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Towards Pragmatic Theories that Underpin the Design of Teacher Professional Development Concerning Technology Use in School Mathematics

Eleonora Faggiano[1], Helena Rocha[2], Ana Isabel Sacristán[3] and Marisol Santacruz-Rodríguez[4] [1*]

[1]University of Bari Aldo Moro, Italy, eleonora.faggiano@uniba.it;
[2]CICS.NOVA, Universidade NOVA de Lisboa, Portugal, hcr@fct.unl.pt;
[3]Center for Research and Advanced Studies (Cinvestav), Mexico, asacrist@cinvestav.mx;
[4] Universidad del Valle, Colombia, mary21391@gmail.com

This chapter aims to make more explicit the grounded or 'pragmatic theories' that inform the design of mathematics teachers' professional development (PD) to exploit technological affordances. It uses aspects of some representative projects that took place in four countries (Colombia, Italy, Mexico, and Portugal) to illustrate lessons learned (e.g., similarities and differences, barriers and opportunities) and provide important insights to inform future PD implementations. To do this, we have identified a set of aspects (and sub-aspects) that emerged in relation to five major themes and reveal our 'pragmatic theories' alongside a consideration of the interconnections between these aspects. Our contribution offers a methodological frame to support future PD designs for teachers of mathematics concerning digital technology uses.

Keywords: professional development, mathematics education, technology use, theory.

INTRODUCTION

Although the complex theoretical ideas that help us to understand and explain how teachers learn to use digital mathematical tools, and the knowledge and practice that ground such use, are reasonably developed, far less is known of effective design of teacher professional learning activity in this respect. This chapter aims to make more explicit the grounded or 'pragmatic theories' that inform the design of professional learning for teachers that intends to support them to exploit technological affordances for the purpose of learning, teaching, and assessing mathematics. We interpret professional learning as the in- and out-of-school activities that teachers engage in, that further promote their professional knowledge, which we consider in the frame of the Mathematical Pedagogical Technology Knowledge or MPTK model (Thomas & Hong, 2013; Clark-Wilson & Hoyles, 2018).

There are no 'global' theories to support the design of professional development (PD) concerning technology use in mathematics education (Clark-Wilson & Hoyles, 2018). However, much 'craft knowledge' on the part of the mathematics teacher

[1]* Authors in alphabetical order (all authors contributed equally).

educator/research communities has emerged, which looks promising, but currently lacks connecting ideas that enable the knowledge to be shared more widely. By exploring the evolution of, and implications from mainly large-scale national professional development initiatives, we aim to arrive at a more theoretical grounding for the topic, supported by case examples that exemplify the most pertinent aspects.

The chapter aims to offer teacher educators and authorities an empirically-informed and research-based approach (our set of pragmatic theories) on how to prepare teachers of mathematics for present and future digitally-enhanced contexts, so they can cope with emergent technologies – mobile technologies, augmented and virtual reality, the use of multiple screens, virtual collaborations, etc. – and develop workable pedagogies for their uses.

SOME REPRESENTATIVE INTERNATIONAL PROJECTS

We begin by reflecting on significant country efforts in Colombia, Italy, Mexico, and Portugal, as a context to examine the respective designs and processes of professional development for teachers of mathematics. Some projects are described below (with comparative analyses provided in Appendix 1); we consider these projects as representative because they were important projects in each of the countries, either because they were very large-scale, or had significant designs or impact.

For Colombia, we selected three nationwide PD programmes with varying objectives: INTC[2] (1998-2004), CIER[3] (2013-2015) and *Computadores Para Educar* (CPE)[4] (2014-on going). INTC sought to integrate the use of digital technologies into mathematics classes in secondary schools. CIER focused on both diversifying the creative uses of digital technologies in the teaching in schools (including mathematics) and supporting the technological capabilities of each region of the country. CPE sought to provide technological tools, training and support to educational communities, in an environmentally-conscious manner by reusing discarded hardware from other sectors.

For Italy, we chose the *Progetto m@t.abel*[5] (2005-2012), launched and supported by the Ministry of Education and developed through a joint collaboration with university researchers. This project consolidated several communities of practice (CoPs) with a blend of teacher training activity and classroom experimentation to offer infrastructural structures (an online platform and support materials) and expert tutor support. Based on

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[2] INTC – *Proyecto "Incorporación de Nuevas Tecnologías al Currículo de Matemáticas de la Educación Básica y Media de Colombia"*: https://www.mineducacion.gov.co/1621/article-81040.html

[3] CIER – *Centros de Innovación Educativa Regional:* http://www.colombiaaprende.edu.co/es/content/centros-de-innovaci%C3%B3n-educativa

[4] https://www.computadoresparaeducar.gov.co/

[5] http://mediarepository.indire.it/iko/uploads/allegati/M7PWITOE.pdf

the experience of m@t.abel, a further (online based) project, DiFiMa[6] (2008-on going), has been implemented at the University of Turin with the aim of reaching mathematics and physics school teachers all over the country, through the use of an online platform. The DiFiMa project is intertwined with the local initiatives developed within the huge national project *Piano Lauree Scientifiche* (PLS)[7] thanks to which, since 2005, all the universities can organise different kinds of initiatives aiming to promote the attractiveness of the scientific university degree courses. As far mathematics teacher education is concerned, many of these initiatives are meant to foster the integration of technology in the teaching and learning processes. In its approach, DiFiMa emphasises the possibilities for technology use for mathematics in schools through periodic face-to-face meetings organised by the university researchers (for local teachers), and, using the digital platform, the organizing team trains voluntary teachers who provide local tutor support on the national scale. In particular, the PD programme offers teaching materials with guidelines and distance support and, more recently, some Massive Open Online Courses (MOOCs) (Aldon et al., 2019).

For Mexico, we chose two national efforts that took place in the early 21st century and were significant because of their scale: the Teaching Mathematics (and Science) with Technology programme for lower secondary school (EMAT, 1998-2007)[8]; and Enciclomedia (2002-2007) for primary school[9]. Both projects aimed to integrate technology into schools and therefore included accompanying teacher training. However, their conceptions and designs were radically different: EMAT was carefully designed by researchers and was planned to be gradually introduced (Sacristán & Rojano, 2009); Enciclomedia was a politically-led decision to create digitised textbooks with accompanying interactive applications and hardware. A third project considered for this chapter, is a local university Master's PD programme for around 100 in-service teachers that was carried out from 2005 to 2009 at the request of one state's government; we call it M.Ed. (Sacristán, Sandoval & Gil, 2008). An overview of the above Mexican projects is given in Trouche et al. (2012). We also mention very briefly, but do not analyse, a more recent project for general technology integration into Mexican schools: the Digital Abilities for All (HDT) programme (2011-2017)[10].

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[6] DiFiMa – *Didattica della Fisica e della Matematica*: https://difima.i-learn.unito.it

[7] https://www.pianolaureescientifiche.it

[8] https://web.archive.org/web/20140605094110/http://www.efit-emat.dgme.sep.gob.mx/

[9] https://web.archive.org/web/20100725200920/http://www.enciclomedia.edu.mx/
https://www.sep.gob.mx/work/models/sep1/Resource/2959/4/images/LB Enciclomedia.pdf

[10] https://sep.gob.mx/work/models/sep1/Resource/2959/5/images/LB%20HDT.pdfhttps://sep.gob.mx/work/models/sep1/Resource/2959/5/images/LB%20HDT.pdf

For Portugal, we selected three PD projects that had a notable impact on Portuguese schools: MINERVA[11] (1985-1994), *Programa de Acompanhamento* or Follow Up of Syllabus implementation (FUS) (1998-2001) and *Plano da Matemática* (PM) (2006-2012). The MINERVA project was the first big project on technology, aiming to integrate technology in the teaching and learning of all subjects at all levels (Ponte, 1994). FUS, which followed the implementation of a new high school mathematics syllabus, intended to promote the teachers' discussions about their practices, addressing namely the role of the students and the use of new materials and resources, where technology was an important element (Teixeira, 2004). PM was a national programme with voluntary teacher enrolment, aiming to improve students' outcomes based on a change of teachers practice, where technology was usually an important element. The specific characteristic of PM was the recognition of the role of the teachers, leaving to them the creation of small CoPs and the decision about the specific focus of each of those communities (GTM, 2019).

THEMES AND ASPECTS ELICITED BY THE PROJECTS

Our method to elicit the themes and aspects from the projects, was twofold. For analysing data from those projects, we were inspired from the work of Tondeur et al. (2012), whose systematic review identified a number of key themes for content and delivery methods to prepare teachers, in general, to integrate technology in their practice. For the analysis, we also took into account some aspects related to teacher's knowledge. For instance, to analyse how teacher training is articulated to mathematics syllabi, we used categories from the Mathematical Knowledge for Teaching or MKT model (Ball, Thames & Phelps, 2008); and, to analyse if the focus of the PD programmes promotes teacher's knowledge for technological integration on math classes, we took into account the various components of the MPTK model (Thomas & Hong, 2013; Clark-Wilson & Hoyles, 2018). More specifically, for analysing the projects in our countries, we started from the classification proposed by Tondeur and colleagues, but keeping in mind some aspects of MKT and MPTK. This was done for each project separately, then compared with the classifications for the other projects, and through a back-and-forth iterative process, we identified our own general set of themes and aspects (those we considered as most important and characteristic) related to the professional development of teachers for technology use in their mathematics teaching practice; this set is organised in a table given in Appendix 1.

In the following descriptions, we use elements of the projects in the four countries, to illustrate lessons learned related to each theme and aspect (e.g., similarities and differences, barriers or opportunities, what works, what does not, what impacts, what is needed/missing) that give pointers –i.e., pragmatic suggestions/theories– for future PD implementations that would aim to develop MPTK. The aspects are related to the following five themes:

[11] MINERVA - Meios INformáticos no Ensino: Racionalização, Valorização, Atualização

1. **Vision/Goals and Policies**: This theme refers to vision, goals and policies that drive a professional development programme, as well as to its scale (e.g., national, regional).
2. **Focus**: This refers to the focus of the professional development programme in terms of whether it is specific for math, general for all school contents, or technology in general, and of who defines that focus.
3. **Strategies and methods**: This refers to strategies and methods of, and for, professional development programmes, and/or policies that affect these.
4. **Constraints, or allowances and incentives**: This refers to aspects that restrict or facilitate participation in a PD programme and/or technology integration.
5. **Evaluating the impact:** This refers to assessing the impact of the PD programme (e.g., on teachers' practice), and how that informs future adaptations (e.g., proposals for improvement) or other PD programmes.

We now describe and discuss the aspects (and sub-aspects) that we identified for each of those themes, adding examples from the national experiences that helped us recognize how those aspects can be implemented for optimal results in future PD programmes. The aspects could be taken into account to figure out our 'pragmatic theories', both for the design, and for evaluating PD programmes, that focus on integrating technology in mathematics education.

1. Vision/Goals and Policies

Some projects, have a vision at a teacher-level (e.g., INTC in Colombia, DiFiMa in Italy, Mexico's M.Ed. and PM in Portugal), that is, their aim is to train teachers to either improve their mathematics teaching in general, or to help them to integrate technology into their practice. Many initiatives are derived from national policies that aim to integrate technologies in general in schools (e.g., MINERVA in Portugal and Enciclomedia in Mexico). Others, either derived from national or local policies, focused more specifically on technology integration for mathematics or science education (e.g., EMAT, in Mexico).

However, top-level initiatives that do not take into account the specific necessities of schools, nor of the different subjects (e.g., mathematics), –as was the case of HDT in Mexico, in the last decade– have shown not to be effective (Trouche et al., 2012; Macías-Herrera, 2014).

However, it seems that when top-level initiatives have been coordinated with subject matter experts (i.e., in mathematics education), and developed programmes that have had more specific goals (such as promoting changes in school cultures and teachers' practices), they have been more effective. Such were the cases of EMAT in Mexico, FUS in Portugal, INTC in Colombia and m@t.abel in Italy. In addition, these projects were 'research-based' (i.e. taking into account research results for their design and implementation), expert-supported (some with connections to academic institutions), and

had associated research projects. Such projects are thus more likely to attend to the needs and specificities of the specific subject area (mathematics) and of schools and teachers. In the EMAT case, in particular, although it was a national programme, it was designed with a scaled implementation from a few schools and regions to expand to a national level over the course of several years, thus allowing for necessary research-informed iterations to the implementation. This scaled approach was also adopted for the PM in Portugal, which started as PD programme for volunteer teachers and ended on a national scale. The INTC project in Colombia was also national, had the support of several regional universities and was associated with several research projects developed in the participating universities. This project focused on promoting change in the practices of secondary school mathematics teachers through strengthening their professional knowledge (e.g. on the design of teaching activities) and thus improving the academic results of their students. Italy's m@t.abel project was also on a national scale and it was an example of a programme that adopted methodologies and content that adhered to current international educational research. Thus, all of the above projects were research-based and intended to bring changes in the teaching practices at a national scale in order to impact on the students learning.

However, our comparative analyses suggest that when project goals are only decided at the top level they can be susceptible to political changes, which we will discuss in more depth in the section on constraints.

2. Focus

When analysing the projects in the various countries, we have to take into account their origin and focus, as not all originate as PD programmes for technology integration into mathematics teaching. Some programmes have a focus on developing the mathematical knowledge of teacher or their pedagogical content knowledge for mathematics teaching (e.g., FUS in Portugal and CIER in Colombia) –that is, teacher training with focus on mathematics; thus, those programmes do not necessarily focus on technological integration although it may be included tangentially. In others cases the mathematical focus is included more profoundly (e.g., m@t.abel and DiFiMa in Italy). Projects that do centre on the use of digital technologies, may do so with focus on the technical aspects of certain technology (e.g., training in the use of Internet, or of a certain app) but with little consideration for the content knowledge and the pedagogical content knowledge. We conclude that this can leave teachers with a lack of know-how on how to implement such technologies for teaching mathematics (and when they do use them, it tends to be only an add-on to the usual teaching, that does not allow them to learn to harness the potential of these technologies), as was the case of Enciclomedia in Mexico (Macías-Herrera, 2014). In Portugal, the MINERVA project was implemented near the advent of digital technologies (mid 1980s). Hence, there was an emphasis on technical issues related to the use of technology, although the development of classroom experiences and the development of tasks that took advantage of the potential of technology were already an important part of the project. In contrast, in the case of FUS, the focus was on the

implementation of a new syllabus, promoting the use of technology, of a different kind of tasks, and the development of a classroom work centred on the students. Then in PM, the idea was to allow teachers to select the material conditions they needed, instead of equipping schools with technologies they might never use, so the teachers were free to choose the focus of the PD most relevant for them.

Our analysis suggests that a 'worst-case scenario' is when a technology integration project does not include an adequate accompanying PD programme and it only focuses on providing schools or students with a particular hardware. This was the case to some extent, in Mexico, of HDT and Enciclomedia (e.g., the provided smartboards were used more as projection screens) (Macías-Herrera, 2014). Such cases have shown that this hardware is wasted and have led to the recognition that only providing resources is insufficient. In fact, this recognition led to the development of the PM project in Portugal.

Some PD programmes focus more specifically on the integration of technology with mathematics teaching and learning (e.g. EMAT in Mexico), and/or specifically on how to articulate the teacher training with the (math-syllabus) curriculum (e.g. FUS in Portugal). This was also the case of the INTC Colombian project for integrating new technologies to the mathematics curriculum of secondary school (Castiblanco, et al., 1999), which included training teachers to harness the potential of digital technologies for their teaching practice (as did the M. Ed. programme in Mexico). In these programmes, although it was important for teachers to learn about the technical aspects of certain tools (hardware/software), there was also emphasis on the following didactical considerations.

Teachers reflected on:

- mathematical ideas in the curriculum, and how to teach them using digital technology (as a tool for math);
- how to design contexts and/or tasks using digital technologies, for their students in schools.

This privileging of the 'teacher's voice' was evident in the cases of PM in Portugal, where the focus was identified based on the teacher's needs and interests; and in Colombia's INTC, where the decision of the specific focus was a negotiation between the teachers enrolled in the programme and the teachers' educators.

The Italian m@t.abel project focused on new methodologies for teaching and learning mathematics as a response to the weaknesses in students' mathematics skills identified by the international OECD-PISA survey. By focusing on new methodologies, the use of digital technologies was an important part, both in terms of teaching in the classroom and of teacher training. The focus of DiFiMa is also on methodologies and, in particular, this project takes into account the necessity for teachers to be supported in exploiting technological affordances, creating collaborative contexts for teachers' work, where they can share their practice and work collaboratively and with the guide of expert teachers and researchers.

3. Strategies and methods

Types of PD programmes and the delivery method

Many of the PD projects (e.g., all the Mexican and Portuguese ones) were implemented only in a face-to-face modality. In recent years, however, due to the availability of more accessible and effective online platforms, distance programmes have also been offered, especially in a mixed modality (combining face-to-face and online delivery methods). This was the case of Italy's m@t.abel and Colombia's CIER projects; in the latter case the format was 3 weeks of virtual activities, one week of face-to-face ones, then 2 further weeks of virtual work. Also, the availability of MOOCs, in which the interactions among trainees and between trainees and trainers occur online and in totally virtual environments, offered the opportunity to enlarge the audience of the courses to a wide, national level. This was the case of the DiFiMa programme at the University of Turin in Italy.

In Mexico, some smaller projects that have attempted to combine face-to-face with online participation, have not been successful in the distance component: teachers in that country are still not so familiar with participating in distance courses or CoP, and/or lack the time or resources for working online (Miranda & Parada, 2011). In Portugal some online experiences are also beginning to be implemented, but they are all small programmes, usually enrolling no more than 20 teachers.

Types of participants and the adoption of the programme

Many of the programmes have focused on in-service teachers. Such is the case for all of the presented Mexican and Portuguese projects and of the m@t.abel project in Italy. Colombia's INTC included both in-service and pre-service teachers (the latter from the regional universities that led the programmes in each region). In INTC and the Italian DiFiMa, participants could be any interested teachers (so participation was voluntary based on the teachers' willingness to be involved in the training process); the focus of both projects was developing classroom teaching experiments, but pre-service teachers could be involved and contribute both in the design phase and in the *a posteriori* analysis of the experiences. Also, in Portugal's PM, the enrolment of the teachers was voluntary, with a significant increase on the number of teachers and students involved across the years (GTM, 2019).

Nevertheless, there are projects at a national level where participation is compulsory (e.g., Mexico's Enciclomedia, Portugal's FUS, and Colombia's CPE). In Enciclomedia, teachers in classrooms that were supplied with the Enciclomedia system, had to be trained in its use, which was compulsory. Colombian PD programmes have generally been voluntary participation; the CPE project, however, was mandatory for many schools in the country, in order to provide digital technologies to schools (e.g. computers, internet). CPE trained teachers in the basic use of technological tools (e.g. opening and using an email account) and supported the designing, by teachers, of classroom tasks. As a result

of this mandatory participation, many schools, teachers and communities in Colombia (many in remote areas) had their first access to digital technologies.

The integration of theory and practice

Some PD programmes are research-based, that is they take into account results of research and theories from mathematics education (and other disciplines) to: inform their implementation, guide the teacher training, and underpin the design of the tasks implemented by the teachers. The Mexican EMAT and the Colombian INTC projects are such examples. In both projects, some materials (e.g., textbooks for students with task ideas) were produced that were derived from research results. In Portugal's FUS, booklets for teachers were written by experts, focusing on mathematical, or more general issues, and discussing theories and the potentials of digital technologies, projects and problem-solving. These booklets included some results from the research on the field.

In INTC, some research results were used in the design of the PD programme. For example, theories related to problem-solving and to the role of the representations in the students' learning (Castiblanco et al., 2004). Years later, the CIER project also included research results and more general theories (e.g., teachers' technological, pedagogical, and content knowledge based on TPACK model from Mishra & Koehler (2006) were widely discussed with the group of teacher-experts in order to identify what elements of those theories might be taken into account to guide the design of digital resources for teaching. Both Colombian projects also included discussions and reflections among the participating teachers to analyse theories, the design of tasks and teaching proposals integrating digital technologies and their implementation. This was also the case in Mexico's smaller M.Ed. programme, and also in the Italian DiFiMa where participating teachers are introduced to some mathematics education theoretical aspects in order to be aware of the framework on which the teaching activities proposed to them have been based. This allows teachers to understand deeply the reasons of the design choices and to implement the classroom activities in an effective way. In addition, in the Colombian projects, some of the teachers' implementations in their classes were observed by expert-teachers (and other participant teachers in the PD programme) and were used for promoting teachers' reflections on their practice and on topics such as the math curriculum. In all the above projects from the three countries, the theoretical integration is also what helps the teachers to analyse with the teacher educators the results of the teaching activities and to reflect on their own professionalism. For these reasons, these projects are interesting examples of theory/practice integration within a PD programme.

Structure of the PD program and how teachers are organised

Some of the PD programmes are based on the knowledge from experts, assuming that teachers are the recipients of that knowledge. Other programmes consider that teachers can be engaged as active agents in the design and development of the PD programme. In some cases (e.g., PM in Portugal), the teachers identify the area where they feel they need to improve in their professional knowledge and are organized in a CoP for each school. In

this model, the teachers from each region get together each month and can seek the support they need from an expert-teacher. Expert-teachers are those that receive training with experts on the field of mathematics education and then become experts that can share their knowledge with their peers. In some cases (e.g., Mexico's EMAT, Italy's m@t.abel, Colombia's CIER), these expert-teachers were required to train other teachers in a 'cascade' model. The teacher training in CIER and EMAT included an interdisciplinary group of experts (e.g., researchers, designers, communicators) who trained a team of expert-teachers who would then train other participating teachers. In the case of m@t.abel, the expert-teachers had the role of tutors in the online platform. In the Portuguese FUS, teachers were selected as expert-teachers to work with the other teachers, guiding the work on the monthly meetings. In some cases (e.g., Portugal's FUS, m@t.abel, EMAT), the expert-teachers were mediators between the national syllabus authors and the schools, taking into account the needs of the teachers.

In the case of Mexico, for Enciclomedia, expert teachers were trained at regional centres during a 3-day course (with no follow-up training, nor support) to lead the training of other teachers to learn the contents of the Enciclomedia system. This was a purely technical training with no didactical training. In the case of EMAT, even though the designers requested follow-up and more continuous teacher support, it was denied by authorities, so training was restricted to up-to-a-week workshops, and uptake by teachers was low (only in places where there were self-appointed coordinators and more continuous support, was the uptake by teachers more successful).

Many of the Colombian PD programmes were implemented as a result of extensive cooperation between institutions. For example, several government entities (e.g., Ministries of Education, of the Environment, and of Telecommunications) and private companies participated in the CPE project. Other programmes such as INTC and CIER, as well as Mexico's EMAT, collaborated with regional universities and also had the support and advice of international experts. In the case of EMAT, the most successful implementations, after the PD programmes, were in places where the regional universities offered long-term support (Sacristán & Rojano, 2009). The INTC programme also added support to its participating teachers through national conferences where they could have contact with the PD programme designers, national and international researchers and with other participating teachers. As a product of these conferences, the MEN (National Ministry of Education) published some books that were distributed nationwide and are still available on their website.

Strategies for teachers' work in the PD

It has been mentioned how some of the PD programmes involved teachers in discussions on related theories and/or classroom implementations. Our analysis suggests that many of the PD programmes (especially those with in-service teachers) involved teachers in the discussion of ideas and classroom experiences. In some cases, this resulted in the development of CoPs; in some other cases, it resulted in teachers working collaboratively

but with a level of collaboration weaker than the one of a CoP. Portugal's PM was guided by what the teachers wanted to do with their students and included weekly meetings of each school's CoP, to include peer work inside and outside of the school, through which the teachers shared their experiences.

In Colombia's CIER, the strategy was to offer digital resources and materials to teachers so that they could work with them in their mathematics classes and subsequently reflect and share their experiences in CoPs implemented during the PD programme. In Italy's DiFiMa and m@t.abel projects, classrooms experiences were also shared and discussed in CoPs (in m@t.abel, in particular, the CoPs combined face-to-face and online training and discussions).

Resources and materials used

For many of the programmes, materials have been created either for teachers to use with their students, or specifically for teacher training. In all of the Mexican programmes, most of the materials and resources used were designed for use in the classroom by the teachers. The main software used in EMAT, were 'universal, open tools' such as Spreadsheets, Dynamic Geometry, Logo and Graphic Calculators that were accompanied with sets of activities (worksheets) to be carried out with students. Teachers were provided with handbooks and books, written by a team of academic experts, with tasks and worksheets (classified according to the compulsory curriculum) for each tool.

In the case of Enciclomedia, the materials and resources were special digitised versions of the compulsory primary-school textbooks with accompanying digital interactive applications (in CD and in a platform). The classrooms were equipped with a computer and a smartboard. Teacher guidebooks described the structure of the interactive system and materials; and teacher training consisted in familiarizing the teachers with the tools and activities, in a purely technical way.

In Portugal, some of the programmes related to the development of support material for teachers and some did not. The FUS project had the strongest focus on such support materials, which took the form of booklets for teachers, which have been described previously. The MINERVA project also produced some materials for teachers, but additionally developed software specific for the teaching of Mathematics.

In the Colombian cases, most of the PD programmes provided equipment to the participating schools: mathematics computer laboratory rooms (in INTC case), hardware, software, Internet connection and technical support (e.g., in CPE case[12]); and website-based digital resources for teachers (e.g., in CIER[13]; and in CPE). In most cases the

[12] http://www.computadoresparaeducar.gov.co/es/contenidos-para-educar-2

[13] http://aprende.colombiaaprende.edu.co/sites/default/files/naspublic/ContenidosAprender/index.html

technological equipment was accompanied by textbooks and other complementary resources for teachers (e.g., worksheets, digital books, applications and videos).

Some PD programs have used videos as part of their teacher training strategy (see also, van den Bogaart, Drijvers & Tolboom, 2019). For example, in the CIER project videos were made and used for two purposes: (1) As part of the resources offered to teachers (through a web page) so that they could use them in their own classes; and, (2) as resources used by teacher educators to promote teacher reflection and knowledge. In the latter case some aspects of a lesson study were included. However, this was neither systematic, nor recurring. Also, in the Italian DiFiMa there have been some, not recurring and systematic, cases of use of videos, conceived as an effective strategy of the teacher training since it allows teachers to reflect on their role through a deep analysis of the outcomes of the teaching activities.

Even though the m@t.abel project ended in 2012, all the materials, developed and used by the researchers and the teachers during the programme, are still available online on the platform managed by the National Agency for School Development and Autonomy (ANSAS - INDIRE) on behalf of the Italian Ministry of Education. In contrast, Mexico's EMAT and Enciclomedia materials are no longer online, although many teachers are still using them (over 10 years after the programmes ended), and for the new 2018 curriculum most of the digital technology recommended activities were taken from EMAT.

4. Constraints, allowances and incentives

Given the national priorities that underpin many of the projects, the subsequent teacher engagement and outcomes are dependent on the related constraints, allowances and incentives that might support their active involvement. One important factor is the top-level government/political decision regarding the resources allocated to the design and implementation of such programmes. In particular, when projects are decided at the top level they can be susceptible to political changes and be abruptly cancelled before they attain their goals, as happened for EMAT, which was terminated before full expansion, despite its success. Similarly, Enciclomedia was suddenly cancelled after only a few years due to political changes, in spite of its massive expenditure. In Portugal MINERVA, FUS or PM were all politically motivated and required considerable financial support, which made them vulnerable to political decisions – FUS was suspended due to financial constraints, although in this case most of the goals of the project had been achieved.

Other factors that affect PD programmes include teachers' opportunities to access technologies, as well as teachers' time constraints, which are well known to impact the integration by teachers of what they have learned in a PD programme (e.g., in terms of the use of technologies) into classroom practice. However, the duration of the PD programme is also relevant. Longer and sustained PD programmes give teachers the opportunity to reflect on the integration of technology to their practice, for instance through using technology in their classrooms during the PD programme, with possible support from the participating experts and peers in that programme. This is, for example,

the case of DiFiMa: the project started in 2008 and has been able to continuously offer a range of evolving opportunities such as online commented materials, MOOCs, blended courses on specific contents or methodologies for teachers at the lower or high secondary school level, online suggestions and support, and periodical conferences in which teachers can also present and discuss their own experiences.

Most PD programmes assume that teachers have access to technology. Hence, most of the projects that we have described, have equipped schools with the relevant technology (Colombia's INTC, CPE, CIER; Mexico's EMAT and Enciclomedia, Portugal's MINERVA and PM projects). In some cases, (e.g., INTC, EMAT and MINERVA), mathematics laboratory rooms (equipped with hardware and software) were created in the participating schools with accompanying training for teachers. In the case of PM, the schools received the equipment asked by the teachers after they had conceptualized their specific project.

The type of teacher enrolment is also an important factor. A voluntary adoption of the programme will result in a group of participants with a deeper recognition of the relevance of the programme. In Portugal's FUS all upper secondary teachers were enrolled, as did all upper primary-school teachers in Mexico's Enciclomedia (where some teachers asked to be transferred, because did not want to participate). In contrast, in Mexico's EMAT, the enrolled schools expanded progressively and teachers' participation was voluntary. This process of progressive inclusion happened also in the case of Portugal's PM where the schools decided if they wanted to participate and how.

Another participation modality that can have impact on the results of a PD programme, is whether it is face-to-face or at a distance. Once again there are different allowances and constraints related to these two options. Face-to-face can result in a deeper involvement of the teachers and in a stronger feeling of belonging to a group. However, this modality has constraints related to the need to be together in the same place, that the distance option does not.

The exclusivity of the time that teachers dedicate to the PD program is also an important factor. In the case of the Colombian CIER project, the participating teachers had a time of exclusive dedication to the program (while the students were on holidays). This enabled the teachers to focus on the program and commit to the training activities. In addition, the program certified their participation and included them in their social networks (e.g., Facebook). At an institutional level, the schools recognised teachers' participation as part of the annual evaluation of teacher performance.

5. Evaluating the impact (of the PD programme)

A PD programme can be assessed in a systematic and/or systemic way, in order to have information about both its impact on teachers' practice and the effectiveness of its components (e.g., focus, methodology, tools). Evaluating a programme's outcomes is particularly important to learn from previous experiences and to improve the design and

implementation of future programmes. However, this aspect is often undervalued and disregarded. Many of the PD programmes we considered were not formally evaluated and, even when this was carried out, the results were not considered as a starting point to design and develop further programmes. For this reason, the impacts of PD programmes are often not shared, neither discussed on national nor international levels.

That is the case of Portugal, where the final reports of the projects are often assumed as classified information and not publicly available. Nevertheless, some characteristics of the PD programme are broadly discussed, allowing for the identification of central points to promote or avoid in future initiatives. This is the origin of ideas such as in FUS, where some teachers were invited to assume an important role in the PD programme as expert teachers. Then in PM, a stronger recognition of the role of the teachers evolved, allowing them to organize and decide the focus of the project in their school.

In the case of the Italian m@t.abel, the impact of the programme has been assessed, even if, similar to Portugal, the results are not publicly available. Notwithstanding, we can assume that the basic assumptions, on which the programme had been designed and developed, have been deemed as successful. Indeed, they constitute the basis for further programmes such as DiFiMa. Moreover, the Italian PD programmes have a very strong connection with research activities and their impacts are also evidenced through the scientific publications that draw data from the teachers' experiences in their classes (e.g., see Taranto, Arzarello & Robutti, 2017; Loisy et al., 2019).

However, some PD programmes have had an associated evaluation project to determine their impact. Such was the case of Mexico's EMAT programme, where schools, teachers and students were surveyed and observed in order to determine the impact of the project (see Trigueros & Sacristán, 2008). The evaluation noted discrepancies between the use that schools and teachers claimed, and what students perceived, which suggested a poor implementation of the programme, perhaps due to insufficient teacher training and support). In addition to an overall evaluation, for EMAT, as well as Enciclomedia and the M.Ed. programme, there were associated academic research projects that analysed parts of the projects (e.g., see respectively, Trigueros & Sacristán, 2008; Trigueros & Lozano, 2012; Sacristán, Sandoval & Gil, 2008; also Trouche et al., 2012).

In the case of the Colombian CPE project, an external evaluation was carried out by another state entity (the National Consulting Centre) to assess its impact, results and sustainability. This adopted a mixed methods approach (using quantitative and qualitative methods) and took into account the data of a sample of selected schools that participated in the project. A summary of this assessment is available online[14]. In this assessment it was concluded that the CPE programme managed to positively impact its participating schools (e.g., helping to reduce student dropout and repetition). Another conclusion was

[14] http://www.computadoresparaeducar.gov.co/sites/default/files/inline-files/Articulo_impacto_Computadores_para_Educar_Colombia.pdf

that the teachers recognise the relevance of the teacher training and they want to continue being involved in further actions of technology integration in the classroom. This resulted in the continuity of the project. However, the assessment results suggest that it is necessary to continue investigating which pedagogical methods are most effective to achieve meaningful experiences in the classroom.

CONCLUDING REMARKS: TOWARDS PRAGMATIC THEORIES FOR PD CONCERNING TECHNOLOGY USE IN SCHOOL MATHEMATICS

The analysis of the various national experiences involves a 'learning over time' that enables us to get insights into what should be taken into account and involved in a professional development programme for mathematics teachers, so that they can exploit technological affordances, be supported and share their practice and work. In this way, we can define better the pragmatic theories underlying teacher professional development concerning technology use in mathematics. In our process of analysis, we identified some common aspects, of various relevance, that leads us to conclude the following set of ideas that both highlights the interconnections between those different aspects and sheds light on the central role of mathematics.

Set of ideas for pragmatic theories for PD design concerning technology use in mathematics

Focusing on mathematics with technology rather than on technology (for math education)

The experiences revealed that, although most of the teachers expressed the need to be informed and trained about the functionalities of the new technologies, it is by focusing on both the mathematical content knowledge and on the pedagogical content knowledge for mathematics teaching that they are able to exploit the opportunity offered by the PD programme and become aware of the limits and potentials of the integration of technologies in their teaching practices (Faggiano, 2009; Clark-Wilson et al., 2014). In addition, our experiences revealed that the teachers need to experience mathematical technologies for themselves 'as learners'. This experience could develop their own professional genesis and best practices with digital technology (Haspekian, 2012) and help them to support their students' instrumental genesis (Trouche, 2004).

Providing schools with digital technology is only part of the solution

Some political decisions in several countries provided materials or hardware (e.g., big initiatives at the end of the 20th century) and these resulted in not successful interventions, as the resources were left mostly unused (e.g., see Trouche et al., 2012). Our experience reveals that the PD programmes, which also offered teachers training and *in-situ* support (technical and pedagogical), are considered more successful than those simply providing schools with digital technology.

Taking into account teachers' needs

The recognition of the knowledge, the relevance and the needs of the teachers is also an important issue in PD. The enrolment of the teachers is deeper and stronger when they see themselves as a relevant part of the project, having the opportunity to discuss what they think is important and being able to define the project based on their needs, their interests and their classroom reality.

Although the underlying learning theories for particular pedagogical approaches may or may not be made explicit for participating teachers, our conclusion is that the success of a PD program is how it responds to the needs of the specific teachers it is directed to.

Enabling connections between different actors: the teachers, the expert-teachers, the researchers and the institutional agency

At the end of the 20th century, partnerships between academia and schools arose, but depended on a good coordination among academic educators, authorities and schools. We have recognized the contributions of experts in mathematics education, and their collaboration with school teachers (in some cases also with expert school teachers) working as authors of the educational activities proposed. This allowed the most interesting and innovative suggestions coming from the international mathematics education research to be collected and dropped into the reality of the school system. In this process, the role of other institutional agency, such as those responsible of the school development and of the national assessment, or the professional councils of mathematicians or teachers, resulted also to be important.

Linking teacher training with classroom practice

Professional development opportunities need to include opportunities for the teachers to plan activities for their students during the PD programme. Indeed, through the design, development and analysis of teaching practices with technology, made during the PD programme in collaboration with expert-teachers and researchers, teachers can develop meta-levels of reflections on both the educational processes and the results of these processes. This would also enable explicit connections to be made with the school curriculum, within CoPs and with experts' support.

Ensuring the duration of the programme and support given to the teacher

It takes time for teachers to effect the transformation of mathematics teaching practices with technology both in terms of the timing and frequency of support (e.g., continuous and sustained support). This support includes (but is not restricted to): technical and logistical aspects, didactical aspects (on the use of digital technologies in the classroom) and opportunities for teachers to participate in teacher communities (e.g., through social networks).

Developing and maintaining fruitful CoPs

Shaping CoPs is a useful strategy for teacher training because it allows greater communication between teachers and sharing their experiences. However, the CoPs are not easy to be implemented and their continuity after the PD program is difficult.

Maintaining political support to the PD programmes

Changes in governments often imply huge changes in educational policies and, in particular, in teacher training programmes (e.g., see Trouche et al., 2012). However, in some cases, it can happen that educational policies give continuity to the PD programme, allowing PDs to be strengthened.

Exploiting the potentials of the newest technological innovations

New technologies are being developed every day and PD programmes should be flexible to incorporate opportunities to reflect on the new affordances available for learning, teaching and teacher training. This can mean, for example, exploring the potentials of touch and multiple screens, augmented reality, eye-tracking. Moreover, there are some other kind of technological innovations that, in our view, deserve more attention such as the MOOCs, or the videos. The latter, for example, can be used not only for classroom experiences to reflect upon, but also for teacher training (e.g., Van den Bogaart, Drijvers & Tolboom, 2019).

Final remarks and call for further investigation

We believe that the process of analysis that we carried out, met our goal of beginning to define a useful framework for the design of future PD programmes, as well as for researchers to analyse existing programmes. But it is also a framework that needs to be developed further and evolve.

On the one hand, as technology and global communications advance, the strategies of the PD programs may need to adapt (e.g., through online programs or MOOCs). A training strategy that was successful a decade ago may no longer be as useful. So the framework needs to adapt.

On the other hand, and more importantly, the development of the framework needs to take into account other experiences from around the world, because professional development practices tend to align to the context and needs of a particular country or region: We believe that similar investigation from countries with different cultural traditions, such as Anglo-Saxon countries, northern Europe countries or Asians countries, needs to be developed. This would allow our set of pragmatic theories to be deepened and further improved.

ACKNOWLEDGEMENT

We are very grateful to Alison Clark-Wilson of the UCL Knowledge Lab, UCL Institute of Education, UK, for her ideas and comments for this chapter.

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APPENDIX 1: ASPECTS AND SUB-ASPECTS OF THE ANALYSED PD PROGRAMMES

		Countries >	Colombia			Italy		Mexico			Portugal		
		Projects >	INTC	CIER	CPE	m@t.abel	DiFiMa	EMAT	Encm	M.Ed.	MINERVA	FUS	PM
Themes	Aspects	Sub-aspects											
Vision / Goals and Policies	What are the political and programme/ initiative goals?	Integrate technology: in general (G) or specifically for mathematics education (ME)	ME	ME	G	ME	ME	ME	G	ME	G	ME	ME
		Promoting change in teachers' practice	X	X	X	X	X	X	X	X	X	X	X
		Deepening teachers' professional knowledge (e.g. MPTK)	X	X	X	X	X	X		X	X	X	X
		Improve student academic results	X						X				X
	Relationship with research	Research findings taken into account for design of PD	X	X		X	X	X		X		X	X
		Integration of PD programme with research project	X	X			X	X		X		X	
	Scale	National (N); Regional (R); Local (university, school, etc.) – (L)	N, L	N, L	N	N, L	N, L	N	N	L	N; L	N	N, L
Focus	Focus on mathematics	Teacher training with focus on math (not necessarily technology).				X	X					X	X
		Teacher training articulated to the mathematics syllabus (focus on MCK and/or PCK for math teaching)		PCK		X			MCK PCK			X	X
		Training in technical aspects of certain hardware/software. (e.g. focus on technical knowledge; no mathematics).		X	X								
	Focus on technology: Teacher training with focus on technology	Training for technological integration (tool for mathematics); i.e. training to use digital technology in mathematics teaching. Focus on: - how some digital tools could be useful for mathematics (T) - developing MPTK	T	MPTK		MPTK	MPTK	T (± MPTK)	T	T, MPTK	T, MPTK	T, MPTK	T, MPTK
	Decision of focus	Focus decided by the teachers enrolled (T) or by outsiders (O)	T, O	O	O	O	T, O	O	O	O	O	O	T

Strategies and methods	Type of PD programme	Delivery/implementation modality: Face-to-face (FF); Online/distance (D), Mixed (face-to-face and online) (M)- If MOOC add D-Mooc or M-Mooc	FF	M	FF	M	M	FF	FF	FF	FF	FF	FF
		Type of participant teachers: In-service (IS); Pre-service (PS); Mixed (M)	M	IS	IS	IS	M	IS	IS	IS	IS	IS	IS
		Adoption of the programme: Progressive (P); Volunteer (V); Compulsory (C)	P, V	P, V	P, C	P, V	P, C	P, C			V	C	V
	Theory/practice integration	Use of research results and theory for design of PD programme	X	X		X	X	X		X		X	X
		Teachers reflect on the theory/practice integration	X	X		X	X			X		X	X
		The PD programme require classroom implementation	X	X	X	X		X	X	X		X	X
		PD trainer observes classroom implementations	X	X			X	X					
	Teacher training strategies	Teachers sharing and reflecting on each others' classroom practices and experiences	X	X		X	X			X		X	X
		Teachers training others teachers		X		X						X	X
		Teachers working collaboratively (CW) or Communities of practice (CoP) Self-support from teachers (SS) Experts and teachers (E-T)	E-T	E-T	CW	CoP	CoP, CW, SS, E-T			CW	E-T	CW, SS, E-T	CoP, SS
	Support or advice	Depth /level: Continuous (C); Local (L); Online (O)	L,	L, O		O	C, L, O	L	L	C, L		C	L
		International support or advice	X	X				X					
		Cooperation between institutions	X	X	X	X		X			X		
	Teacher training methods and resources	Digital resources: - Making use of what is already available (Digital tools/ Integrating digital tools with non-digital ones) (A) -Using/presenting/training in new digital resources (N) -Type of use (as content-C; for delivery-D; for teacher support-TS)	A, C	A, N,	A, C	A, N, C, TS	A, N, C, TS	A, C	N, C	A, C	A, N C, D	A, N D	A D

		Use of videos -for teacher training produced by trainers (TT) -by teachers for reflection of own practice (or of students) (S) for discussion with peers (P)		S, P			S						
		Development of some kind of technology (app, digital textbook, videos, etc): by experts (E) for teachers' use in their classrooms; or teachers (T) involved in producing technology (apps)	E	E	E				E		E		
		Production of school materials (activities) integrating technology by Experts (E) or by Teachers (T)	T	T	T	E	E, T	E	E	T	E, T	E, T	T
	Assessment	Development of skills and competences of teachers related to the students' assessment		X								X	
		Teachers' self-assessment	X	X	X					X		X	X
Constraints or allowances and incentives	Government and/or institutional policies (context)	Impact of government/institutional policies on: design (D), continuity of programme (C), availability of resources (RA) etc.	C, CA	D, C, RA	C, RA			C, RA	D, C, RA		D, C, RA	D, C, RA	D, C, RA
	Access	Level of access that teachers have to technology (availability of resources): Low (L); Medium (M); High (H)	M	H	L			M	H	L	M	M	H
	Time	Length of programme (in years: ½, 1, 2, etc)	2	1	2	6	11	8	4	5	10	3	6
		Teachers' time for involvement in programme: Low (L), Medium (M), High (H)	H	H	M	M	M	L	M	M	M	H	H
		Teachers' time for implementation in class: Low (L), Medium (M), High (H)	M	H	L	M	M	L	L	M		H	H
	Affect/ motivation/ perceptions	Did the programme seek to include aspects deemed to be motivating for teachers (T) and/or for their students (S), to participate and learn?		T			T	T, S	S	T	T	S	T, S
		Did the designers/trainers make explicit to participants their perceptions on the role and use of digital technologies in math classes?	X			X	X	X		X	X	X	

		(If info available – see assessment): Did teachers' perceptions regarding the use of DT in mathematics classes affect their participation in the programme and/or implementation of what was learned in their practice? Yes(Y), Partially(P), No(N)				Y	P	Y		Y	Y	Y	Y
Evaluating the impact (of the PD prog.)	Systematic and systemic evaluation of the programme	Was there evaluation of its impact on practice (Evaluation of the teachers within and after the programme) Yes(Y), Partially(P), No(N)	P			P	P	P		P	N	N	N
		Were the components of the training programme evaluated ? Yes(Y), Partially(P), No(N)	P	P		P	N	P			P		
	Proposals for improvement	Were proposals for improvements for future implementations made? Yes(Y), Partially(P), No(N)	P	Y			P	P			P		

Oggetto: Letter of Acceptance "Mathematics Education in the Digital Age: Learning, Practice and Theory"
Mittente: "Clark-Wilson, Alison" <a.clark-wilson@ucl.ac.uk>
Data: 02/12/20, 18:47
A: "eleonora.faggiano@uniba.it" <eleonora.faggiano@uniba.it>

Institute of Education

UCL

2nd December 2020

Eleonora Faggiano
University of Bari Aldo Moro

Dear Eleonora,

This letter confirms that you are a co-author of the following chapter(s) that has(have) been accepted for publication in the edited volume entitled "Mathematics Education in the Digital Age: Learning, Practice and Theory". The book is a Volume in the ERME Series *New Perspectives on Research in Mathematics Education* and it is scheduled to be published by Routledge in May 2021.

Towards Pragmatic Theories that Underpin the Design of Teacher Professional Development Concerning Technology Use in School Mathematics

On behalf of the editorial team, can I thank you for your contributions to this important volume.

Kind regards,

Alison Clark-Wilson

Dr Alison Clark-Wilson
Principal Research Felllow
UCL Knowledge Lab,

UCL Institute of Education
20 Bedford Way, London WC1H 0AL
+44 (0)20 7612 6000 | enquiries@ioe.ac.uk | www.ucl.ac.uk/ioe