Math education master students focusing on teaching mathematics with digital resources

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A teaching activity on rotations has been used in a math education master course in order to provide students with an insight into mathematics teaching with digital resources. The design of the activity was framed by the Theory of Semiotic Mediation taking into account related research results concerning the synergy between manipulatives and digital artefacts at school level. The aim of the study described in this paper is to investigate the potentialities of the designed teaching activity in helping prospective teachers to reflect on the role and the use of digital resources in high-school mathematics teaching and learning.

Keywords: mathematics teaching with digital resources, Theory of Semiotic Mediation, prospective teacher education.

INTRODUCTION

Integration of digital resources in mathematics teaching and learning is one of the main research topics in mathematics education, at least for the last twenty years (Trgalová et al., 2018). The issue is addressed from different prospective such as: design and development of resources; mathematics curriculum development and task design; benefit for students' learning; and, more recently in particular, mathematics teacher education and professional development (Clark-Wilson et al., 2014). With respect to the latter, many research studies are being devoted to this area: to identify the specific knowledge and expertise that is required to efficiently/effectively teach mathematics using digital resources; and to design and evaluate teacher's education in mathematics and professional development programs, aiming to enhance this knowledge and expertise. Within this research field, this paper aims at contributing to investigate the prospective teachers' interaction with digital resources with a dual purpose: on the one hand we focus on their personal reflection on mathematical meanings through the accomplishment of a sequence of tasks involving different kind of resources; on the other, we pay attention to their professional development process in reflecting on the integration of digital resources in mathematics teaching.

To do this, we present a teaching activity and its implementation in a teaching experiment, involving master students in Mathematics (here conceived as prospective teachers). The activity, concerning rotation around a centre in the plane, is described within the framework of the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), highlighting the role of the synergic use of digital and non-digital resources. Some key episodes of the implementation of the teaching activity are described and analysed in order to answer a first research question: can the combined, intentional and controlled use of digital and non-digital artefacts within the activity enhance students' mathematical content knowledge? Moreover, we show some evidence supporting the hypothesis about the potentiality of the use of the activity to develop prospective teachers' professional knowledge and skills. This aims to answer the following research question: may the activity enhance the students' knowledge and expertise on the use of digital resources in mathematics teaching and learning?

THEORETICAL FRAMEWORK

The Theory of Semiotic Mediation (TSM) offers a theoretical framework suitable to design teaching sequences embedding digital resources and to analyse data in order to gain insight into the students' learning process. According to TSM, personal meanings, emerging from the activities carried out with an artefact, may evolve into mathematical meanings, which constitute the objective of the teaching intervention. Fostered by specific semiotic activities, the evolution can occur, in peer interaction during the accomplishment of the task and in collective discussions, orchestrated by the expert guidance of the teacher. Some research studies based on TSM (Mariotti, 2009) have pointed out the fundamental role of the teacher in fostering the construction of mathematical meanings throughout the students' learning process. Within TSM, the design of tasks develops on the base of a fine grain a priori analysis of the solution processes, and specifically on the identification of the schema of utilization that are expected. Moreover, Faggiano, Montone and Mariotti (2018), showed that a potential synergy may occur between the use of different digital and non-digital artefacts, providing a rich support to the development of mathematical meanings. For the purposes of the math education master course at stake, the TSM was chosen to offer to the students a framework to develop their reflection on the role and the use of digital resources in mathematics teaching and learning.

METHODOLOGY

Participants, procedure and data collection

The study was conducted with 12 master students in Mathematics, within a mathematics education course. They were familiar with the notion of rotation as an isometric transformation of the plane in itself, with one fixed point, called centre of the rotation. The teaching experiment was developed in three lessons of two hours each. In the first session students were asked to work in groups of four on a sequence of three tasks on rotation, involving different kind of resources. The second session was devoted to a collective discussion, conducted by the teacher, aimed at allowing personal meanings to emerge and evolve towards the meaning of rotation. The aim of the last session was to collectively discuss with the students the experience they had with the teaching sequence of tasks and to bring them to reflect on the way the activity, conveniently changed, can be developed in a high-school class in order to teach rotations through a synergic use of digital and non-digital resources.

Group work and discussions were videotaped, transcribed and analysed together with the students' protocols. In this paper we present some of the collected data with the aim to provide elements to answer to our research questions.

Overview of the teaching sequence

The aim of the teaching sequence was to focus on the properties of the rotation as a one-to-one correspondence between the points of the plane, described by the centre of rotation, the angle of rotation, and the direction of the turn, and which preserves the distances of each rotated point from the centre. The tasks have been designed according to TSM and combining the use of different artefacts in order to exploit the synergy among them.

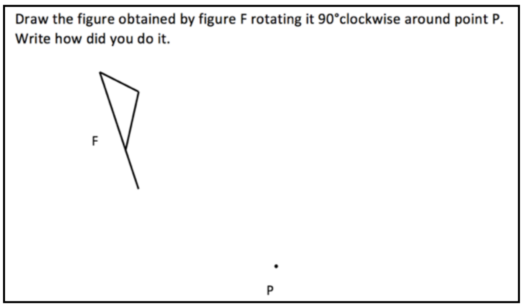

Figure 1a: Task 1

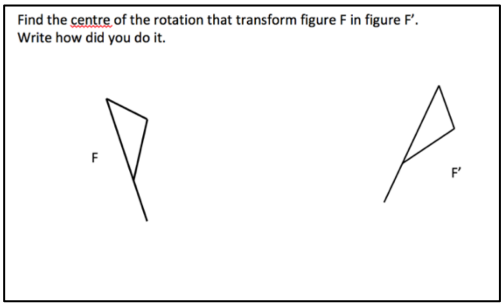

Figure 1b: Task 3

The first task (Fig. 1a) required students to draw the figure obtained by rotating 90° clockwise a given figure around a given (external) centre and to explain the way they did it. In order to carry out the request, the students were given tools such as the protractor, a ruler, set squares and a compass. The aim of this first task was to draw students' attention to the following aspects of the concept of rotation: the idea of *rotation as a circular rigid movement* of a figure around a point –called centre– by a certain angle, expressed by the use of the compass –to preserve the distances from the centre– and the use of the protractor –to rotate all the points by 90°; the idea of *rotation as a punctual correspondence*, expressed by the identification of the rotated points in the intersection points of the arches –drawn with the compass– with the rays –drawn from the centre creating 90° angles; the idea of *rotation as an isometric transformation* which, in particular, preserves the distances and the amplitudes of the angles and transforms segments into congruent segments, expressed by the process to join the obtained rotated points to draw the rotated figure.

In the second task students were required to use a Dynamic Geometry Environment (GeoGebra). They were asked: to construct the segment A'B' obtained by rotating 60° clockwise a given segment AB around a given point P, using the tool/button “Rotate around point”; to observe what moves and what doesn’t move when dragging the extremes of the two segments or the point P and to explain the reasons why it happens. The aims of this second task were: to draw students' attention once more to the meaning of rotation as a punctual correspondence of the points of the plane and as an isometric transformation –by dragging the extremes of the segment AB and observing the resulting movement of the segment A'B'; to highlight the fundamental role played by the centre of rotation, during the construction of the required rotation –by dragging the point P and observing the resulting movement of the segment A'B'.

The third task (Fig. 1b) does not require the use of GeoGebra. Students were given two congruent figures drawn on a piece of paper and were asked to find the centre of the rotation that transforms one figure into the other. The aim of the third task was to draw students' attention to the centre of the rotation as the unique point at the same distance from each pair of corresponding points of the figures. That is, the centre of rotation can be found as the intersection point of the perpendicular bisectors of two segments joining a point of one figure to the corresponding point of the other figure.

RESULTS AND DISCUSSION

The transcripts and the protocols, presented and discussed in this section, were chosen for their features to give evidence of: the emergence and evolution of signs showing the students' enhancement of their mathematical knowledge; and the students' understanding of the role and the use of digital resources in high-school mathematics teaching and learning.

Students' accomplishment of the three tasks

During the first lesson students worked in groups of four, but each of them received his/her own sheet to work on. In order to obtain the 90° clockwise rotation of the given figure around the given point P, all of them immediately started using the compass, pointed in P, to draw four arches passing through the vertices of the given figure. Then they drew the four segments joining each of the vertices with the point P. Finally, as the required rotation was 90°, they drew, starting from P, the perpendicular lines to these four segments. To do that, some of them used the protractor and others the set squares. The rotated figure was identified joining the intersection points of the arches with the perpendicular lines (see. Fig. 2a).

A tablet was given to each of the groups in order to accomplish the second task. As requested, they: opened the given GeoGebra file; used the tool/button “Rotate around point” to create the 60° clockwise rotation of the given segment AB around the given point P; dragged A and/or B and then dragged P; discussed about what happened; wrote their observations concerning the dragging of the elements. The quote below is what one of the students wrote:

> - Dragging the point A, point B and the rotated point B' remain fixed, differently from the rotated point A'. Moreover, the length of the segment remains the same. Varying A, segment AB varies too, such as segment A'B'.
>
> - Dragging point P, segment A'B' translates so that: the 60° inclination between the line through line segment AB and the line through line segment A'B' is preserved, and PA=PA' and PB=PB'.

The text shows the student's comprehension that dragging the extremes of the segment AB gives as a result the movement of the segment A'B, that endows rotation with the meaning of a punctual correspondence and of an isometric transformation. Moreover, the dragging of point P and the observation of the resulting movement of the segment A'B', has allowed the role of the centre of rotation to be highlighted.

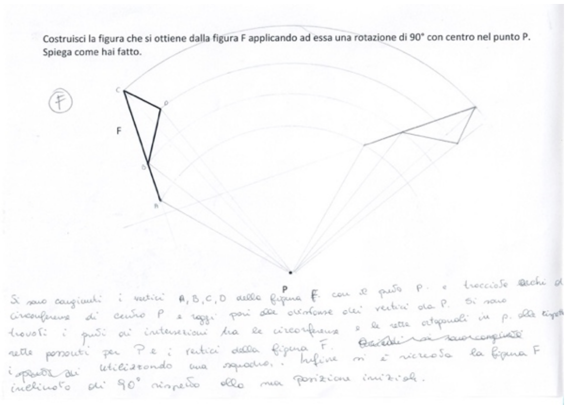

Figure 2a: One of the students' protocols of Task 1

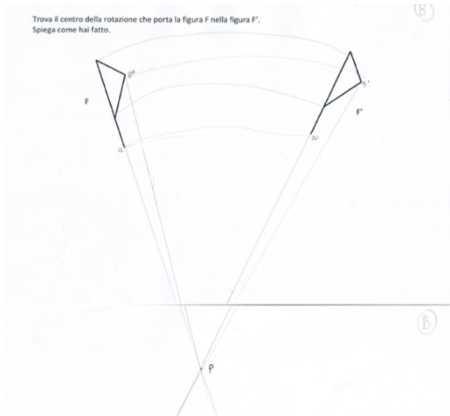

Figure 2b: One of the students' protocols of Task 3

In the final part of the lesson students worked on the third task to identify the centre of rotation that transforms one of the given figures into the other. This resulted in being the most challenging task. In one of the groups, for example, students started reflecting on the idea that a rotation preserves the distances between the centre and each pair of the corresponding points. However, in order to identify the centre, they extended the line through one of the segments of the figure and focused on one of the vertices. Along this line, they looked for the point which has the same distance from the corresponding point of the second figure. In this way, the centre was outside the piece of paper and they decided to take another sheet and extend the lines on that one (see Fig. 2b). To conclude, students didn't succeed in accomplishing the third task, and this became the main focus of the next class discussion.

Class discussion of the three tasks

According to the TSM, during the second lesson, the teacher initiated a class discussion with the aim to focus on the aspects of the rotation on which students were required to reflect on, in order to fully construct the mathematical meaning. In particular, the students' attention was brought to the perpendicular bisector of the segment joining a point of one figure to the corresponding point of the rotated figure. As the centre of the rotation belongs to the perpendicular bisector for any pair of corresponding points, indeed, this allows us to find the centre as the intersection point of two different perpendicular bisectors. For this purpose, students were firstly asked to reflect on how to accomplish the first task without any tools. The required folding of the paper allowed the students to focus on the triangles obtained joining two corresponding points between them and with the centre. In this way, it was easy to pay attention to the fact that these triangles are isosceles.

At this point, the teacher moved on to discuss the second task. Asking students to report on what they had done working in groups during the first lesson, she pointed on what moves and what doesn't move when dragging A or P. Students were able to make the right hypothesis concerning the movement of A' with respect to dragging of A. The discussion on how A'B' moves when dragging P was based on the use of the trace (Fig. 3a). The teacher's aim was to let students give meaning to the "translation" of the segment A'B'. As the following quote shows, students claimed that the segment AB has to have always the same direction as the segment A'B':

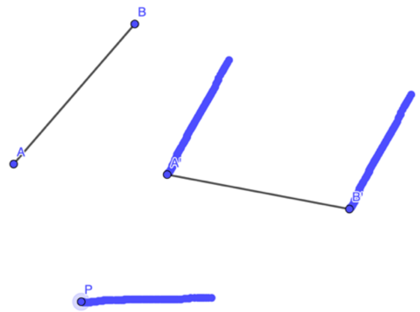

Figure 3a: GeoGebra screenshot taken during the discussion – dragging P on the left

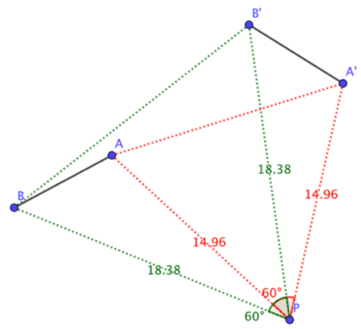

Figure 3b: GeoGebra screenshot taken during the discussion – highlighting the triangles

Giusy: A'B' has to move along the direction of 60° and has to move down as much as I drag on the left in order to keep constant the distances PA' and AP

Felice: it is right that it goes down because if it had gone up the angle would have been contracted

Susanna: indeed, we paid attention to the distances but not to the angles, so [for the segment] to go up the angle contracts, as he said, and the angle is not preserved of course

The discussion continued with the teacher aiming to focus once more on the triangles obtained joining two corresponding points between them and with the centre. Different colours were used to draw the segments joining different pairs of corresponding points with the centre (see. Fig. 3.b). It was an easy consequence, thus, to perceive the centre as the intersection point of two different perpendicular bisectors of any two pairs of corresponding points. This allowed the request of the third task to be accomplished according to the following observations:

Stefano: if we consider the circumferences on which the points move, the centre of the circumference that transform this point into this other will be on the perpendicular bisector of the chord

Susanna: once the points are connected, we should find the midpoint of these segments

Giusy: and so, it comes that what is given by the intersection of the perpendicular bisectors is the centre of the rotation

The excerpt shows how, through the mediation of the digital artefact, the students realised that the centre of rotation can be found as the intersection point of the perpendicular bisectors of two segments joining a point of one figure to the corresponding point of the other figure.

Class discussion on the use of a similar activity to introduce rotations at high-school level

During the last session, students were asked to reflect on the experience they had had with the teaching activity. They recognized that the activity had helped them to give meaning to the notion of rotation, focusing not only on its definition as an isometric transformation but also on its properties. During the discussion, in particular, they paid attention to the relationship between rotations and axial symmetries: it happens, indeed, that, in the attempt to obtain the rotated figure, one of the students folded twice the paper and used a pin. Then the students were required to reflect on the way the activity, conveniently changed, can be developed in a high-school class in order to teach rotations through a synergic use of digital and non-digital resources. They recognised that GeoGebra, and in particular the trace tool, reveals itself to be effective in order to highlight the notion of rotation as an isometric transformation, which preserves the distances and the amplitudes of the angles and transforms segments into congruent segments. However, they believed it was important to start the sequence of tasks with paper and pencil. In particular, they started discussing on the idea to introduce the activity starting from the request to do a double axial symmetry with respect to two lines having in common a point, and they thought to make use of a further artefact, a transparency:

Teacher: so, I ask you to do the two axial symmetries, and then? What are the questions to pose?

Stefano: then I put a transparency on

Mario: what is happened to the figure?

Stefano: putting the transparency on and tracing the starting figure, try to overlap rotating

As the quote above shows, the idea was to guide students to build the notion of rotation as a combination of two axial symmetries with their axes that meet in the centre of the rotation, using paper, pencil and a transparency.

Then they moved to the use of GeoGebra in order to exploit its potential to focus on the dependence of the final figure from the starting one and from the intersection point of the symmetry axes. It also emerges that using GeoGebra to look at the rotation as a double axial symmetry can allow high-school students to easily recognise that rotations preserve the distances of any pair of corresponding points from the centre. However, they underlined the importance of guiding students to focus on the triangles obtained joining two corresponding points between them and with the centre, as they recognised this aspect to also have been important in their experience. It is by means of this observation that they were able to find the centre of the rotation, given the figures, and the third task, as it was, was considered to be a valuable activity in concluding the sequence.

CONCLUSION

The episodes of the implementation of the teaching activity, discussed in the last section, reveal that the combined, intentional and controlled use of the digital and non-digital artefacts enhanced students' mathematical content knowledge. Indeed, the mediation of the artefacts resulted to be fundamental in order to let personal meanings emerge during the interaction with the artefact in the accomplishment of the tasks. These meanings evolved towards mathematical meanings throughout collective discussion. The most evident aspect is the one concerning the characterization of the centre as intersection of two perpendicular bisectors.

Moreover, results show how the activity was useful for the prospective teachers to develop professional knowledge and skills. The teaching activity was the occasion for them to discuss specific mathematical and pedagogical aspects. For example, the notion of rotation as a combination of two axial symmetries was seen as important to help high-school students to highlight the properties of the rotation. The role of digital resources emerged while the prospective teachers interacted with it and reflected on their experience. They experienced how digital resources can foster the construction of meanings and can be integrated by the teacher in order to serve her didactic objectives.

To conclude, we can say that the activity seemed to have enhanced the students' knowledge (in terms of awareness of the various aspects and the properties of the geometrical concept) and expertise on the use of digital resources in mathematics teaching and learning (in terms of semiotic potential of the resources).

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