Letter

Directional emission and photon bunching from a qubit pair in waveguide

Maria Maffei, Domenico Pomarico,* Paolo Facchi, Giuseppe Magnifico, Saverio Pascazio, and Francesco V. Pepe
Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy
and INFN, Sezione di Bari, I-70125 Bari, Italy

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Waveguide quantum electrodynamics represents a powerful platform to generate entanglement and tailor photonic states. We consider a pair of identical qubits coupled to a parity invariant waveguide in the microwave domain. By working in the one- and two-excitation sectors, we provide a unified view of decay processes and show the common origin of directional single-photon emission and two-photon directional bunching. Unveiling the quantum trajectories, we demonstrate that both phenomena are rooted in the selective coupling of orthogonal Bell states of the qubits with photons propagating in opposite directions. We comment on how to use this mechanism to implement optimized post-selection of Bell states, heralded by the detection of a photon on one side of the system.

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Introduction. In recent years, the field of waveguide quantum electrodynamics [1–17] has endeavored toward the implementation of networks to communicate and manipulate information encoded in itinerant photons [18–23]. In this context, it is crucial to achieve selective and tunable directional propagation of photons. In the optical domain, this task is easily achieved by exploiting the locking of the photon polarization with the direction of propagation in the so-called chiral waveguides [24–26]. In the microwave domain, where this effect cannot be exploited, destructive interference between fields emitted by a pair of identical two-level systems (qubits) has been identified as a promising strategy [27–43]. One-dimensional arrays of multiple emitters have been extensively investigated as well [44–60], including systems in the optical domain [61,62].

The most natural description of the a pair of identical emitters in a parity-invariant waveguide uses centrally symmetric and antisymmetric states of the propagating electromagnetic field [29,63]. However, such a natural formulation does not correspond to a simple experimental detectability of the two kinds of photons, symmetric or antisymmetric, unless specific interferometric techniques are employed. Yet, describing the dynamics in terms of photon propagation directions gives new insights into the system physics and the possibility to implement new procedures.

An independent emission of photons propagating to the left or to the right of the emitters can be achieved only for certain specific values of the distance between the emitters and additionally requires the implementation of a control coupling between them: two identical qubits placed a quarter wavelength apart and connected via a suitable control coupling can emit and absorb single photons directionally [64,65]. This happens as orthogonal Bell states of the qubits get coupled selectively with different photon propagation directions (see Fig. 1). Remarkably the same mechanism can be used to generate two-photon N00N states [66–72].

In this letter, we provide a unified view of the decay processes of a pair of qubits in the one-excitation and two-excitation sectors, showing the common origin of directional emission and bunching phenomena [63,73–76]. Differently from the existing theoretical literature, our results are not built on the solution of the qubits master equation but rather on that of the closed light-matter dynamics [77–79]. The joint system state shows that the state of the emitted photons and their entanglement can be tuned by changing the qubits distance and the strength of the control coupling.

The unveiling of the quantum trajectories of the joint system shows that the emission of a two-photon N00N state with directional bunching can be regarded as an avalanche process: the first photon is emitted toward the left or right with equal probability, hence conserving the initial parity symmetry; then, according to its direction, the qubits are projected onto a different Bell state that consequently is forced to emit the second photon in the same direction. We then show that the mechanisms underlying left/right photon emission can be used to implement optimized post-selection of Bell states, heralded by the detection of photons on one or the other side of the qubits pair.

Model and dynamics. We consider a pair of identical qubits coupled to the same one-dimensional waveguide in different points, at a distance d from each other. The bare Hamiltonian of qubit $j \in \{1, 2\}$ is $H_j^{(0)} = \omega_0 \sigma_j^\dagger \sigma_j$, where $\sigma_j = |g_j\rangle\langle e_j|$, with e and g labeling the excited and ground state, respectively. For notation shortness, we will denote the states of the tensor product emitter basis as $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, and $|gg\rangle$. The atoms are also coupled among each

*Contact author: domenico.pomarico@ba.infn.it

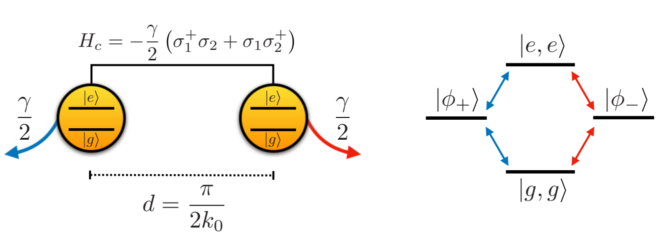

FIG. 1. A pair of identical qubits of frequency ω_0 are placed at a distance d along a parity-invariant waveguide with linear dispersion relation $\omega = v_g k$. Besides being coupled by field-mediated interactions, the qubits interact through a control coupling H_c of strength J. In the case shown in figure, with $d = \pi/(2k_0) = \pi v_g/(2\omega_0)$ and $J = -\gamma/2$, the Bell state $|\phi_+\rangle$ ($|\phi_-\rangle$) absorbs/emits only left- (right-) propagating photons. The system thus behaves as an effective four-level system with optical selection rules.

other directly by an energy-exchange interaction described by the Hamiltonian $H_c = J(\sigma_1^+\sigma_2 + \sigma_1\sigma_2^+)$, which is called, for reasons that will be clear in the following, the *cancellation coupling*. A possible strategy to implement such a term consists of buffering the emitter-waveguide coupling with interacting resonant cavities [40,64,66]. The electromagnetic field propagates along the waveguide with a linear dispersion relation (in the relevant bandwidth around ω_0), with constant group velocity v_g. Hence, in the interaction picture with respect to the bare Hamiltonians of the qubits and the field, the coupling between the atoms and the waveguide photons reads, within the rotating wave approximation,

$$V_I(t) = \sqrt{\frac{\gamma}{2}}\{\sigma_1[b_R^\dagger(t) + b_L^\dagger(t)] + \sigma_2[e^{-i\omega_0\tau} b_R^\dagger(t-\tau) + e^{i\omega_0\tau} b_L^\dagger(t+\tau)]\} + \text{H.c.} \tag{1}$$

Here, $\tau = d/v_g$ is the time of flight between the qubits placed at $x = 0$ and $x = d$, and $b_\ell(t)$ with $\ell \in \{R, L\}$, where L and R stand for left- and right-propagating photons, are the annihilation operators (quantum noise), verifying $[b_\ell(t), b_m^\dagger(t')] = \delta_{\ell,m}\delta(t-t')$ [80]. We assumed that the coupling rate γ, equal for the two propagation directions, is constant over the relevant bandwidth (first Markov approximation) [81], with the rotating wave approximation holding true for $\gamma \ll \omega_0$ [82,83].

In the following, we will assume that the qubit distance d has an order of magnitude of the atomic wavelength. This condition implies that $\tau \sim \omega_0^{-1} \ll \gamma^{-1}$—that is, the time of flight between the qubits is much smaller than the typical lifetime of their excited states. Within this regime, we can neglect the propagation delay between the qubits in the quantum noise operators replacing $b_R^\dagger(t-\tau)$ and $b_L^\dagger(t+\tau)$ with $b_R^\dagger(t)$ and $b_L^\dagger(t)$, and thus the dynamics of the two qubits can be described by a GKLS master equation [84–86].

Now, let us consider the quantum noise increment operators defined as $dB_\ell(t) \equiv \int_t^{t+dt} ds\, b_\ell(s)$, with $[dB_\ell(t), dB_m^\dagger(t')] = \delta_{\ell,m}dt$ for $t = t'$ and 0 otherwise, and the associated increment of the number operator $dN_\ell(t) \equiv \int_t^{t+dt} ds\, b_\ell^\dagger(s) b_\ell(s)$ [80]. The stochastic differential equation of the unitary propagator in Itō form reads [84,87]

$$dU(t) = \left\{\left(-iH + \frac{1}{2}\sum_\ell \mathcal{J}_\ell^\dagger \mathcal{J}_\ell\right) dt + \sum_\ell [\mathcal{J}_\ell dB_\ell^\dagger - \mathcal{J}_\ell^\dagger dB_\ell + (e^{i\omega_0\tau} - 1)\, dN_\ell]\right\} U(t), \tag{2}$$

where $H = H_e + H_c$, with $H_e = (\gamma/2)(\sigma_1^+\sigma_2 + \sigma_1\sigma_2^+)$ being an effective qubit-qubit energy-exchange interaction mediated by the electromagnetic field, and

$$\mathcal{J}_R = -i\sqrt{\frac{\gamma}{2}}(\sigma_1 + e^{-i\omega_0\tau}\sigma_2), \quad \mathcal{J}_L = -i\sqrt{\frac{\gamma}{2}}(\sigma_1 + e^{i\omega_0\tau}\sigma_2) \tag{3}$$

are the jump operators associated with the right and left emission, respectively.

Importantly, the combination of H_e with the cancellation coupling H_c determines a new effective Hamiltonian dynamics

$$H = H_e + H_c = \frac{\gamma}{2}(\sin(\omega_0\tau) - g_c)(\sigma_1^\dagger\sigma_2 + \sigma_1\sigma_2^\dagger), \tag{4}$$

with $g_c = -2J/\gamma$. Therefore, the choice $g_c = \sin(\omega_0\tau)$ cancels the exchange interaction between the emitters, $H = 0$, leaving the dissipation as the only non-trivial part of the dynamics (2) induced by the coupling with the waveguide field.

The Schrödinger equation (2) is invariant under point reflection through the center $x = d/2$; hence, to obtain selective directional propagation, the inherent central symmetry of the dynamics needs to be broken by preparing the system in an asymmetric initial state [64]. Considering the form of the light-matter coupling in (2), one could naively expect that preparing either of the states

$$|\psi_L\rangle = \frac{|eg\rangle - e^{i\omega_0\tau}|ge\rangle}{\sqrt{2}}, \quad |\psi_R\rangle = \frac{|eg\rangle - e^{-i\omega_0\tau}|ge\rangle}{\sqrt{2}}, \tag{5}$$

which are selectively annihilated by the jump operators (3) (i.e., $\mathcal{J}_R|\psi_L\rangle = 0 = \mathcal{J}_L|\psi_R\rangle$) would provide pure directional emission. However, this is not the case, as the two states are generally coupled to each other by the effective Hamiltonian (4). The following analysis will show that fully directional emission occurs only in exceptional cases, characterized by specific values of emitter distance and by fine-tuned cancellation couplings. Moreover, we show that these conditions are identical to those in which two-photon emission from a doubly excited emitter state is fully bunched in direction.

One-excitation sector. Let us first consider the case where the emitters are prepared in a pure single-excitation state $|\psi\rangle = a_{eg}|eg\rangle + a_{ge}|ge\rangle$ so that the system state at a later time t reads

$$|\Psi(t)\rangle = [a_{eg}(t)|eg\rangle + a_{ge}(t)|ge\rangle] \otimes |0_R 0_L\rangle + |gg\rangle \otimes \int_0^t ds[f_R(s)b_R^\dagger(s) + f_L(s)b_L^\dagger(s)]|0_R 0_L\rangle, \tag{6}$$

where $|0_R 0_L\rangle$ is the waveguide field vacuum. The coefficients of the excited qubit states are given by the matrix elements

$$a_{eg}(t) = \langle eg|\mathcal{K}(t)|\psi\rangle, \quad a_{ge}(t) = \langle ge|\mathcal{K}(t)|\psi\rangle \tag{7}$$

of the Kraus operator $\mathcal{K}(t) \equiv \langle 0_R 0_L | U(t) | 0_R 0_L \rangle$ acting on the qubits, whose analytical expression can be shown to be [87]

$$\mathcal{K}(t) = \begin{pmatrix} e^{-\gamma t} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\left(e^{-\frac{1}{2}\mu_+ t} + e^{-\frac{1}{2}\mu_- t}\right) & \frac{1}{2}\left(e^{-\frac{1}{2}\mu_+ t} - e^{-\frac{1}{2}\mu_- t}\right) & 0 \\ 0 & \frac{1}{2}\left(e^{-\frac{1}{2}\mu_+ t} - e^{-\frac{1}{2}\mu_- t}\right) & \frac{1}{2}\left(e^{-\frac{1}{2}\mu_+ t} + e^{-\frac{1}{2}\mu_- t}\right) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

Here, $\mu_\pm = \gamma_\pm \pm i\delta$, where

$$\gamma_\pm = \gamma[1 \pm \cos(\omega_0 \tau)], \qquad \delta = \gamma[\sin(\omega_0 \tau) - g_c] \tag{9}$$

respectively correspond to the imaginary and the real part of the self-energy eigenvalues in the limit of linear dispersion relation and including the cancellation coupling, as shown in Ref. [29]. The single-photon amplitudes $f_\ell(t)$ in Eq. (6), with $\ell \in \{R, L\}$, are given by the matrix elements

$$f_\ell(s) = \langle gg | \mathcal{K}(t-s) \mathcal{J}_\ell \mathcal{K}(s) | \psi \rangle, \tag{10}$$

whose analytical expression is reported in the Supplemental Material [87].

Besides the trivial eigenvectors $|ee\rangle$ and $|gg\rangle$, the matrix $\mathcal{K}(t)$ is generally diagonalized by the real-coefficient Bell states $|\psi_\pm\rangle = (|eg\rangle \pm |ge\rangle)/\sqrt{2}$ with decay rates $\gamma_\pm$, as well as relative energy splitting δ determined by the Hamiltonian (4). Since they do not break central symmetry, the states $|\psi_\pm\rangle$ cannot give rise to any prevalence of emission in one direction.

The most suitable candidates for directional emission would be the states (5), but as one can observe from the form of $\mathcal{K}(t)$ in Eq. (8), the dynamics generally entails transitions between them, thus hindering purely directional emission. A remarkable exception is represented by the following cases, which we can call *controlled antiresonances*,

$$\omega_0 \tau = \left(n + \tfrac{1}{2}\right)\pi, \quad g_c = (-1)^n, \quad \text{with } n \in \mathbb{N}, \tag{11}$$

in which the antiresonance condition on $\omega_0\tau$ makes the quantities $\gamma_\pm$ equal to the isolated-qubit decay rate, whereas the cancellation coupling is used to suppress the Hamiltonian evolution in the single-excitation sector, thus making $\mathcal{K}(t)$ diagonal. In these conditions, the right- and left-emitting states (5) specialize to the *orthogonal* Bell states $|\phi_\pm\rangle = (|eg\rangle \pm i|ge\rangle)/\sqrt{2}$: for even-$n$ (resp. odd-n) antiresonances, one finds $|\psi_R\rangle = |\phi_+\rangle$ and $|\psi_L\rangle = |\phi_-\rangle$ (resp. $|\psi_R\rangle = |\phi_-\rangle$ and $|\psi_L\rangle = |\phi_+\rangle$). Due to the cancellation condition $g_c = (-1)^n$, no coherent transition between the two states $|\phi_\pm\rangle$ occurs, and the preparation of either of them at the initial time generates pure directional emission. In this case, the two qubits can be regarded as a four-level system with optical selection rules [40], as depicted in Fig. 1(a), corresponding to an even-n controlled antiresonance.

In general, the directionality of the emitted field can be quantified through the ratio $r_1(|\psi\rangle) = \mathcal{P}_L^{(\psi)}/\mathcal{P}_R^{(\psi)}$ with $\mathcal{P}_{L/R}^{(\psi)} = \int_0^\infty dt |f_{L/R}^{(\psi)}(t)|^2$ being the probability that the state $|\psi\rangle$ emits toward left/right. The states $|\psi_{L/R}\rangle$ of Eq. (5) yield

$$r_1(|\psi_L\rangle) = \frac{1 + (g_c - \sin(\omega_0\tau))^2 + \sin^2(\omega_0\tau)}{1 + (g_c - \sin(\omega_0\tau))^2 - \sin^2(\omega_0\tau)}, \tag{12}$$

and $r_1(|\psi_R\rangle) = 1/r_1(|\psi_L\rangle)$. Hence, as expected, the emission is purely directional (i.e., $r_1(|\psi_L\rangle) = \infty$ and $r_1(|\psi_R\rangle) = 0$) provided that $\omega_0\tau$ and g_c verify the controlled antiresonance condition in Eq. (11). It is interesting to compare the preceding result with the one obtained using the initial one-excitation states that break spatial inversion symmetry but are factorized (i.e., $|eg\rangle$ and $|ge\rangle$). In this case, one finds

$$r_1(|eg\rangle) = r_1(|eg\rangle)^{-1} = 3 - 2\frac{g_c^2 + \cos^2(\omega_0\tau)}{1 + g_c(g_c - \sin(\omega_0\tau))}. \tag{13}$$

Therefore, despite the cancellation coupling, if the initial states are not tailored for pure directional emission, the directionality ratio can never exceed the value $r_1 = 3$ [63].

Two-excitation sector: entangled photons and Bell state post-selection. When the qubits are prepared in the doubly excited state $|ee\rangle$, which is obviously centrally symmetric, the state at time t comprises three different amplitudes describing the following: (i) both emitters remaining excited, (ii) the emitters being in a single-excitation state and one photon being emitted, and (iii) the emitters being in the state $|gg\rangle$ and two photons being emitted. The dynamics of such an evolution is strongly influenced by what occurs in the one-excitation sector, especially concerning the alternation of resonances (i.e., the cases $\omega_0\tau = n\pi$ with $n \in \mathbb{N}, n \neq 0$, when one of the two Bell states $|\psi_\pm\rangle$ is stable) and antiresonances [63]. Even though the Schrödinger equation (2) enables us to determine the dynamics at any time (see [87]), let us focus on the asymptotic regime $t \gg \gamma^{-1}$, when only the amplitudes of type (3) survive (i.e., the emitters are found in $|gg\rangle$) and the field state reads

$$|\Xi\rangle = \sum_{\ell,m \in \{R,L\}} \int_0^\infty dt_2 \int_0^{t_2} dt_1 [\lambda_{\ell,m}(t_1,t_2) b_\ell^\dagger(t_1) b_m^\dagger(t_2)] |0_R 0_L\rangle, \tag{14}$$

with normalization achieved in the limit $t \to \infty$. The two-photon wavefunctions $\lambda_{\ell,m}(t_1,t_2)$ with $t_1 \leqslant t_2$ are given by

$$\lambda_{\ell,m}(t_1,t_2) = \lim_{t\to\infty} \langle gg | \mathcal{K}(t-t_2) \mathcal{J}_m \mathcal{K}(t_2 - t_1) \mathcal{J}_\ell \mathcal{K}(t_1) | ee \rangle. \tag{15}$$

The properties $\lambda_{R,R}(t_1,t_2) = e^{2i\omega_0\tau} \lambda_{L,L}(t_1,t_2)$ and $\lambda_{R,L}(t_1, t_2) = \lambda_{L,R}(t_1,t_2)$ imply that the probabilities $\mathcal{P}_{l,m} = \int_0^\infty dt_2 \int_0^{t_2} dt_1 |\lambda_{l,m}(t_1,t_2)|^2$ are invariant under exchange of the propagation direction, as expected by the central symmetry of the initial state of the system.

Hence, the ratio between the probabilities of antiparallel emission, $\mathcal{P}_{\uparrow\downarrow} = \mathcal{P}_{L,R} + \mathcal{P}_{R,L}$, and that of parallel emission,

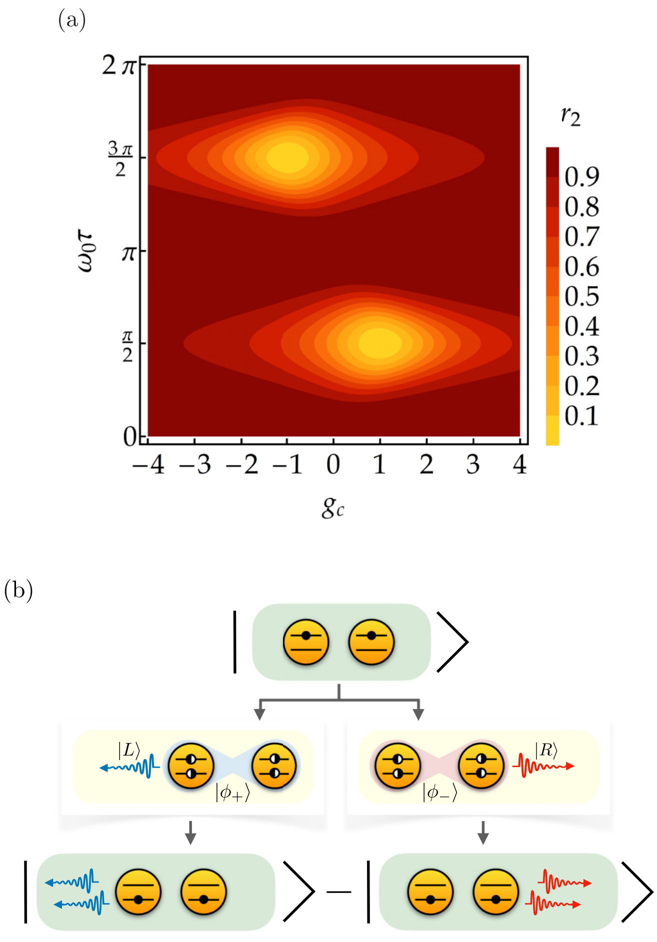

FIG. 2. (a) Plot of the ratio r_2 in Eq. (16), as a function of $\omega_0\tau$ and g_c. As expected, $r_2 = 0$ (purely parallel emission) at the points verifying the even-n (resp. odd-n) controlled antiresonance condition, $\{\omega_0\tau, g_c\} = \{\frac{\pi}{2}, 1\}$ (resp. $\{\omega_0\tau, g_c\} = \{\frac{3\pi}{2}, -1\}$). (b) Pictorial representation of the quantum trajectories underlying the N00N state generation: when the controlled antiresonance condition is verified, the doubly excited state undergoes a sequence of transitions selectively coupled with the two directions of propagation [see Eq. (18)].

$\mathcal{P}_{\parallel} = \mathcal{P}_{L,L} + \mathcal{P}_{R,R}$, is equal to

$$r_2 = \frac{\mathcal{P}_{1\downarrow}}{\mathcal{P}_{\parallel}} = \frac{(2-\sin^2(\omega_0\tau))[1+(\sin(\omega_0\tau)-g_c)^2]+\sin^4(\omega_0\tau)}{(2-\sin^2(\omega_0\tau))[1+(\sin(\omega_0\tau)-g_c)^2]-\sin^4(\omega_0\tau)}. \tag{16}$$

In the absence of cancellation coupling, r_2 always lies between the value 1/3, reached at antiresonance, and the value 1 reached at resonance (see the small-coupling limit in Ref. [63]). Thus, in this case, antiparallel emission can never be suppressed, in accordance with the findings in the single-excitation sector. Instead, by adding a cancellation coupling that verifies the controlled antiresonance condition (11), one achieves pure directional bunching of a two-photon emission corresponding to a vanishing r_2 [see Fig. 2(a)]. In this case, $\lambda_{R,L}(t_1, t_2) = \lambda_{L,R}(t_1, t_2) = 0$, and $\lambda_{R,R}(t_1, t_2) = -\lambda_{L,L}(t_1, t_2) = -i\gamma(-1)^n e^{-\gamma(t_1+t_2)/2}$ (see the Supplemental Material [87] for the explicit derivation). Hence, the system asymptotically approaches the state $|gg\rangle \otimes |\Xi\rangle$, with the field in the two-photon N00N state

$$|\Xi\rangle = \frac{|2_R 0_L\rangle - |0_R 2_L\rangle}{\sqrt{2}}, \tag{17}$$

where we have introduced the Fock state notation $|n_R n_L\rangle = (n_R! n_L!)^{-1/2}(c_R^\dagger)^{n_R}(c_L^\dagger)^{n_L}|0_R 0_L\rangle$ associated to the mode operators $c_{R/L} \equiv \int_0^\infty dt \sqrt{\gamma} e^{-\gamma t/2} b_{R/L}(t)$ [88,89], with $|0_R 0_L\rangle$ being the field vacuum. Note that due to the phases acquired at point inversion, the preceding state is centrally *symmetric*, as well as the initial state $|ee\rangle$.

Looking at Eq. (15), the entangling mechanism appears transparent. As $\mathcal{K}(t)|ee\rangle = e^{-\gamma t}|ee\rangle$ [see Eq. (8)], the first jump operator acts on $|ee\rangle$ and projects it onto one of the states $|\phi_\pm\rangle$, which is then forced to emit the second photon toward the same direction as the first one. Therefore, in the cases (11), the decay of the $|ee\rangle$ occurs with equal probability through the uncoupled channels [see Fig. 2(b)]:

$$|ee\rangle \to \begin{cases} |\psi_R\rangle \otimes |1_R 0_L\rangle \to |gg\rangle \otimes |2_R 0_L\rangle \\ |\psi_L\rangle \otimes |0_R 1_L\rangle \to |gg\rangle \otimes |0_R 2_L\rangle \end{cases} \tag{18}$$

with $|\psi_R\rangle = |\phi_+\rangle$ and $|\psi_L\rangle = |\phi_-\rangle$ ($|\psi_R\rangle = |\phi_-\rangle$ and $|\psi_L\rangle = |\phi_+\rangle$) for even-$n$ (odd-n) antiresonances.

Then, under the assumption of ideal photodetection, if two detectors are placed on the left and on the right sides of the emitters, the observation of the first photon by the left detector (resp. the right one), occurring after an average time $(2\gamma)^{-1}$, unambiguously selects the state $|\psi_L\rangle$ (resp. $|\psi_R\rangle$). The selected state does not decay until the second photon is observed after an additional average time of γ^{-1}. Therefore, the two-excitation decay, Eq. (18), makes one of two orthogonal Bell states available, with certainty, within an average time γ^{-1}. During this time, one can think of implementing strategies to adiabatically decouple the emitters from the field, hence preserving the Bell state from decaying. Triggering different operations, depending on which detector clicks, it is possible to select one of the two equiprobable system trajectories.

Let us note that the same post-selection scheme can give access to a Bell state also within resonance conditions. In the n-th resonance, only one decay channel is open:

$$|ee\rangle \to |\psi_{(-1)^n}\rangle \otimes |1_{(-1)^n}\rangle \to |gg\rangle \otimes |2\rangle, \tag{19}$$

with $|1_\pm\rangle = (|1_R, 0_L\rangle \pm |0_R, 1_L\rangle)/\sqrt{2}$ being one-photon states satisfying central symmetry (antisymmetry) and $|2\rangle = (|2_R 0_L\rangle + |0_R 2_L\rangle + \sqrt{2}|1_R 1_L\rangle)/2$. In this case, as before, the first emitted photon is observed by one of the two detectors after an average time $(2\gamma)^{-1}$. Regardless of which detector clicks, the qubits are left in the Bell state $|\psi_+\rangle$ (resp. $|\psi_-\rangle$) for an even-n (resp. an odd-n resonance). However, such a state subsequently decays *twice as fast* than $|\phi_\pm\rangle$ at antiresonance. This halves the time for possible operations to decouple the emitters from the field to preserve the Bell state.

Finally, when the system is neither in resonance nor antiresonance, one can still post-select one of the states $|\psi_\pm\rangle$, with uneven relative probabilities $\gamma_\pm/(2\gamma) = (1 \pm \cos(\omega_0\tau))/2$, but this would require an interferometric detection scheme to distinguish between symmetric and antisymmetric single-photon states $|1_{\pm}\rangle$. Moreover, it is worth noting that in this case, the most probable state to be detected is also the one that decays faster afterward.

Outlook. We presented an analytic description of the dynamics underlying the directional emission of single photons and the generation of two-photon N00N states from a pair of qubits in the waveguide. We displayed the common root of these phenomena emerging in the same antiresonance conditions, hence highlighting the primary role played by central symmetry. The proposed approach emerges as the ideal candidate to achieve exact modeling and characterization of arrays of multiple qubits and multi-level systems (qudits) whose dynamics and collective properties are determined by the symmetries, particularly in the microwave domain [21]. Furthermore, by describing the closed-system dynamics through a collision model, our analysis can be extended to include a proper description of time delays, feedback [53,90], and scattering phenomena [38,91], which could also be also interesting for protocols involving two-photon correlations [92,93].

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