings, emerging through the modelling activity, towards mathematical meanings, which are the objectives of the teaching intervention. In this paper, we will report on a research study that aims to understand the ways students transit between the simulation of real phenomenon and the mathematical representation, as the modelling activity is introduced by a dynamic digital artefact. We exemplify our approach by analyzing a teaching experiment concerning the quadratic function. For this purpose, we describe a designed and implemented modelling activity, which is based on the simulation of a rolling ball over an inclined plane – Galileo experiment – through the use of a dynamic digital artefact. We have chosen the Galileo experiment because the mathematical model of the spaces traversed by the ball is quadratic. We have chosen to simulate the real phenomenon using a dynamic digital artefact because it has the potentiality to precisely display the mathematical representation of the model and the real phenomenon using multiple-linked representations. The teaching experiment has been analyzed with the aim of revealing how the use of the digital artefact contributes to the construction of mathematical meanings, and to show how the semiotic mediation approach may help us to shed some light on the students' twofold transition between the real world and the mathematical world.

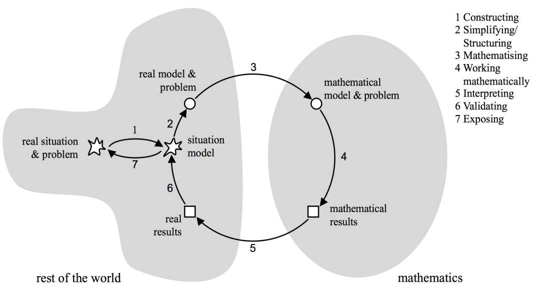

Figure 1: The Blum's modelling model (Blum & Leiß, 2007)

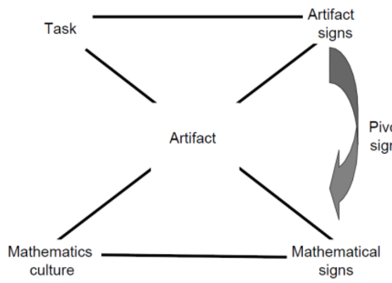

Figure 2: The semiotic mediation approach

Theoretical Framework - Semiotic Mediation Approach

In the context of using artefacts, Bartolini Bussi and Mariotti (2008) have modelled learning process by taking advantage of the potential of artefacts. The model aims to describe how meanings related to the use of a certain artefact can evolve into meanings recognizable as mathematical. The semiotic mediation approach assumes that social interaction and semiotic processes play a key role in learning, particularly in situation in which learners are encouraged to use the artefact in order to solve a given task. This approach considers learning to be an alignment between the personal meanings arising from the use of a certain artefact for the accomplishment of a task and the mathematical meanings that are deployed in the artefact. The relationship that the artefact has with the personal meanings emerging from its use and the mathematical meanings that might be evoked by such use is described as a double semiotic relationship and defines the *semiotic potential of the artefact*. On one hand, we concentrate on the use of the artefact to accomplish a task, recognizing the construction of knowledge within the solution of the task. On the other hand, we analyze the use of the artefact, distinguishing between the construction of the personal meanings arising in individuals from their use of the artefact in accomplishing the task (top part of Figure 2) and the mathematical meanings; meanings that an expert recognizes as mathematical (bottom part of Figure 2) when observing the students' use of the artefact in order to complete the task (left triangle in Figure 2). Personal meanings emerging from the activities carried out with an artefact may evolve into mathematical meanings, objectives of the teaching intervention. This evolution can occur in the peer interaction during the accomplishment of the task and in the collective discussions conducted by the teacher. In this long and complex construction process of shared mathematical meanings, it is possible to identify evolution paths (called *semiotic chains*) which are described by the appearance and enchainment of different types of signs: artefact signs, mathematical signs and pivot signs. The *artefact signs* refer to the artefact and its use, are produced through the social use of the artefact (upper right vertex in Figure 2) and may evolve into signs (called mathematical sign) that refer to the mathematics context. The *mathematical signs* are related to the mathematical meanings shared in the institution to which the classroom belongs (right side of Figure 2). Through a complex process of evolution of the artefact sign into a mathematical sign, other types of signs, which Bartolini Bussi and Mariotti called *pivot signs*, play a crucial role. The authors suggest that the characteristic of these signs is their shared polysemy: the pivot signs with their hybrid nature, both referring to the use of the artefact and to the mathematical domain, are characterized by their function in the evolution process, fostering the move from artefact to mathematical signs with their intrinsic ambiguity. An a-priori analysis of the semiotic potential of the artefact with respect to the given task has a double role: it can help the teacher in her important role to foster the evolution of signs during the class discussion; it can also be the basis on which researchers can identify students' construction of meanings through the evolution of signs.

Method

Setting and procedure of the teaching experiment

The study took place in a high school in Turin (Italy), in which a senior teacher introduces the Galileo experiment with her eighteen 9th grade students. A short introduction to the activity and its design principles were previously given to the teacher. The teacher was followed by the researchers as she taught one lesson of 1.5 hour. In order to introduce the students to the chosen topic, a short video clip (Inclined plane experiment) concerning the Galileo experiment about a ball rolling on an inclined plane is provided, and students were asked, before the lesson, to watch the video at their home, to observe and make conjectures concerning what they have observed. The students were initially required to work in small (triads or four students) groups, sharing a worksheet containing the task and a computer for the simulation of the Galileo experiment. The teacher walked around and interacted with the students, when she believed it was needed. As soon as the students accomplished the task in the small groups, the teacher conducted a general discussion with the whole class.

Task given to the students and a-priori analysis of the semiotic potential

The design principles of the tasks are motivated by mathematical modelling approach and by the choice to exploit the potential of digital artefacts. The task presents a simulation of the real world phenomenon and, through the use of a dynamic digital artefact, offers mathematical representations of the dynamic aspects of the phenomenon to be analyzed. The main idea of the design is to create different situations by varying some aspects of the phenomenon while keeping the others invariant. Figure 3 shows the written modelling task given to the students. It blends the simulation of the rolling ball on the inclined plane with the numerical representation. At the beginning of the lesson the task was given to the students requesting them to explore the mathematical model through the dynamic digital artefact (Figure 3). In the default situation of the simulation, the inclination plane angle is invariant, while the spaces traversed by the ball, when the “Start” button is clicked, vary as the time elapsed. Then, according to the task, students are asked to conjecture and then verify how the plane inclination affects the ball movement and the mathematical model. This can be done, in particular, focusing on the differences of the distance and looking for the equation that describes the movement. These insights may lead the students to grasp the intended object of learning, namely, the quadratic function and its properties.

Task 1

Your task is to explore how the change of the plane’s inclination may affect the ball movement.

A. How the change of the inclination affect the ball movement?
B. Change the plane inclination to verify or refute the conjectures you raised in (A). Do your conjectures changes? If yes, why? If not prove your conjectures.
C. Can you find equations that describe the ball movement? Why or why not? Justify your answer.

Figure 3: The Galileo experiment task

An a-priory analysis of the semiotic potential of the artefact with respect to the given task (Tab. 1) is fundamental in order to identify the appearance and evolution of signs during the teaching-learning activity, as it describes what is expected to emerge in the classroom.

Table 1: The semiotic potential of the artefact with respect to the task

Mathematical meanings	Semiotic potential
- The table represents a correlation between two sets of values (time vs distance). - The third column represents the differences between the distances. - The differences between two consecutive differences in the third column are constant. - The model of spaces traversed by the ball is a quadratic function of the form $s(t) = at^2$.	- When clicking on the start button, the ball rolls down on the plane and the table is filled: this may evoke the idea that there is a correlation between the time and the spaces traversed by the ball and that there is also a constant correlation between two consecutive differences in the third column. - When changing the angle, the values in the second column (except for the last), the last value in the first column, and the values in the third column change as well, but the differences between two consecutive values always remain constant: this may evoke the idea that the inclination affects the ball’s movement and that the model of the spaces traversed by the ball is a quadratic function.

Data collection and method of analysis

To collect the data, the learning experiment was entirely video recorded. The students and their actions on the computers were captured. The collective discussion conducted by the teacher, after the students completed the task, was video recorded as well. Segments of the recorded clips of the students' transition between the real-world phenomena and the mathematical representation were identified and analyzed at a macro level according to the Blum's model. Thereafter, to have an in-depth insight into the learning processes, a micro level analysis was done using the semiotic mediation approach. At this level, the artefact signs and pivot signs used in the transition processes were identified and their evolution toward mathematical meanings were also considered and analyzed. In particular, the analysis was developed with the aim to identify semiotic chains in order to reveal the unfolding of the semiotic potential of the artefact throughout the modelling activity and the collective discussion.

Results and discussion

Herein we present results coming from the students' interactions in one of the small working groups and from the subsequent whole class discussion conducted by the teacher.

As students were asked to see the Galileo experiment clip at home, when the first lesson starts they already have their own personal conjectures on the phenomenon. According to the task, they are asked to explore if and how the change of the plane inclination may affect the ball movement, to share their hypotheses and verify them. According to the observations done at home, they agree on the hypothesis that the more the plane is inclined the faster the ball moves. In order to verify their conjecture, they use the provided digital artefact and take notes of the values of the spaces traversed as the time elapses. That was obtained when they started the simulation with the given (24°) inclination plane angle. They realize that the difference between two consecutive numbers in the third column is always 4. Then, they change the angle to 40° and realize that the numbers in the third column are the differences between the corresponding two consecutive numbers in the second column. They take note of the values and change the angle to 45°. At this point, they are watching the first value in the second column and, while the others are comparing it with the one obtained before, one of the students, A., observes that:

00.26.27	A.:	Yes, true, look, I told you that in one second it [the ball] always makes more distance.

But it seems that this observation is not useful for them to look for the relation among the variables - time and distance - so A. focuses on the last row and then a pressure of speeches and gestures follows:

00.26.51	A.:	Here [*pointing to the last number in the first column*], if you see, instead of being 5.8, it's 5.5… changes… it is faster.
00.27.07	B.:	Yes, true, because…
00.27.08	C.:	The more it is inclined [*she moves her hands in the collective space in front of the screen: the left hand, raised with the palm facing up, would represent the final point they are focusing on and the right hand, waved up and down, would represent the increasing of the plane inclination*], the faster it [the ball]

is... than this here [*pointing on the screen and on the paper on which they have copied the data before*].

00.27.12	B.:	The distances increase but this last datum [*pointing on the last difference on screen*] is not useful for us because it [the ball] arrives down before… let's try 50°.

Finally, when the angle is 50°, as the last number they find in the first column is smaller, they are convinced they have verified their hypothesis and then start writing their shared conclusion: "increasing the plane inclination and keeping the same time, distances increase and the ball moves faster if the plane is more inclined; the example shows that to reach the same final distance, 108, if the plane inclination is 40° it takes 5.8 seconds, if the plane inclination is 45° it takes 5.5 seconds". In the students' interaction described above it is clear how they are trying to interpret the artefact signs, obtained when accomplishing the task with the digital artefact, in order to give meanings to them in terms of the situation under study. In the following part of the lesson they attempt to find the way to mathematically express the relationship which describes the ball movement. They recognize that the graph of the distances, as times elapsed, cannot be modelled by a straight line. The work at this point is stopped by the teacher who decides to start the whole class discussion.

At a macro level, it is worth noting how in this initial phase of the activity students' discourses move from the real world to the mathematical world. As a matter of fact, the teacher then needs only four minutes to summarize the situation. And when she asks if there is something that changes when passing from the video clip to the digital artefact the students immediately, and with judgment, answer negatively. The students' answer reveals that the students consider the artefact as a mean for verifying the conjecture raised when they watched the clip. At this point, it is not clear what is the added value of the digital artefact in constructing meanings. However, the micro level analysis, as we will present below, highlights the specific contribution of the artefact in the process of the construction of meanings.

A micro level semiotic analysis, indeed, reveals how through the use of the digital artefact during the transition phase of the modelling activity, the signs emerge and evolve. The word "faster" referring to the ball when the inclination is increased, for instance, is a pivot sign which evokes, on the one hand the increasing of the numbers in the second column, on the other hand the decreasing of the last number in the first column, which represents the time needed to traverse the distance until the arrival, "to reach the same final distance".

Hereinafter, the teacher (T.) conducts the class discussion with the aim of focusing on identifying the variables to be used to describe the situation:

01.00.20	A.:	If when you incline the plan you increase the time… you take less time… <u>the ball takes less time</u> to traverse that distance…
01.00.34	T.:	the ball takes less time to traverse that distance...
01.00.36	A.:	the same distance, but as the plane is more inclined, <u>the ball is faster.</u>
01.00.42	T.:	ok, so, to the same distance… [*she use her hands making an iconic gesture to represent a given quantity for the distance*].
01.00.48	A.:	but <u>time is smaller</u>.

01.00.49	T.:	Time is smaller [*she moves on her right as to represent a shifting on something else, namely what changes in the time*] And the same time?
01.00.52	A.:	the distance… the distance is bigger.
01.00.58	T.:	the distance is bigger...
01.02.20	T.:	How can we read the table?
01.02.25	D.:	when the time is 1 the space is 2.13, when the time is 2 the space is 8.51 and so in a given time the ball traverses a given space.
01.02.58	T.:	and this space always increases more… as it happens in the video… and so how can I write a relation between this space and the time? Is it a line?
01.03.25	E.:	No. Because otherwise it would have had all the same differences.
01.03.26	T.:	and here?
01.03.27	E.:	And here the differences are not the same.

From the modelling point of view, it is evident here that A. is trying to figure out what he has observed varying the inclination of the plane using the digital artefact in terms of movement of the ball. He is in the transition phase from the mathematical world to the real world. However, A. and his classmates still need to understand how to mathematically express the movement of the ball. The request to interpret the table, and to express the relationship between the time and the distances, allows them to focus on the third column, namely on the differences between the distances.

A semiotic analysis of the episode reveals some more details of the learning processes. Worthy of note is, for instance the semiotic chain in Figure 4 (identified in the transcript above) which highlights the unfolding of the semiotic potential of the digital artefact, through the emergence and enchainment of signs, and will bring to the quadratic function.

The ball takes less time → The ball is faster → The time is smaller ↓

The differences are not the same ← The space always increase more ← The distance is bigger ←

Figure 4: An identified semiotic chain

While students face the real world situation accomplishing the task through the mediation of the digital artefact, indeed, the signs they use are evolving. The focus on the difference of the distances would allow them later on to conjecture that the spaces traversed are proportional to the squares of the times according to a factor which depends on the inclination of the plane.

Final remarks

Literature about modelling mainly focuses on the transition between the real world and the mathematical world. However, in our view, the way students move throughout the transition is not sufficiently addressed. This study attempts to deeply focus on the transition processes between the real and the mathematical worlds. Although these transitions are crucial in order to understand the overall modelling process, we believe that they are challenging and need a detailed analysis. Our results suggest that a macro level analysis, based on the Blum's model, is not sufficient to reveal the contribution of technology in modelling activities. However, a semiotic micro level analysis can bring to the fore the role of the digital artefact in the complex students' transition processes. The semiotic mediation approach, indeed, helped us to highlight the complexity of the evolution from the artefact signs to the mathematical signs. In this evolution process the students used a variety of signs, some of them refer to the real phenomena while the others refer to the mathematical representation. Through this process the students also produce pivot signs which have a polysemy of meanings, namely they may refer both to the real situation and at the same time to the mathematical representation. Revealing the evolution of signs has not only a theoretical importance but also a practical one: the semiotic chain which bridges the gap between the real situation and the mathematical representation, indeed, can help teachers in their mediating process of the students' learning. This happened, for instance, in the episode we analyzed when the teacher fostered A.'s thought and signs to emerge and to be shared. In this way she guided the class to move from considering the final "real" effect - the ball is faster - to focusing on the distances (traversed) and the "mathematical" information given by their differences.

Acknowledgment

This study was partly supported by the Israel Science Foundation (Grants No. 1089/18).

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