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### Abstract

Investigations of the developed mathematical model of a vibratory mill made it possible to find out regularities for the complex movement of its operating mechanisms and to obtain a graphic interpretation for the kinematic, power and energy characteristics of the vibratory system to be developed. On the basis of theoretical pre-requisites, a series of experimental studies were conducted that provided an opportunity to obtain amplitude-frequency, velocity and energy characteristics of the developed machine, to determine their impact upon the kinetics of the fine grinding process of the agricultural raw material. As a result, rational operating parameters of the examined machine were established with minimal energy input.

**Keywords** Vibratory mill; Fine grinding; Mathematical model; Operating mode.

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## **Highlights**

- Developed vibratory mill of angular oscillations, which allows to increase the grinding efficiency of raw material.
- Theoretical studies of the process of fine grinding bulk raw materials confirmed by experimental tests.
- Rational operating parameters established with minimal energy input.

1 **Theoretical and experimental investigation of a vibratory mill for fine grinding of grain**

2

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24 **Abstract**

25 Investigations of the developed mathematical model of a vibratory mill made it possible to find out  
26 regularities for the complex movement of its operating mechanisms and to obtain a graphic  
27 interpretation for the kinematic, power and energy characteristics of the vibratory system to be  
28 developed. On the basis of theoretical pre-requisites, a series of experimental studies were  
29 conducted that provided an opportunity to obtain amplitude-frequency, velocity and energy  
30 characteristics of the developed machine, to determine their impact upon the kinetics of the fine  
31 grinding process of the agricultural raw material. As a result, rational operating parameters of the  
32 examined machine were established with minimal energy input.

33

34 **Key words:**

35 Vibratory mill, Fine grinding, Mathematical model, Operating mode.

36

## 38 1. Introduction

39 Grain refinement is the oldest technology used by humanity to produce foodstuffs. The main  
40 technological task of grinding grain is to obtain a homogeneous mixture with a desired degree of  
41 grinding of the ingredients (Linch, 2005; Butkovsky, 1990). On the other hand, the refining process  
42 at the mill allows to separate components, such as the parts of the wheat grain: the endosperm  
43 containing mostly starch and proteins, the germ composed mostly of lipids and proteins and the  
44 bran containing mainly dietary fiber (Liu et al., 2015; Marquart et al., 2007). At present, there are  
45 widely used various types of mills and crushers for the production of flour for food and animal  
46 fodder (Savinyh et al, 2016, Sysuev et al, 2016; Cho et al., 2013). As example, the production of  
47 whole grain flour is mainly affected by the milling process and actually the predominant techniques  
48 for grinding whole grain flours are stone mill, roller mill, ultra-fine mill and hammer mill (Kihlberg  
49 et al., 2004; Kent and Evers, 1994). Research in the intensification of grain refinement is reflected  
50 in many works (Bulgakov et al, 2017, Sysuev et al, 2015; Rosenkranz et al., 2011; Rajamani et al.,  
51 2000); however, the operation of a vibratory mill for fine grinding has been so far studied not  
52 enough. As a result of the execution of the vibratory action, there occur a significant decrease in the  
53 coefficient of internal friction, an active increase in the specific surface area of the material when  
54 interacting with the technological filler, efficient creating of essential gradients in the rates of  
55 internal deformations in the micro- and macro-volumes of the product on condition that its  
56 structural mechanical characteristics are changed.

57 When implementing the fine grinding process, the vibratory action makes it possible to substantially  
58 increase the shock-erasing effect due to the possibility of technological variation of its strength and  
59 friction components; on the one hand, to increase the destruction rate of the material particles under  
60 the impact of cyclic loads, but on the other hand, – as a result of dynamic interaction between each  
61 other, it ensures their ability of active abrasion (Doumanidis et al., 2016; Janovich et al., 2016; Mori  
62 et al., 2004; Gonzalez, 1995). To the main shortcomings of the traditional machines for the  
63 production of fine and highly disperse material one should relate significant specific energy input

64 into the processing of raw materials, low performance characteristics of the operating mechanisms  
65 due to the active abrasion of their operating surfaces, and lowering of their technological efficiency  
66 as a result of adhesion of products with increased humidity (Kaletnik, 2016, Nasir, 2005,  
67 Yaroshevich, 2011).

68 In order to eliminate the above-mentioned shortcomings in the intensification of the processes of  
69 grinding agricultural raw materials and to increase the operational and technological parameters of  
70 machines for their implementation, it is proposed to apply a complex vibromechanical impact,  
71 which is the purpose of this scientific work, justifying its topicality and prospects for its  
72 implementation.

73 The purpose of this scientific work is to develop a mathematical model of a vibratory mill and to  
74 carry out experimental studies on the substantiation of the technological parameters of its operation  
75 under the condition of an intensified grain refinement process, and to increase the operational and  
76 technological indicators of machines on the basis of a complex vibro-mechanical impact.

77

## 78 **2. Materials and methods**

79 An experimental vibratory mill of angular oscillations was used for the studies (Janovich, 2015;  
80 Franchuk, 1970) but the theoretical investigations were carried out on the basis of its equivalent  
81 scheme. Estimation of the energy consumption of the developed machine, its amplitude-frequency,  
82 kinetic and operational parameters was carried out on the basis of experimental studies, but the  
83 analysis of the kinematic characteristics of the vibrator (vibration acceleration, vibration velocity  
84 and vibrodisplacement) was conducted applying the developed software for commutation.

### 85 *2.1. The measure chain*

86 In order to determine the amplitude-frequency characteristics, an accelerometer analyser was  
87 developed, based on the 3-axis accelerometer LIS3DH (STMicroelectronics, USA) (Fig. 1), whose  
88 main features were: dynamically user-selectable full scales of  $\pm 2g/\pm 4g/\pm 8g/\pm 16g$ , capability of  
89 measuring accelerations with output data rates from 1 Hz to 5.3 kHz. .



90 The principle of operation of this device is the following: after connecting the sensor to the surface  
91 of the grinding chamber of the mill, the drive mechanism of the machine is switched on, due to  
92 which the resulting vibrations of the grinding chamber initiate inclusion of the integrated  
93 accelerometer that starts registration of the amplitude-frequency characteristics of the machine  
94 under study. To record the rotation speed of the drive shaft, the UNI-T UT372 (Uni-Trend  
95 Technology Limited, China), wireless tachometer was used, whose main technical features were:  
96 measurement 10 to 99,999 RPM, accuracy 0.04% +/- 2dgt. The control and change of the rotational  
97 speed of the motor shaft was carried out with the help of the autotransformer AOSN-20-220-75,  
98 which is intended for operation with alternating current. To determine the energy characteristics of  
99 the machine studied, the electronic wattmeter EMF-1 was used.

100 The dispersion ability of the material was determined through the method of mechanical separation  
101 of the parts – by the sieve analysis on a laboratory sieve analyzer A-20 (labTime, Russia), whose  
102 main technical features were: diameter of screen 200 mm, number of screen 4 to 10, amplitude of  
103 oscillations- 2 to 4, frequency of oscillation 1500 rad·s<sup>-1</sup>, granularity 0.008 to 5 mm. It is used as  
104 standard screen machine grading all kinds powder material particle. In order to determine the  
105 relative humidity of the material, a Wile 55 moisture meter was applied, which can be used to  
106 measure the relative humidity of various types of grain and seeds, the data being stored in the  
107 memory of the instrument. Equipment Wile 55 (Farmcomp Oy, Finland) can measure a wide range  
108 of moisture, grain 8 to 35% and oil seeds 5 to 25%. Wile 55 is the basic tool for precise quick  
109 measurement of grain moisture. It has an easy-to-read LCD display and automatic temperature  
110 compensation that utilizes the internal temperature sensors for both grain and the temperature of the  
111 device itself.

112 Besides, it is equipped with a conditional scale showing the rough values of the measurements and  
113 can be used to measure the humidity of the samples not entered into the database of the moisture  
114 meter. To determine the specific surface area of the treated material during the vibratory grinding  
115 process with the particle size being 25-1000 microns, equipment PSH-9 was used, the action of

116 which is based on measuring the air permeability of a layer of material through which air is leaking  
117 under pressure close to the atmospheric one. Equipment PSH-9 (Company Hranat, Russia) had the  
118 following main features: range of specific surface area of the material 300 to 50000 m<sup>2</sup>·g<sup>-1</sup>, time of  
119 one analysis 3.5 min, accuracy 0.5%.

120 To analyse the quality of the crushed raw material, a laboratory sampler was applied for sampling  
121 by the point selection method. To control the supply of material, a mobile vibratory dispenser PG-2  
122 (Company Vybrotekhnyk, Russia) was used, whose main technical features were: size of the source  
123 material 0.05 to 5 mm, productivity 400 kg·h<sup>-1</sup>, amplitude of oscillation of the tray 2 mm, bunker  
124 volume 9 dm<sup>3</sup>.

125 Thus, the presented structural design for carrying out laboratory and production tests allows us to  
126 determine precisely enough the main technological parameters of the angular oscillations of the  
127 developed mill and to evaluate the quality of the material obtained within the range of experimental  
128 studies.

## 129 *2.2. The developed vibratory mill*

130 The principle of operation of the developed vibratory mill (Janovich, 2014) is the following (Figure  
131 2): when the electric motor 1 is switched on, the rotation torque is transmitted through the elastic  
132 clutch 2 to the drive shaft 3 with unbalances 4 arranged on it, rotation of which ensures generation  
133 of a combined force and moment imbalance of the oppositely arranged (on the periphery from the  
134 two sides) in relation to the central axis 5 grinding chambers 6 filled to 75% with the grinding  
135 elements in the form of steel balls. The grinding chambers, in their turn, are interconnected by  
136 traverses (crosspieces) 7. This design solution of the machine allows to provide angular oscillations,  
137 ensuring a significant dynamic state of the technological filler (promoting active collision of the  
138 grinding elements), and, as a consequence, it provides a possibility to increase the grinding  
139 efficiency of the raw material being processed continuously and entering through the feed pipes 8  
140 from the hopper 9. Destruction of the raw material particles takes place due to cyclic loads created

141 by the crushing elements and their dynamic interaction with each other. When the particle size of  
142 the material to be crushed is reduced under the impact of the centrifugal forces and reversal loads, it  
143 is transported along the grinding chamber to the separating section where particles, smaller than the  
144 separation holes of the sieve, are discharged from the mill through the pipe 10; the particles of the  
145 material that have a larger size – are crushed due to the active impact of crushing elements.

146 As an electric drive of the developed vibratory machine, an electric motor with a power of 0.75 kW  
147 and an operating rotation speed of  $1500 \text{ min}^{-1}$  was used. Preliminary studies of the experimental  
148 mill witnessed that its efficiency is  $220 \text{ kg} \cdot \text{h}^{-1}$ . To study the regularities of the process of fine  
149 grinding of bulk raw materials, it is proposed to determine the internal structure of the dynamical  
150 system under investigation and to find linear and nonlinear effects, and on this basis to formulate a  
151 mathematical model of the vibratory system (Dreizler and Ludde, 2010; Maitra and Prasad, 1985;  
152 John and Stephens, 1984,).

### 153 *2.3. The mathematical model*

154 In order to create a mathematical model of angular oscillations of the vibratory mill (based on the  
155 D'Alembert-Euler principle), an equivalent scheme was designed, characterised by five degrees of  
156 freedom (Fig. 3).

157

158 Accordingly, the differential equations of the movement of the operating mechanism of the machine  
159 are described by five independent generalised coordinates, i.e.:

160  $\rho$  – the radial coordinate of positioning the grinding chamber relative to axes  $Oz$ ,  $Ox$ , m;

161  $\varphi$  – the displacement angle of the grinding chamber with respect to the vertical plane, rad;

162  $\alpha$  – the arrangement angle of the vibrodrive relative to the container, rad;

163  $\varphi_1$  – the displacement angle of the vibrator drive depending on the change of the position of the  
164 grinding chamber, rad;

165  $\varphi_3$  – the angle of rotation of the unbalance relative to its axis of rotation, rad.

166  $O_{xz}$  – the fixed coordinate system;

167  $O_1$  – the centre of the container mass;

168  $c_x, c_z$  – the spring stiffness in the direction of the corresponding axis,  $N \cdot m^{-1}$ ;

169  $F_c$  – the force of elasticity, N;

170  $F_k$  – the centrifugal force, N.

171

172 Considering the peculiarity of the angular movement of the grinding chamber of the machine, the  
173 mathematical model of the dynamic system is formed in the polar coordinate system. The  
174 prerequisite for theoretical investigations is a search of operational and design parameters of the  
175 developed machine in which the vertical component of the vibrations of its grinding chambers will  
176 acquire its maximum value. It is possible to find out the relationship between the design and  
177 technological parameters using the initial Lagrange equations of the second kind.

178 The kinetic energy of the grinding chamber will be equal to:

179 
$$T_c = \frac{1}{2} (m_c V_{O_1}^2 + J_c \dot{\phi}_1^2). \quad (1)$$

180 where  $m_c$  – the mass of the container, kg;

181  $J_c$  – the inertia moment of the container,  $kg \cdot m^2$ ;

182  $\dot{\phi}_1$  – the angular velocity of the container,  $rad \cdot s^{-1}$ ;

183  $V_{O_1}$  – the velocity of point  $O_1$  in the polar coordinate system:

184 
$$V_{O_1}^2 = \dot{\rho}^2 + (\rho \cdot \dot{\phi})^2. \quad (2)$$

185 In its turn, the kinetic energy of the unbalance will be equal to:

186

187 
$$T_{unb} = \frac{1}{2} (m_{unb} V_{O_3}^2 + J_{unb} \dot{\phi}_3^2), \quad (3)$$

188 where  $m_{unb}$  – the mass of the unbalance, kg;

189  $J_{unb}$  – the inertia moment of the unbalance,  $kg \cdot m^2$ ;

190  $\dot{\phi}_3$  – the angular velocity of unbalance,  $rad \cdot s^{-1}$ ;

191  $V_{O_3}$  – the velocity of the centre of masses of unbalance, that is determined using the polar  
 192 coordinate system, where the pole to the system is located at this point. The speed  $V_{O_3}$  is then  
 193 determined according to the following vector equation:

$$194 \quad \bar{V}_{O_3} = \bar{V}_{O_1} + \bar{V}_{O_2O_1} + \bar{V}_{O_3O_2}, \quad (4)$$

195 where  $\bar{V}_{O_1}$  – the vector velocity of point  $O_1$  in the polar coordinate system;

196

197  $\bar{V}_{O_2O_1}$  – the vector velocity of point  $O_2$  relatively point  $O_1$ ;

198

199  $\bar{V}_{O_3O_2}$  – the vector velocity of point  $O_3$  relatively point  $O_2$ ;

200

201 In this case, the expression for the kinetic energy of the unbalance has the form:

$$202 \quad T = \frac{1}{2}m_c(\dot{\rho}^2 + \rho^2\dot{\varphi}^2) + \frac{1}{2}J_c\dot{\varphi}_1^2 + \frac{1}{2}m_{unb}[\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{\varphi}_1^2l_2^2 + \dot{\varphi}_3^2l_3^2 + \\ 2\dot{\rho}\dot{\varphi}_1l_2\cos(\varphi - \varphi_1 - \alpha) + 2\dot{\rho}\dot{\varphi}_3l_3\sin(\varphi - \varphi_1 - \varphi_3) + 2\rho\dot{\varphi}\dot{\varphi}_1l_2\sin(\alpha + \varphi_1 - \varphi) + \\ + 2\rho\dot{\varphi}\dot{\varphi}_3l_3\cos(\varphi - \varphi_1 - \varphi_3) + 2\dot{\varphi}_1l_2\dot{\varphi}_3l_3\sin(\alpha - \varphi_3)] + \frac{1}{2}J_{unb}\dot{\varphi}_3^2. \quad (5)$$

203 As a result of the partial derivatives found for the corresponding generalised coordinates, the  
 204 generalised forces of the system and their functional transformations, we obtain a general form of  
 205 the required Lagrange equations of the second kind that fully determine the movement regularity of  
 206 the grinding chamber of the experimental mill:

$$207 \quad (m_c + m_{unb})\ddot{\rho}\varphi_1 - m_c\rho\dot{\varphi}^2 + m_{unb}[-\rho\dot{\varphi}^2 + \ddot{\varphi}_1l_2\cos(\varphi - \varphi_1 - \alpha) + \\ (\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3\sin(\varphi - \varphi_1 - \varphi_3) + \dot{\varphi}_1^2l_2\sin(\varphi - \varphi_1 - \alpha) - (\dot{\varphi}_1 + \dot{\varphi}_3)^2 \times \\ \times l_3\cos(\varphi - \varphi_1 - \varphi_3)] = -(c_x\cos^2\varphi + c_z\sin^2\varphi)\rho; \quad (6)$$

208

$$209 \quad (m_c + m_{unb})\rho^2\ddot{\varphi} + 2(m_c + m_{unb})\rho\dot{\rho}\dot{\varphi} + m_{unb}[\rho\dot{\varphi}_1l_2\sin(\alpha + \varphi_1 - \varphi) + \\ + \rho\dot{\varphi}_1^2l_2\cos(\alpha + \varphi_1 - \varphi) + \rho(\ddot{\varphi}_1 + \ddot{\varphi}_3)l_3\cos(\varphi - \varphi_1 - \varphi_3) + \\ + \rho(\dot{\varphi}_1 + \dot{\varphi}_3)^2l_3\sin(\varphi - \varphi_1 - \varphi_3)] = (c_x - c_z)\rho^2\sin\varphi\cos\varphi; \quad (7)$$

210

$$\begin{aligned}
& J_c \ddot{\varphi}_1 + J_{unb} (\ddot{\varphi}_1 + \ddot{\varphi}_3) + m_{unb} [\ddot{\varphi}_1 l_2^2 + (\ddot{\varphi}_1 + \ddot{\varphi}_3) l_3^2 + \ddot{\rho} l_2 \cos(\varphi - \varphi_1 - \alpha) + \\
& + \ddot{\rho} l_3 \sin(\varphi - \varphi_1 - \varphi_3) + \rho \ddot{\varphi} l_2 \sin(\alpha + \varphi_1 - \varphi) + \rho \ddot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \\
211 \quad & + (2\ddot{\varphi}_1 + \ddot{\varphi}_3) l_2 l_3 \sin(\alpha - \varphi_3) - 2\dot{\rho} \dot{\varphi} l_2 \sin(\varphi - \varphi_1 - \alpha) + 2\dot{\rho} \dot{\varphi} l_3 \times \\
& \times \cos(\varphi - \varphi_1 - \varphi_3) - \rho \dot{\varphi}^2 l_2 \cos(\alpha + \varphi_1 - \varphi) - \rho \dot{\varphi}^2 l_3 \sin(\varphi - \varphi_1 - \varphi_3) - \\
& - (2\dot{\varphi}_1 + \dot{\varphi}_3) \dot{\varphi} l_2 l_3 \cos(\alpha - \varphi_3)] = -c_z \varphi_1 - m_{unb} [l_2 \sin(\alpha + \varphi_1) + l_3 \cos(\varphi_1 + \varphi_3)];
\end{aligned} \tag{8}$$

$$\begin{aligned}
& J_{unb} (\ddot{\varphi}_1 + \ddot{\varphi}_3) + m_\theta [(\ddot{\varphi}_1 + \ddot{\varphi}_3) l_3^2 + \ddot{\rho} l_3 \sin(\varphi - \varphi_1 - \varphi_3) + \rho \ddot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \\
213 \quad & + \ddot{\varphi}_1 l_2 l_3 \sin(\alpha - \varphi_3) + 2\dot{\rho} \dot{\varphi} l_3 \cos(\varphi - \varphi_1 - \varphi_3) + \dot{\varphi}_1^2 l_2 l_3 \cos(\alpha - \varphi_3) - \rho \dot{\varphi}^2 \times \\
& l_3 \sin(\varphi - \varphi_1 - \varphi_3)] = M_{unr} - m_{unb} \dot{\varphi}_3^2 l_2 l_3 \cos(\varphi_1 + \varphi_3);
\end{aligned} \tag{9}$$

$$\begin{aligned}
214 \quad & \\
215 \quad & M_{unr} = \frac{2M_r (\omega_w - \omega_{\max}) (\omega_w - \omega)}{(\omega_w - \omega)^2 + (\omega - \omega_{\max})^2}; \tag{10}
\end{aligned}$$

216

217 where  $M_r$  – the rotation torque on the drive shaft, N·m;

218  $M_{unr}$  – the moment of the support on the drive shaft, N·m;

219  $\omega, \omega_{\max}, \omega_w$  – the respective initial, maximal and operating angular velocity of the drive shaft,  
220 rad·s<sup>-1</sup>.

221 Assuming that  $M_r = \text{const}$ , we consider that  $\omega = \omega_3$ ,  $\omega_w = \text{const}$ ,  $\omega_r = \text{const}$ . We are looking for  
222 solutions like:

$$223 \quad \rho(t) = \rho_0 + \rho_1(t), \tag{11}$$

224 where  $\rho_0 = \text{const}$  – the initial value  $\rho$ , m;

225  $\rho_1 = \varphi_3(t) = \omega t$  – the value of  $\rho$  as the function of time  $t$ , m;

226  $\varphi(t) = \omega t + \varphi_0(t)$  – the value of angle  $\varphi$  is due to the rotational movement of the  
227 unbalance.

228 As a result of solving and simplifying the obtained equations, based on the trigonometric  
229 connection of the polar and Cartesian coordinate systems, the obtained regularity of the movement  
230 of the grinding chamber of the mill will assume the form:

$$\left. \begin{aligned}
z(t) &= -\frac{m_{unb}l_3 \sin \omega t}{m_c + m_{unb}} + l_2 \sin \left( \frac{3\pi}{2} + \frac{(m_{unb}m_c l_2 l_3 \cos(\omega t - \alpha)) \cos(\omega t - \alpha)}{(J_c + J_{unb} + m_{unb}(l_2^2 + l_3^2))(m_c + m_{unb})} \right) \\
x(t) &= -\frac{m_{unb}l_3 \sin \omega t}{m_c + m_{unb}} - l_2 \cos \left( -\frac{(m_{unb}m_c l_2 l_3 \cos(\omega t - \alpha)) \cos(\omega t - \alpha)}{(J_c + J_{unb} + m_{unb}(l_2^2 + l_3^2))(m_c + m_{unb})} \right)
\end{aligned} \right\} \quad (12)$$

231

232

#### 233 2.4. Data analysis

234 The processing of the experimental data was carried out using the methodology of planning a  
235 multifactor experiment and a variational analysis of the factor space in the Microsoft Excel,  
236 Statistika 10.0, FluidLab software environments. To assess the impact of the technological  
237 parameters of the developed vibrator upon the efficiency of the grinding process of a bulk raw  
238 material on condition that energy input is minimised, a statistical method of rotatable central  
239 compositional planning of a multifactor experiment was applied (Protasov, 2005).

240 Among the basic criteria for the evaluation of the grinding process of a bulk raw material, there  
241 were chosen the energy consumption of the vibrator  $N$ , kW·h, and the specific surface area of the  
242 initial raw material  $S$ , cm<sup>2</sup>·g<sup>-1</sup>, characterised by the impact of four most significant factors that  
243 determine the kinetics of the particular treatment: vibro-acceleration  $a$ , m·s<sup>-2</sup>, as a complex  
244 parameter of dynamic state of the vibratory system; the diameter of the balls  $d$ , mm and the degree  
245 of loading of the grinding chambers  $E$ , % with the technological filler.

246 The choice of the variation ranges of the factors of functions was made in such a way that any of  
247 their aggregates, as envisaged by the experimental design, could be embodied in these intervals and  
248 did not lead to contradictions.

249

### 250 3. Results and discussion

251

252 Analysis of the obtained dependencies (12) in the FluidLab software environment made it possible  
253 to establish that the maximal dynamic state of the system is provided at a 290-degree angle of the  
254 vibratory drive and the angular velocity of the drive shaft  $120 \text{ rad}\cdot\text{s}^{-1}$  (Fig. 4). With these parameters  
255 the ratio of the components of the amplitude of the oscillations is  $A_z/A_x = 2.5$  times (Fig. 5). In this  
256 case, the displacement amplitude of the grinding container along axis  $Oz = 5.4 \text{ mm}$ , along axis  
257  $Ox = 2.2 \text{ mm}$ .

258 Also, it should be noted that the received graphic dependences of the amplitude of oscillations from  
259 the angle of placement of the vibration drive, have a symmetric distribution relative to the x-axis.  
260 The point of their symmetry is 180 degrees.

261  
262 To verify the obtained analytical results of the theory, a number of experimental studies were  
263 carried out on the efficiency of fine grinding in the developed machine. It was established that the  
264 total amplitude of oscillations  $A$  at  $\omega = 44 \text{ rad}\cdot\text{s}^{-1}$  without a technological load ( $E=0\%$ ) and at  
265  $E=50\%$  and  $E=75\%$  reaches  $3.8 \text{ mm}$ , whereas in the resonance of the investigated vibratory system  
266 in the range  $\omega = 50\text{-}57 \text{ rad}\cdot\text{s}^{-1}$  the value of the required parameter without a technological filler  
267 ( $E=0\%$ ) is  $A = 4.5 \text{ mm}$ ; at  $E=50\%$   $A = 4.2 \text{ mm}$ ;  $E=75\%$   $A = 4.0 \text{ mm}$ . After that curve  $A$  in the range  
268  $\omega = 63\text{-}110 \text{ rad}\cdot\text{s}^{-1}$  stabilises by  $3.7 \text{ mm}$ , regardless of the degree of loading of the technological  
269 filler (Fig. 6).

270  
271 The obtained data made it possible to evaluate the efficiency of the developed design in the context  
272 of minimising the dissipative properties of the investigated vibratory system, the criterion for the  
273 estimation of which being the change in the magnitude of the oscillation amplitude depending on  
274 the loading of the grinding chamber of the machine with the technological filler. This trend is due to  
275 the presence of a peripherally located drive shaft with relatively oppositely placed grinding  
276 chambers of the mill, the forced force of which considerably exceeds the inertial characteristics of  
277 the technological filler. The machine is characterised by angular oscillations of the container



278 relative to the centrally located axis. Therefore, the obtained data on the dominance of the vertical  
279 component do not contradict the concept of the development of the investigated vibrator and ensure  
280 an effective dynamic state of processing the raw material. The analysis of the dependence of the  
281 energy consumption of the mill upon its angular velocity and loading with the technological filler  
282 witnessed (Fig. 7) that the filling degree has no significant effect on the summary energy input.  
283 Thus, in the absence of a technological filler ( $E=0\%$ )  $N = 0.65 \text{ kW}\cdot\text{h}$ ; at  $E=50\%$   $N = 0.68 \text{ kW}\cdot\text{h}$ ; at  
284  $E=75\%$   $N = 0.75 \text{ kW}\cdot\text{h}$ . On the basis of the obtained results one can make a conclusion that  
285 introduction of the peripheral placement of the vibrator drive is one of the key factors for the  
286 minimisation of energy consumption in the vibratory mills of this design solution while maintaining  
287 a significant power potential in the processing of bulk materials. In order to estimate the quality  
288 indicators of the vibratory grinding process of a bulk material processed in the mill, a number of  
289 experiments were carried out to change the dispersion ability of the treated raw materials, caused by  
290 the force of the technological filler (peas, the oat and wheat grains with a moisture content of 8-11%  
291 were used in the experiments). Evaluation of the grinding efficiency was based on the determination  
292 of the specific area  $S$ , the particle size distribution  $\Delta S$ ,  $\mu\text{m}$ , and the proper specific area of the  
293 material particles, which characterises the ratio of their geometric sizes before and after the  
294 treatment. When analysing the change in the specific area of the initial fraction of wheat under the  
295 condition that a vibratory mill of angular vibrations (Fig. 8) is used, it is evident that increase in  $S$   
296 depending on the angular velocity of the drive shaft does not differ significantly, and at  
297  $\omega = 110 \text{ rad}\cdot\text{s}^{-1}$ ,  $t = 55 \text{ s}$  it is  $4300\text{-}4500 \text{ cm}^2\cdot\text{g}^{-1}$ . This trend is caused by a specific form of  
298 vibration, which considerably intensifies the process of active abrasion of the particles of the  
299 material processed.

300 At the same time, with the increase in the specific area of the material, there occurs active decrease  
301 in the proper area of the particles of the processed material and, as a consequence, the time of  
302 technological treatment, depending on the physical-mechanical parameters of the incoming raw  
303 material. On the basis of the obtained data about the kinetics of the pea, corn and wheat grinding

304 process at  $\omega = 110 \text{ rad}\cdot\text{s}^{-1}$  (Fig. 9), one can conclude that the approximate time of their processing  
305 will be  $t = 45 \text{ s}$  at the particle size is: for grinding of peas  $\Delta S = 200\text{-}240 \text{ }\mu\text{m}$ , for grinding of corn  
306  $\Delta S = 100\text{-}125 \text{ }\mu\text{m}$ , for grinding of wheat  $\Delta S = 50\text{-}100 \text{ }\mu\text{m}$ .

307 Along with the obtained kinetic curves of the process under investigation, depending on the  
308 amplitude-frequency characteristics of the developed machine, there are also obtained a series of  
309 dependencies of the influence of the geometric parameters of the technological filler  $d$  upon the  
310 value of the specific area  $S$  of the crushed material (Fig. 10).

311 Based on the obtained data, one can conclude that the use of grinding balls with a diameter  
312  $d = 10\text{-}18 \text{ mm}$  is most expedient for coarse grinding  $S = 3000 \text{ cm}^2\cdot\text{g}^{-1}$ , but, when the degree of  
313 grinding is increased to  $S = 4500 \text{ cm}^2\cdot\text{g}^{-1}$ , most purposeful it is to use balls with a diameter  
314  $d = 3\text{-}9 \text{ mm}$ , which promotes significant increase in the number of their collisions and, as a  
315 consequence, leads to active processing of the raw material. Analysis of the dependence of the  
316 specific area of the initial material upon the degree of loading of the grinding chamber with the  
317 technological filler testified that the highest value of  $S = 4300\text{-}4500 \text{ cm}^2\cdot\text{g}^{-1}$  is observed at  
318  $E = 60\text{-}75\%$ . However, the filling percentage of the grinding chamber with the technological filler is  
319 always connected with increased energy consumption; therefore the choice of optimal operating  
320 conditions should be made by means of a compromise solution for the specific area of the initial  
321 fraction and the efficiency of the experimental machine.

322

323 In order to evaluate the efficiency of the developed machine, a series of experiments were carried  
324 out to study the productivity of the developed angular oscillation mill, depending on the kinematic  
325 characteristics of the drive shaft and the diameter of the sieve holes, under the condition of  
326 previously established operational and design parameters of the developed machine (Fig. 11). When  
327 analysing the dependence of the productivity of the vibratory mill of angular oscillations with a  
328 varying diameter  $d_s$ , it is worthwhile noting the narrow range of its values  $P = 100\text{-}110 \text{ kg}\cdot\text{h}^{-1}$  at  
329  $\omega = 100\text{-}105 \text{ rad}\cdot\text{s}^{-1}$ . However, a significant increase in the productivity  $P$  is observed increasing the

330 angular velocity of the drive shaft  $\omega = 110-125 \text{ rad}\cdot\text{s}^{-1}$ , which leads to intense transportation of the  
331 processed material in relation to the grinding chamber.

332

333 Thus, the maximal value of productivity in the operating mode of the developed mill  $\omega = 110 \text{ rad}\cdot\text{s}^{-1}$   
334 is: for  $d_s = 0.5 \text{ mm}$   $P = 150 \text{ kg}\cdot\text{h}^{-1}$ , for  $d_s = 1 \text{ mm}$   $P = 160 \text{ kg}\cdot\text{h}^{-1}$ , and for  $d_s = 2 \text{ mm}$   
335  $P = 168 \text{ kg}\cdot\text{h}^{-1}$ . Evaluation of the investigated process of grinding of the loose raw material was  
336 carried out on the basis of the results of the analysis of the selected samples (Fig. 12).

337

338 After processing the results of the obtained data in the statistical environment Statistica 10.0,  
339 dependencies of the evaluation criteria upon the investigated factors were obtained in the form of  
340 the second-order multiple regression equations. For the vibratory mill of angular oscillations, the  
341 regression models will have the following forms, respectively for the specific surface area of the  
342 initial raw material  $S, \text{ cm}^2\cdot\text{g}^{-1}$  (13) and the energy consumption of the vibrator  $N, \text{ kW}\cdot\text{h}$  (14), and  
343 for the concerning indicators the following values: the multiple determination coefficient –  
344  $D = 0.82$ ; the coefficient of multiple correlation –  $R = 0.90$ ; the criterion Fisher's –  $F = 5.7$ . Analysis  
345 of the above indicators confirms the high level of adequacy of the regression models.

346 
$$S = -204 - 38.5 \cdot a + 584.5 \cdot d + 55 \cdot E + 1 \cdot a^2 - 80 \cdot d^2 - 0.6 \cdot E^2 + 4.1 \cdot E \cdot d, \quad (13)$$

347 
$$N = 987.3 - 19 \cdot a + 122 \cdot d - 6.6 \cdot E - 0.3 \cdot a^2 - 8.5 \cdot d^2 + 0.7 \cdot a \cdot d - 0.7 \cdot d \cdot E. \quad (14)$$

348 The results of the graphical interpretation of the produced equations are shown respectively in  
349 Figure 13 and Figure 14.

350 By the results of experimental studies and on the basis of the constructed response surfaces of the  
351 obtained regression equations efficient parameters of their operation were determined. At the same  
352 time, a compromise value was found by the Cramer method in the Mathcad 15 mathematical  
353 environment. Thus, at the angular velocity of the drive shaft  $110 \text{ rad}\cdot\text{s}^{-1}$  and the moisture content of  
354 the material 8-11%, the rational values of the vibration acceleration of the grinding chamber are  
355  $45-50 \text{ m s}^{-2}$ , the diameter of the holes in the separation surface is 0.5 mm; the specific area of the

356 material is  $5000 \text{ cm}^2 \cdot \text{g}^{-1}$ ; the productivity –  $220 \text{ kg} \cdot \text{h}^{-1}$ ; energy consumption –  $0.75 \text{ kW} \cdot \text{h}$ , with the  
357 specific energy consumption being equal to  $0.003 \text{ kW} \cdot \text{h} \cdot \text{kg}^{-1}$ .

358

#### 359 **4. Conclusions**

360

361 A mathematical model of a vibratory mill has been developed as a foundation of the theory of  
362 angular oscillations, which made it possible to obtain a functional dependence of the movement of  
363 its grinding chambers and to establish that the maximal dynamic state of the technological filler and  
364 the surface treatment is ensured at an angular velocity of the drive shaft of  $120 \text{ rad} \cdot \text{s}^{-1}$  and the angle  
365 of the vibrodrive placement of 290 degrees. With these parameters the ratio of the amplitude  
366 components of oscillations is 2.5 times.

367 The conclusions of theoretical studies of the process of fine grinding bulk raw materials using a  
368 vibratory mill of angular vibrations have been confirmed by experiments. It has been established  
369 that at  $120 \text{ rad} \cdot \text{s}^{-1}$  and the angle of placement of the vibration drive equal to 290 degrees, the limits  
370 of the rational operating parameters of the vibratory machine will have the following values: vibro-  
371 acceleration  $45\text{-}50 \text{ m} \cdot \text{s}^{-2}$  the vibration intensity  $16 \text{ m}^2 \cdot \text{s}^{-3}$  the grinding chambers 50-60% with a  
372 diameter of the crushing balls 3-5 mm. It has also been established that these technical solutions  
373 will make it possible to produce a material with a specific surface of  $5000 \text{ cm}^2 \cdot \text{g}^{-1}$  at a productivity  
374 of  $220 \text{ kg} \cdot \text{h}^{-1}$  and specific energy consumption of  $0.003 \text{ kW} \cdot \text{h} \cdot \text{kg}^{-1}$ .

375

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377

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380

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460 **Figure captions**

461

462 Figure 1. General view of the device: 1 – microport for attaching the sensor of the accelerometer; 2  
463 – power battery; 3 – memory card; 4 – power button; 5 – adaptive microport for reading data; 6 –  
464 accelerometer housing; 7 – accelerometer

465

466 Figure 2. General view of the vibratory mill of angular oscillations: 1 – electric motor; 2 – flexible  
467 coupling; 3 – shaft; 4 – unbalances; 5 – central axis; 6 – grinding chamber; 7 – traverses  
468 (crosspieces); 8 – feed pipes; 9 – hopper; 10 – discharge pipe; 11 – bearing supports; 12, 13 –  
469 bearing units assemblies; 14 – frame

470

471 Figure 3. Equivalent scheme of the vibratory mill of angular oscillations

472

473 Figure 4. Kinetic characteristics of the grinding chamber of the mill, depending on the angle of  
474 placement of the vibratory drive  $\alpha$

475

476 Figure 5. Dependence of the amplitude of oscillations  $A$  on the angle of placement of the vibrator  
477 activator  $\alpha$ : a)  $A_x$ , horizontal component of the oscillation amplitude; b)  $A_z$ , vertical component of  
478 the oscillation amplitude; c)  $A$ , ratio of the aforesaid components of the amplitude

479

480 Figure 6. a) Total amplitude of oscillations  $A$  and b) vibration acceleration  $\alpha$  of the machine as  
481 function of the angular velocity  $\omega$  of the drive shaft and the loading of the technological filler  $E$ : 1)  
482 without a technological filler,  $E=0\%$ ; 2) at  $E=50\%$ ; 3) at  $E=75\%$ ;

483

484 Figure 7. Dependence of energy consumption  $N$  of the mill of angular oscillations upon the angular  
485 velocity  $\omega$  and loading of the technological filler  $E$ : 1) without a technological filler,  $E=0\%$ ; 2) at  
486  $E=50\%$ ; 3) at  $E=75\%$

487

488 Figure 8. Dependence of the specific area of the wheat particles upon the angular velocity of the  
489 drive shaft  $\omega$  and time  $t$  being in the grinding chamber of the mill: 1)  $\omega = 90 \text{ rad}\cdot\text{s}^{-1}$ ; 2)  $\omega = 100$   
490  $\text{rad}\cdot\text{s}^{-1}$ ; 3)  $\omega = 110 \text{ rad}\cdot\text{s}^{-1}$

491

492 Figure 9. Dependence of the particle size of the processed material upon the time of its being in  
493 the grinding chamber at  $\omega = 110 \text{ rad}\cdot\text{s}^{-1}$ : 1) grinding of peas; 2) grinding of corn; 3) grinding of  
494 wheat

495

496 Figure 10. Dependence of the specific area of the wheat particles processed in the mill of angular  
497 vibrations depending on time  $t$  and diameter  $d$  of the crushing balls: 1) 50 s; 2) 40 s; 3) 30 s; 4)  
498 20 s; 5) 10 s

499

500 Figure 11. Dependence of the output of the angular oscillation mill upon the angular velocity of  
501 the drive shaft  $\omega$  and the diameter of the screen holes  $d_s$ : 1)  $d_s = 0.5 \text{ mm}$ ; 2)  $d_s = 1.0 \text{ mm}$ ; 3)  $d_s =$   
502  $2.0 \text{ mm}$

503

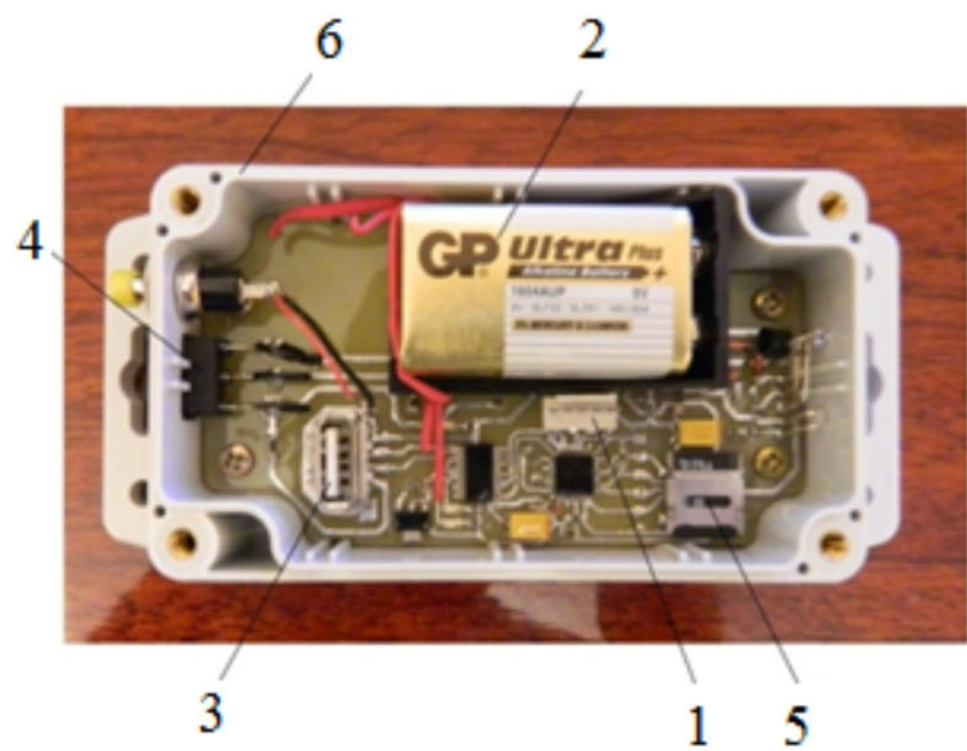
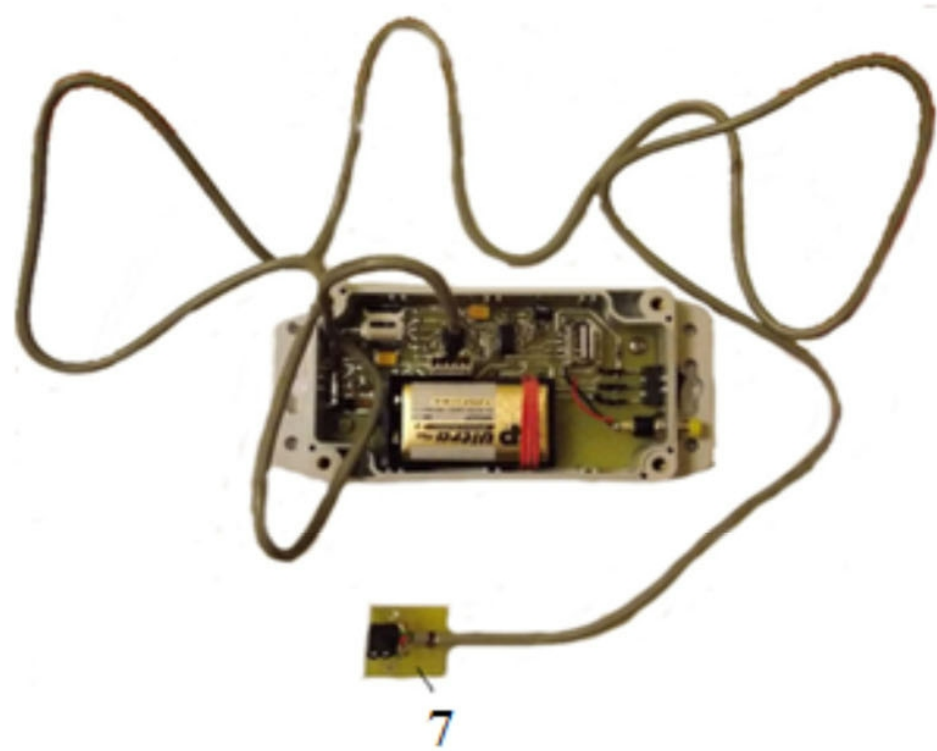
504 Figure 12. The obtained samples of the raw material before and after treatment: a) corn; b) peas;  
505 c) rye; d) wheat

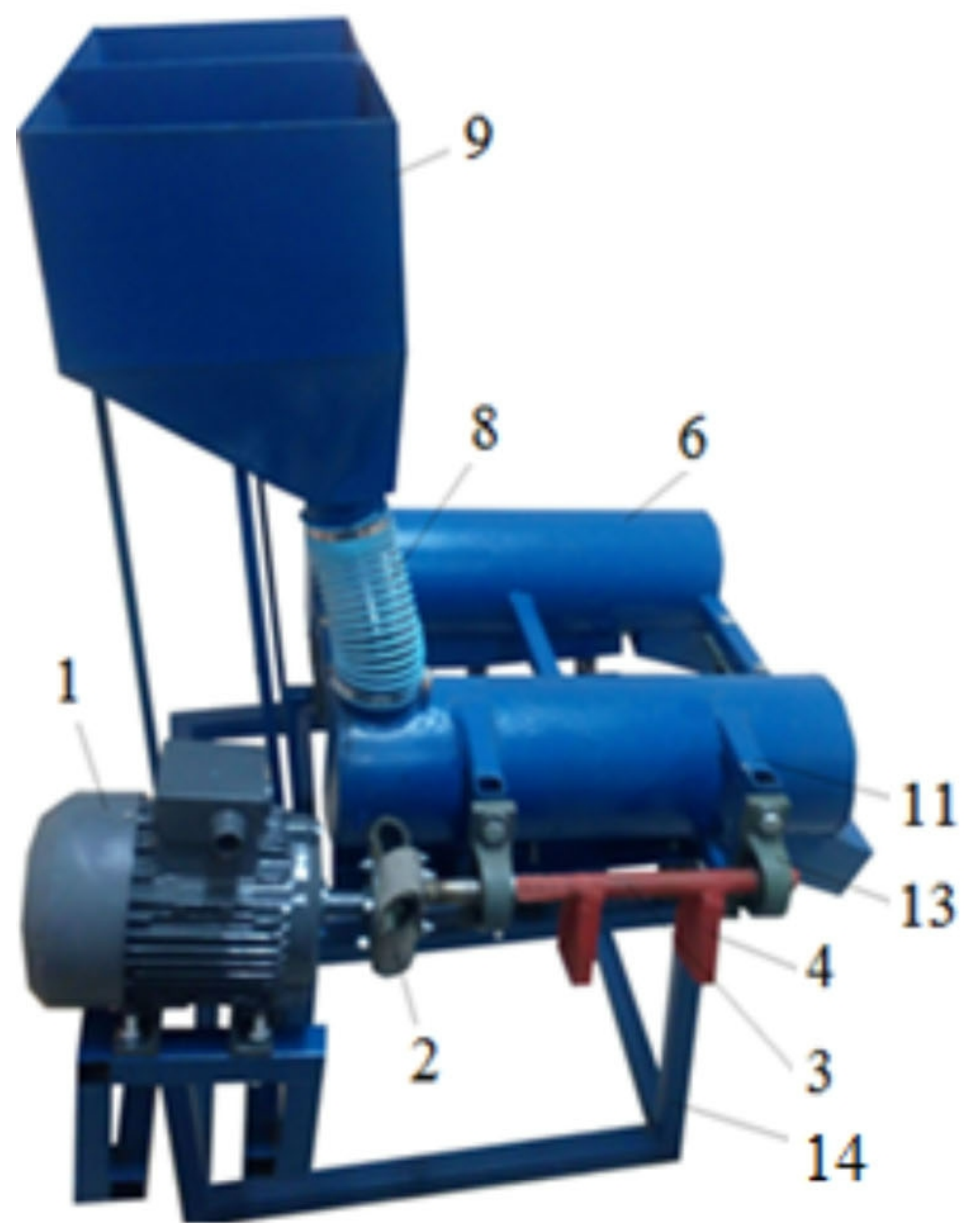
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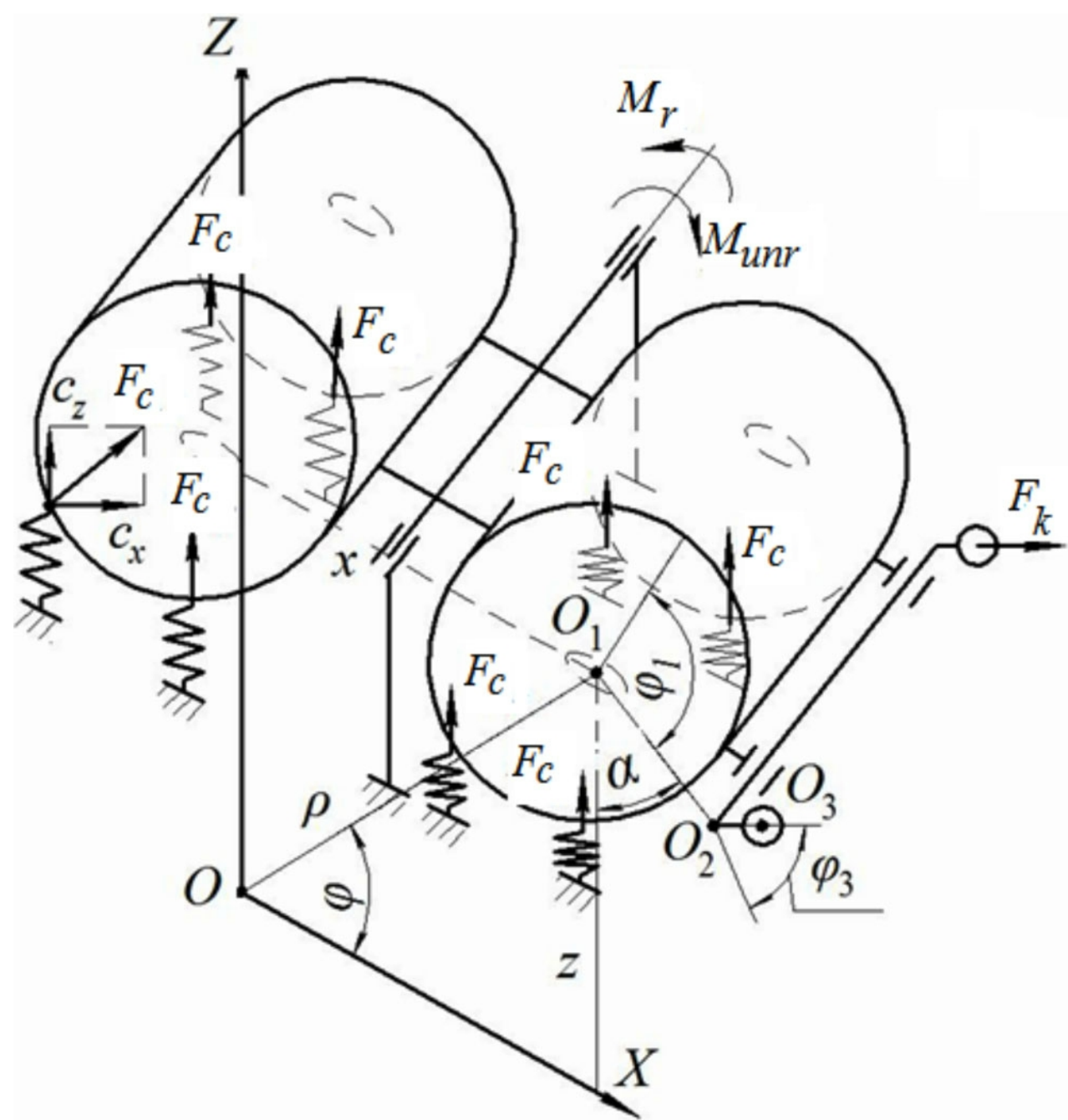
507 Figure 13. Specific area of the material  $S$  as a result of its processing in the vibratory mill of  
508 angular oscillations depending on: a) degree of loading with the technological filler  $E$  and  
509 vibration acceleration  $\alpha$ ; b) degree of loading with the technological filler  $E$  and the diameter of  
510 the crushing bodies  $d$

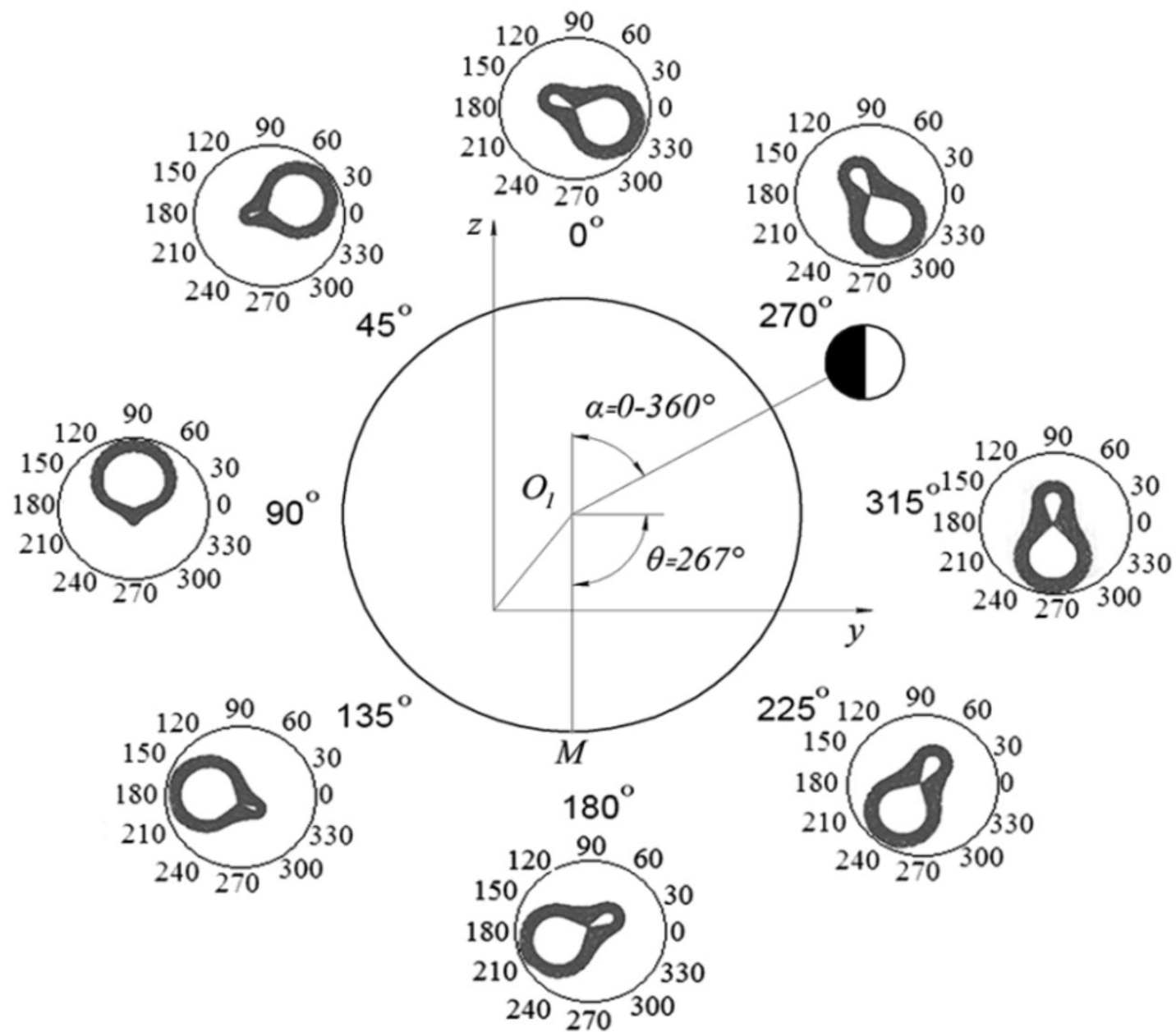
511 Figure 14. Energy consumption of the vibratory mill of angular oscillations depending on: a)  
512 diameter of the crushing bodies  $d$  and vibration acceleration  $\alpha$ ; b) degree of loading with the  
513 technological filler E and the diameter of the crushing bodies  $d$

514

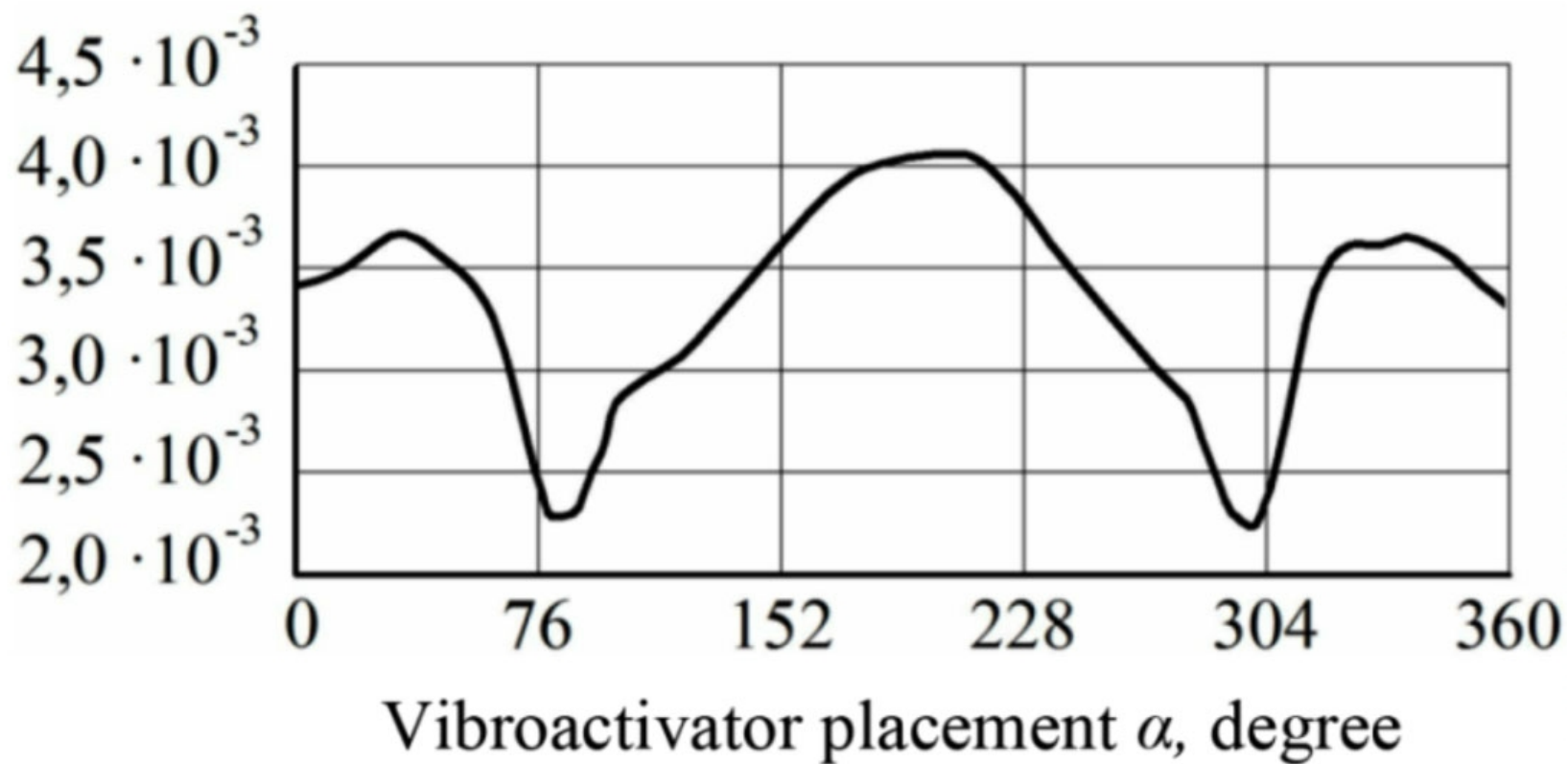




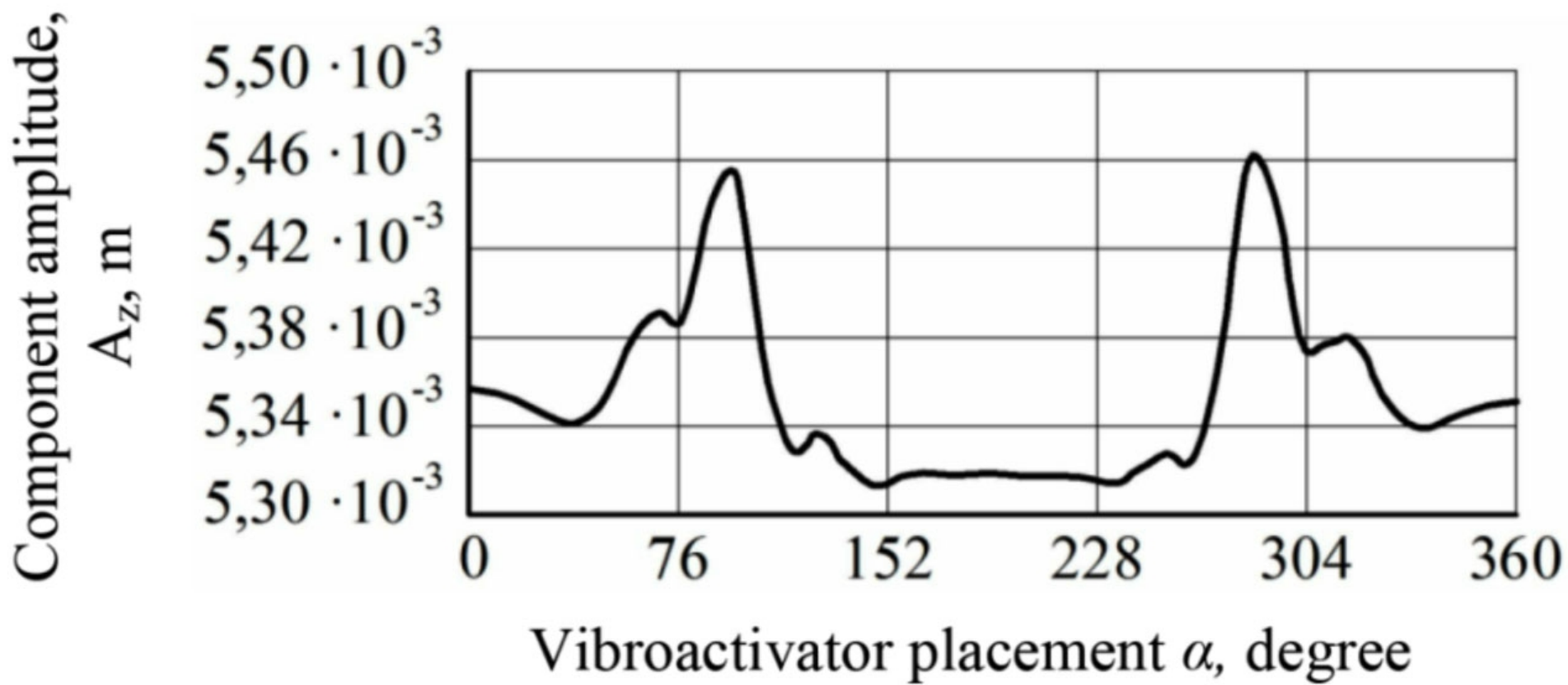




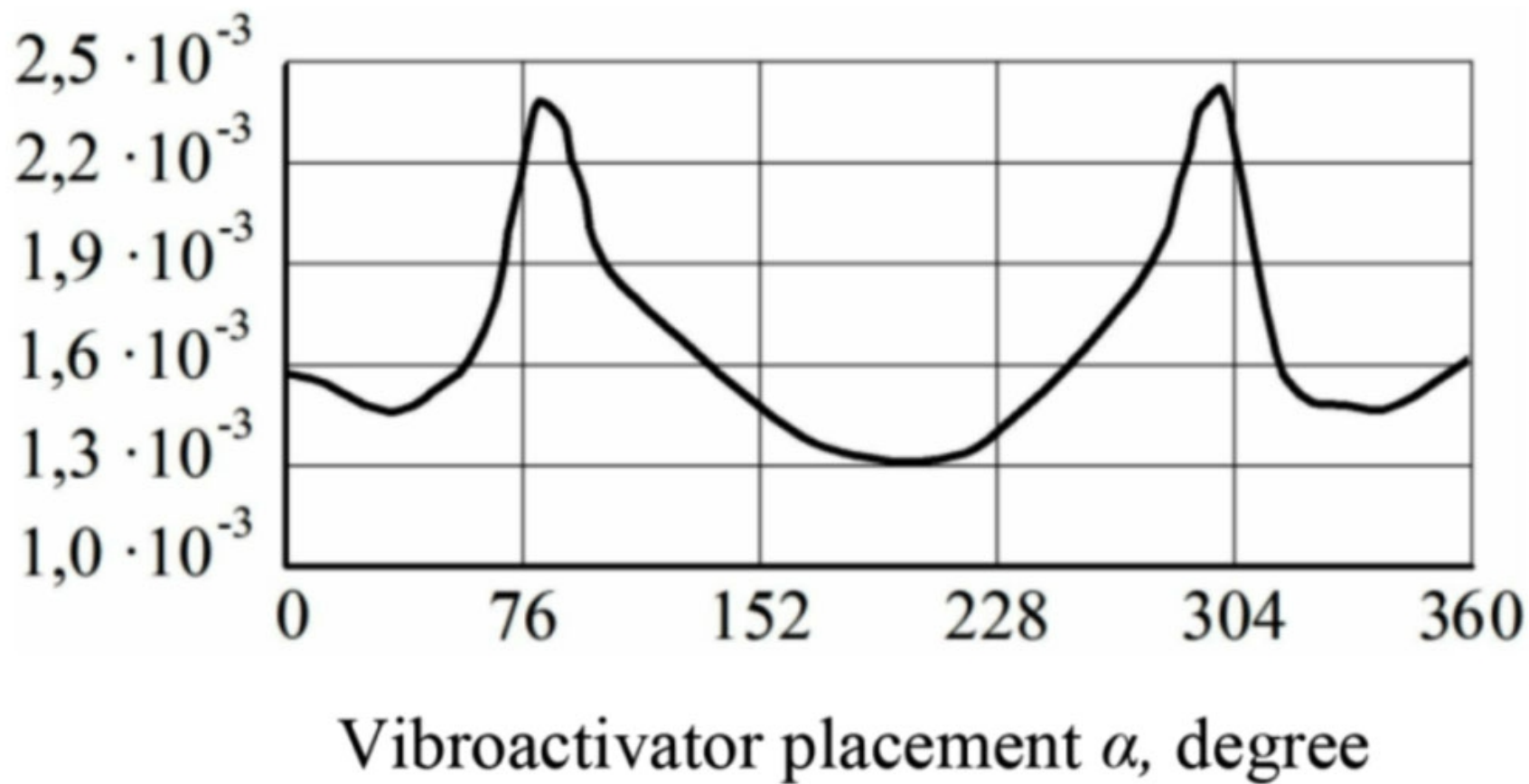
Component amplitude,  
 $A_x, m$

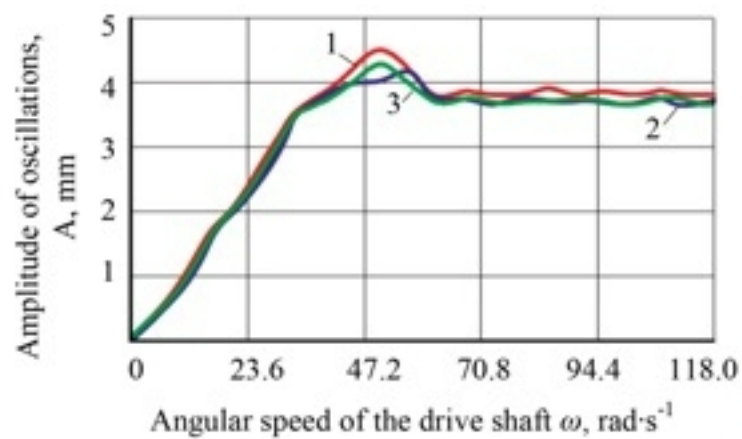


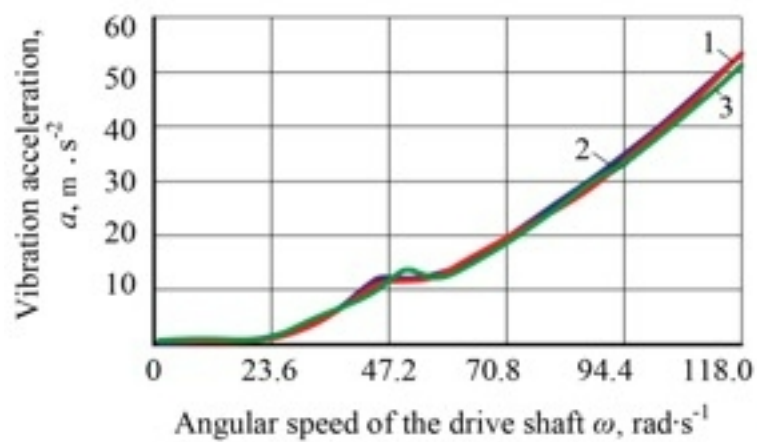


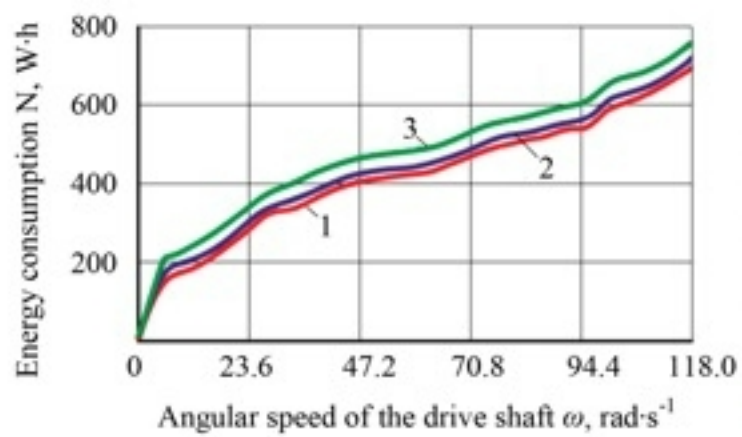


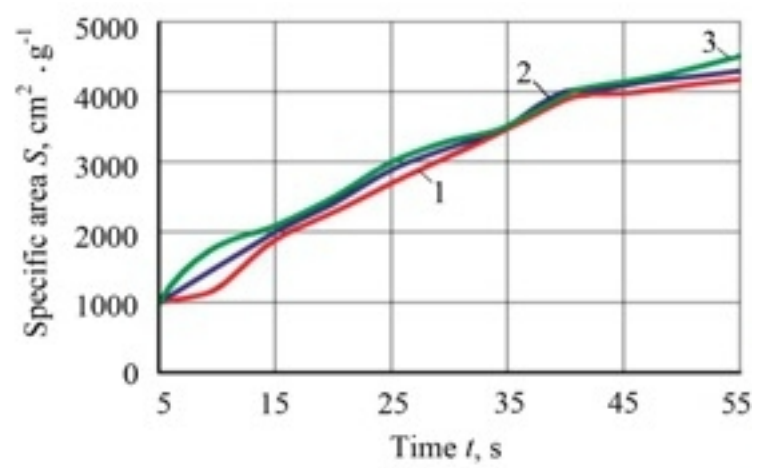
Amplitude of oscillations,  
 $A$ , m

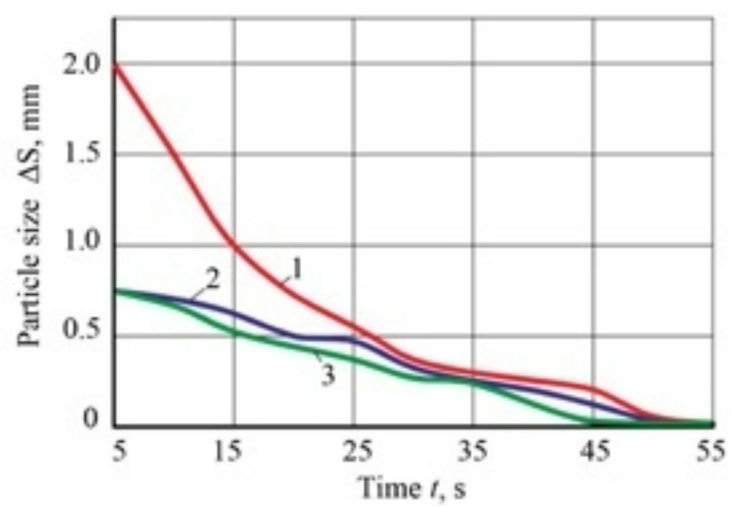


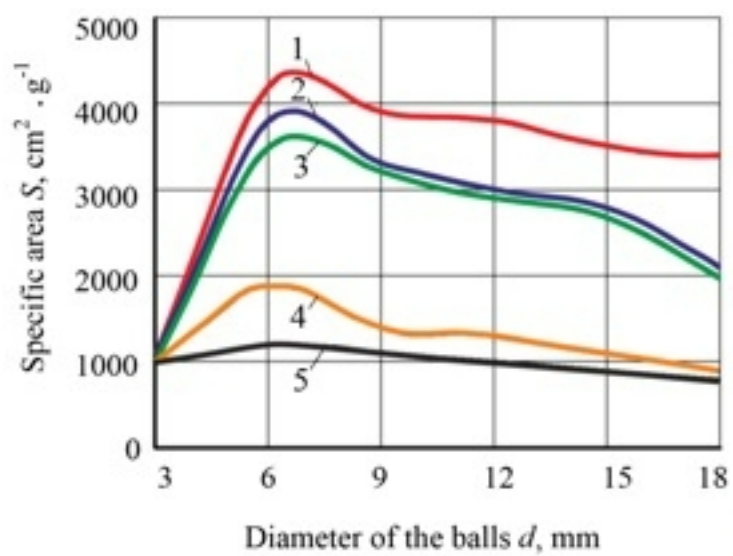




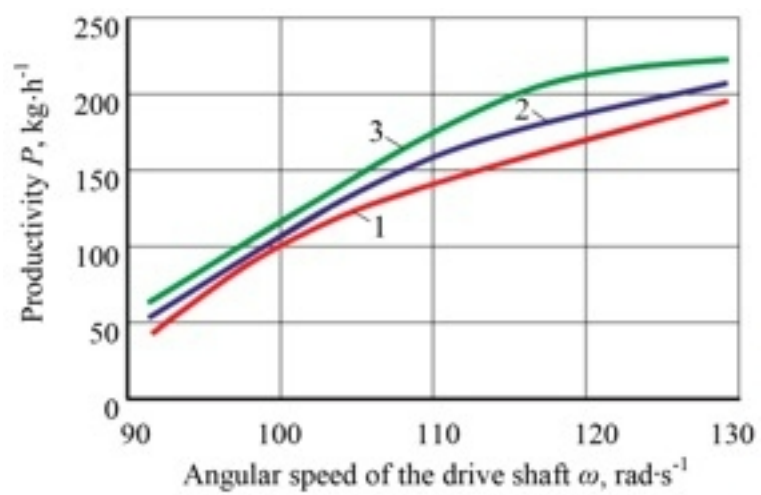














$$S = 1800 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 5300 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 650 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 4300 \text{ cm}^2 \cdot \text{g}^{-1}$$

a)

b)



$$S = 980 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 4100 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 980 \text{ cm}^2 \cdot \text{g}^{-1}$$



$$S = 4200 \text{ cm}^2 \cdot \text{g}^{-1}$$

c)

d)

