ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme D
Lisbon, Portugal, 3-7 February 2020

THE COLLABORATIVE WORK ON SCENARIO DESIGN AS A TOOL TO FOSTER TEACHERS' PROFESSIONAL DEVELOPMENT

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In this paper we introduce the idea of scenario design as a method to support teacher professional development (PD). Specifically, we refer to the Meta-Didactical transposition framework to analyse data from a PD programme implemented in Italy, with the aim of highlighting how the collaborative work on scenario design could foster the evolution of teachers' meta-didactical praxeologies. To strenghten this analysis, we present teachers' reflections about the effects that their collaborative work on the scenario design had on their teaching practices and, in general, on their PD.

In this paper we introduce the idea of the *scenario design* as a way of extending the typical activity of task design. We define the scenario design as a process of envisioning possible implementations of mathematical classroom activities in which the interventions of the students and the teachers are made explicit. In particular, this process consists, not only in designing the tasks for students and the teaching methodology, but also in hypotesising possible students' answers to the tasks and hypothetical excerpts of classroom discussion, containing teachers' interventions. The product of this process is an ordered set of scenes - herein called scenario - representing the foreseen development of the classroom activity (i.e. teachers' interventions aimed at supporting students' learning processes, highlighting and discussing their difficulties, activating students' reflections…).

The scenario design has been implemented with a small group of in-service secondary school teachers, who volunteered to take part in a eight months PD Programme focused on teaching methodologies for mathematics education. Herein, our aim is to investigate how the teachers' collaborative work on scenario design triggered and fostered their professional development, while they worked together and also worked with mathematics teacher educators within University Professional Development (PD) Programmes. To develop this analysis, we will refer to the Meta-Didactical Transposition framework (Arzarello et al. 2014).

Theoretical Framework: the Meta-Didactical Transposition

Meta-Didactical Transposition (MDT) is a theoretical framework used to describe mathematics teacher professional development as a process that comprises a number of variables and their possible changes over time, in a dynamic way. It was first introduced in Italy, and then more widely by the international community (Arzarello et al. 2014, Aldon et al., 2013, Prodromou et al., 2018). It is based on Chevallard's Anthropological Theory of Didactics (Chevallard, 1991) - which is contextualised in the teaching of mathematics at school - and considers the relationships and reciprocal influences of teachers and researchers involved in professional development, within their target institutions. The term meta-didactical refers to the fact that important issues related to the didactical transposition of

knowledge are faced at a meta-level. During an educational programme, teachers usually encounter a new didactical paradigm, and, after careful reflection on it, they may reconsider and discuss their practices, and they may also change them. This change is described in terms of evolution of teachers' praxeologies, made of four main components (according to Chevallard, 1991): on the pragmatic side, the task and technique and, on the theoretical side, the technology (term used by Chevallard to mean justification) and theory. If a didactical praxeology refers to teacher's activity into the class, a meta-didactical praxeology refers to teacher's activity in PD programmes. In our case, the task for teachers, given by the researcher (acting as a teacher educator), is to design a scenario. This task has to be solved taking into account specific criteria (the techniques), and justifying the tecniques referring to theories from mathematics education (the technology and theory components).

The possible change in teaching practices is described in terms of changing of the status of the components of teachers' praxeologies. These components, in fact, can pass from external - located outside the teacher's personal world (Clark & Hollingsworth, 2002) - to internal – constitutive of teacher's professional world of practice. Teachers' praxeologies evolve due to a double dialectic (Arzarello et al., 2014) that encapsulates two interrelated processes: (1) a first dialectic, at the didactic level, in the classroom, highlighted through the analysis of teachers' practices; (2) a second dialectic, at the meta-didactic level, within PD programmes, engendered through the shared reflections, developed by teachers and researchers when interpreting the first dialectic.

Method

The PD programme

The PD programme consists of six meetings which took place in the Mathematics Department of Turin University. One of the authors was the teacher educator of the programme who interacted with the teachers. Other two researchers took part to the meetings as observing participants. The first meeting was devoted to presenting theoretical aspects to frame mathematics teaching methodologies. The second meeting was devoted to a "ready made activity", aimed at supporting the teachers' appropriation of the theoretical aspects shared in the first meeting, involving them in a classroom activity where they played the role of students, while the teacher educator played the role of the teacher. The task for the teachers for the next meeting was to work in pairs or groups of three to design a scenario on a given mathematical task for students (see Figure 1). This work has been done using a software to create comics (Lesson Sketch, https://www.lessonsketch.org).

Task for teachers

You are requested to work in pairs or groups of three.

- Design a scenario for implementing the task for students in your class, writing an hypothetical excerpt of a classroom discussion, to be developed starting from possible students' answers.
- The focus of the teacher's interventions should be on fostering a comparison between students' answers, supporting their learning processes, highlighting and discussing their difficulties, and activating reflections .
- Use the Lesson Sketch software to construct the scenarios.
- Share your prepared scenarios with the researchers.

Task for students

On a segment AB, whose length is 10 cm, choose a point P, between A and B, and construct two squares on the same side with respect to the line AB: one with side AP, the other with side PB.
Explore how the perimeter of the entire figure obtained from the two squares varies as the position of P changes on the segment AB.

Figure 1: The task for teachers and the task for students

The third meeting was devoted to discuss the scenario prepared by the teachers. Between the third and the fourth meetings, teachers were requested to re-design the scenarios according to the feedback given during the third meeting. Also, the fourth and fifth meetings were devoted to a collective discussion on the re-designed scenarios. After the fifth meeting, the activities were implemented by teachers in their classes and the final meeting (the sixth) was devoted to a discussion about the way the final scenario mirrored with what really happened when the teachers used it as a script for their classrooms' activities and to final reflections about the PD process.

Participants, data collection and data analysis

Four teachers from scientific oriented secondary school (9 and 10 grades) and one teacher from middle school (6-8 grades) participated in this study. All the teachers, except for the middle school teacher, were familiar with this kind of PD programmes in which researchers and teachers share and discuss didactical ideas together. The mean teaching experience of the participating teachers is 25 years. The group met six times, from October to June. The meetings, conducted by three researchers and teacher educators, were video-recorded and the videos were transcribed. The written scenarios were also collected and analyzed. In the last meeting, the teachers were interviewed and they were asked to reflect on their experience on collaborative scenario design and on the comparison between the scenario design and the actual implementation of the task in their classes. The interview was also recorded and transcribed later. To analyse the data, we describe the changes of the designed scenarios in the course of the PD programme in order to infer the evolution of teachers' praxeologies. Moreover, we refer to the teachers' reflections at the end of the programme to strenghten our inferences.

Results

The data we are going to analyse refer to the third, fourth and sixth meetings of the researchers and the teachers who attended the PD programme. With regards to the third and fourth meetings, we will concentrate on the discussion developed on the scenario designed together by two teachers, Paola and Silvia, who planned to implement the activity in a 10-grade class (upper secondary school with a scientific orientation). Because of space constraints, we will focus on the discussion developed on some scenes of the scenario and on the re-design of these scenes by the teachers.

As stated above, before the third and fourth meetings, the teachers' meta-didactical praxeology is referred to the task of designing a scenario. The teachers do not have a meta-didactical praxeology for scenario design at the beginning of the PD programme (since they never engaged with this kind of task), so we can assume that the techniques at their disposal, after the first two meetings, are the initial criteria for the scenario design (see the task for teachers in Figure 1). In the same way, we can assume that the theoretical part of their initial praxeology is composed by the theoretical frame shared during the first and second meetings (theoretical reflections on the design of tasks to support students' inquiry and on the role played by the teacher in implementing this kind of activities).

At the beginning of the discussion developed during the third meeting, the teachers refered to their initial scenario as a "a pre-draft", because they were aware that it should be improved to become in tune with what was required by the researchers (task for teachers, Figure 1). The teacher educator displayed, through the video-projector, the different scenes of Paola and Silvia's scenario, organized in a PowerPoint presentation.

In *scene three* the teacher poses to the class the question in the task for students (see Figure 1), and, in *scene four*, a pair of students observe: "If P moves away from A, then the perimeter decreases", and the teacher asks to the rest of the class: "Do you agree?". Paola declared that they chose to start with this students' intervention because it reflects the possible result of an initial partial exploration of the relation between the perimeter and the length of the segment AP.

In *scene five*, the students answer with "yes" or "no" to the question posed by the teacher in the previous scene ("Do you agree?"). Specifically, four students answer "yes", highlighting that they do not grasp that the perimeter is not always decreasing, and only one student answers "no". The teacher educator posed a question about what the teacher should do if the answers given by the students were not in tune with this hypothesis: "*if there isn't this 'No', what can you do? And if they all say 'no'*?". In the subsequent discussion the teachers agreed that the scenario should be constructed as a sort of '*tree of possibilities*', with the aim of identifying possible different ways of developing the classroom discussion, according to the actual interventions of the students. This was the first new criterion for the scenario design that was shared during the third meeting. The teacher educator also suggested a possible strategy to make the students discuss answers that were not given by them: the teacher can say that, in another class, a student proposed the answer that she wants to discuss.

In *scene seven*, Paola and Silvia envisioned different students' interventions to support the "yes/no" answers given in the previous scene. Among these interventions, in particular, they hypothesized that a group of students may construct an incorrect symbolic expression "40-2*x*", to describe the relationship between the length of AP (x) and the perimeter of the figure, and may add: "*if x increases, the perimeter decreases*". The discussion of this scene focuses on two aspects: the importance of making students discuss their interpretations of the algebraic expressions and the ways to involve the students, who did not use an algebraic approach, in the discussion.

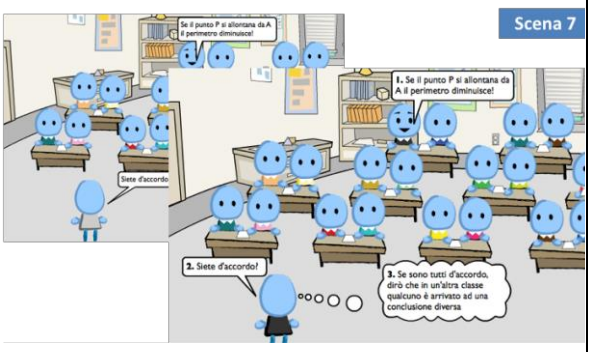
In the subsequent part of the third meeting, another important focus was on the need of making explicit the motivations associated to the different interventions of the teacher, according to the aim of the classroom discussion. As a result of these reflections, a second new criterion for the scenario design was shared, that is the idea of inserting '*thought balloons*' to highlight the reasons why the teacher proposes specific interventions during the discussion: "…*we could insert both what the teacher says and what she thinks*". The aim of this choice is to to make explicit what is implicit, to foster the sharing, at a meta level, of the scenarios.

The third meeting ended with a task for the subsequent meeting: going on in the scenario design and re-design, taking into account all the reflections developed during the discussion between teachers and researchers. The influence of this discussion was evident in the re-design of Paola and Silvia's scenario, discussed during the fourth meeting.

A first evidence of this influence was that Paola and Silvia inserted, in the re-designed scenario, new scenes aimed at highlighting the thinking of the teacher, in particular with reference to choices about the teaching methodology. For example, the thinking of the teacher is envisioned in *scene seven* (*scene four* in the previous version of the scenario), where the teacher thinks about what to do in case all the students agree with an incorrect solution proposed by a group of students ("If P moves away from A, then the perimeter decreases"). The thinking of the teacher displays another important effect of the discussion developed in the third meeting about the ways in which the teacher should react to

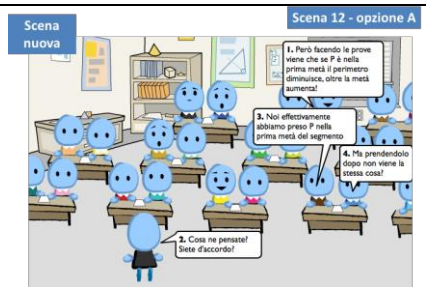
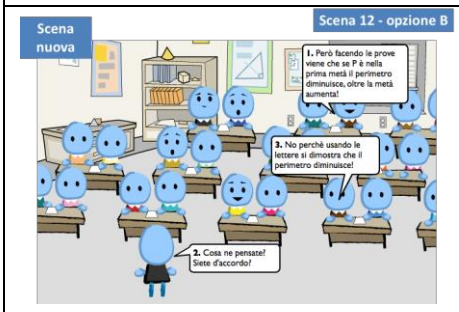
specific students' interventions in order to focus their attention in discussing the most important elements that could support their understanding (in Table 1, the evolution of scene seven).

Table 1: The evolution of scene seven

The two versions of scene seven	Translation of the balloons
	*First version of the scene*: Group: If P goes away from A, the perimeter decreases. Teacher: Do you agree? *Second version of the scene*: Group: If P goes away from A, the perimeter decreases. Teacher: Do you agree? Thinking of the teacher: If they do all agree, I will say that in another class some students reached a different conclusion.
Acknowledgments: The scenes have been constructed by teachers using the depicted tool and the ThExpians B character set in LessonSketch environment (www.lessonsketch.org). Intellectual property for LessonSketch, Depict and the ThExpians B character set belongs to The Regents of the University of Michigan.	

Silvia and Paola also adopted the other new criterion, discussed during the third meeting: designing the scenarios as '*trees of possibilities*'. *Scene 12*, for example, is a new scene introduced by them and represents a first example of '*ramification*' within the scenario. In fact, the two teachers designed two versions of scene 12 (see Table 2), in which the students comment on the results obtained by the group who proposes the expression 40-2*x* as a representation of the relation between the perimeter and the length of AP (in *scene seven* in the previous version of the scenario).

Table 2: An example of 'ramification'

Scene 12 – Option A	Translation of the balloons
	1. Making the numerical examples, we obtained that in the first half (of the segment AB) the perimeter decreases, while in the second half it increases. 2. Teacher: What do you think about it? Do you agree? 3. We actually chose P in the first half of the segment. 4. But, if we choose P in the second half, isn't it the same?
Scene 12 – Option B	
	1. Making the numerical examples, we obtained that in the first half (of the segment AB) the perimeter decreases, while in the second half it increases. 2. Teacher: What do you think about it? Do you agree? 3. No, because using the letters we proved that the perimeter decreases!
Acknowledgments: The scenes have been constructed by teachers using the depicted tool and the ThExpians B character set in LessonSketch environment (www.lessonsketch.org). Intellectual property for LessonSketch, Depict and the ThExpians B character set belongs to The Regents of the University of Michigan.	

In *scene 12 - option A*, one student of the group who directly tried to work on a symbolic expression understands the main problem connected to the incomplete representation they constructed, that is the lack of reference to the whole range of examples. In *scene 12 - option B*, the students who proposed the incorrect expression both rely on their construction, without analysing the expression in depth. Paola and Silvia insterted also other examples of '*ramifications*' (also ramifications with three branches) in their re-designed scenario. Reflecting about the possible 'ramifications' within the scenarios could represent a way to foresee, at the meta-didactic level, gaps between the teacher's intention and what comes to the students' minds (namely, similar to what Aldon (2014) defines bifurcations at the didactical level).

The evolved scenarios include many aspects connected to teaching-learning processes: mathematical aspects (envisioning students' learning processes when facing the task for students and their possible answers), aspects related to the teaching practice (envisioning possible ways in which the students and the teacher could interact), and aspects connected to teachers' justifications of specific didactical choices (making the thinking of the teachers explicit).

The analysis of the evolution of the scenario designed by Paola and Silvia highlights a corresponding evolution of their meta-didactical praxeology referred to the task of carrying out a scenario design. In particular, the technique-component (initial criteria for the scenario design) evolves within the space shared by teachers and researchers during the meetings: as a result, new criteria (the use of 'thought baloons' and the idea of scenario as a 'tree of possibilities') are integrated in the initial criteria, hence the technique-component evolves. Moreover, teachers modified their scenarios taking into account new elements that correspond to ways of questioning themselves when they face the scenario design: in which ways could the discussion be prompted? What could be the best starting point for the discussion? How could I react to specific students' answers? What should I do if this intervention is not proposed by students?

Also the task-component and the technology and theory – components evolve throughout the whole PD programme. An evidence of the evolution of these components of the teachers' praxeology could be highlighted if we focus on the reflections on the scenario design that the teachers proposed during their final meeting (the sixth). The new personal justifying discourses introduced by teachers, in fact, pose the task at a different meta-level. On one hand, they reflected on the role played by the scenario design in supporting their actual interaction with students, as witnessed, for example by this reflection: "*...while the students were working, I re-read my notes, going to see what I had written that I would do, ... it was useful for me to have clear what I wanted to ask and where I wanted to go"*.

On the other hand, teachers also reflected on the role of scenario design as a tool to support their professional development: "*... we have acquired a different way of working, an ease in working, even on topics that have not been analysed or experimented in depth, in a different way, improvising a little more"*. This demonstrates that teachers are aware that they interiorized the way of questioning themselves that characterises the practice of scenario design and that this new habit of mind enables them to become more flexible during their teaching practice.

Final discussion

In this paper we have shown how the collaborative work between teachers and researchers on the scenario design has fostered the evolution of teachers' meta-didactical praxeologies. The task, the

techniques and the theoretical components of teachers' praxeology evolved within the space shared by teachers and researchers during the meetings, both in terms of changing, and of new-entering. This result has been specifically confirmed through the analysis developed in the previous section and by the reflections introduced by teachers during the last meeting, which highlight their awareness of this process. This enables researchers to develop further reflections about the role of scenario design as a methodological tool. The teachers, in fact, said that, thanks to their work on scenario design, they become able to better govern it, because the scenario evolved through their work on it, as the following reflection highlights: "*Having designed everything together makes you feel the activity yours. You handle it well. It is not like replicating something that has been built by others"*. This observation shows that teachers, working together, acquired new competences in developing the scenario design. This fact, from the research point of view, means that their praxeology components passed from external to internal ("yours" in teacher's word). Similar results were highlighted when the same PD programme was implemented in Australia, where the process of scenario design entailed teachers' reflections on their teaching practices while systematically studying the development of scenarios.

The reflections developed by the teachers during the last meeting of the PD programme highlight also their awareness about the fundamental role of their collaboration in the scenario design, as testified in this reflection: "*We also worked together on it. This is the richness of this work… It is different to share with someone and find out more elements*". Moreover, they also declared that an important added value was their collaboration with researchers because "…*from the interventions researchers do, you see that there is a different eye because it is that of someone who sees things from above*".

Another reflection concerns the complexity of the task of scenario design for teachers. All the teachers, in fact, declared that this task was really challenging for them, in terms of both time and mental commitment. As researchers, we have to think that, if, from one side, engaging teachers in scenario design prompts a shared (meta-didactical) praxeology, from the other side, there should be a corresponding didactal praxeology to be used in the class. In fact, scenario design is a process directed to transform design in actions. We are convinced that scenario design is not a guarantee that every scene will be played by the teachers in their classrooms. A pilot study we conducted in Israel aimed at examining the adaption of the scenarios in the classrooms revealed that playing the scenarios several times by the teachers, and reflecting on them after each time, improves the adaption between what is played and what is designed. Furthermore, we also learned that the teaching experience of the teachers determined the number of times the teacher may play the scenario to improve its adaption.

From the researchers point of view, this work on scenario design suggests further theoretical researches. One of these concerns the project of extending the well known idea of instrumental genesis to something that could be called 'meta-instrumental genesis', basing on the fact that the scenario design aims at teachers' education and not at direct students' learning.

Essentially, scenario design can be framed basing on the theoretical lens of the instrumental approach (Rabardel & Bourmand, 2005; Guin, Ruthven, & Trouche, 2005), which, as it is well known, distinguishes between an artefact, available for a given user, and an instrument that is developed by the user through the processes of instrumentation and instrumentalization. A scenario design is a process through which the teachers and the researchers interact with each other, representing, through the scenario (as artefact), what may happen in the classroom when introducing a certain didactical

situation. The scenario design allows forecasting and commenting possible narratives in the way students and teachers can interact with each other while facing a didactical situation in the classroom. The teachers' reflections on these narratives engender a double-level dialectic that supports an evolution of scenarios and, simultaneously, of the techniques used to develop the scenario design, which corresponds to an evolution of the teachers' meta-didactical praxeologies during the PD programme. This process, as the process that occurs in the instrumental genesis, has a dual nature. On the one side, teachers and researchers guide the way the scenario is built and used (instrumentalization). On the other side, the effect of the scenario design is to dynamically transform what could be a usual a-priori or a-posteriori analysis (Artigue, 2009) into the elaborations of fresh scenario's utilization schemes for the didactic situations at stake. Hence the collaborative work on the scenario design stimulates a process of instrumentation by both teachers and researchers.

The further step of our research will be, therefore, to investigate the idea of meta-instrumental genesis, referring also to notions already introduced to analyse the process of instrumental genesis that characterize the teachers' work on artifacts/instruments (double instrumental genesis, by Haspekian, 2014) and on resources/documents (documentational genesis, by Gueudet & Trouche, 2009). Moreover, we aim to study how the relationship between meta-didactical transposition and didactical transposition could support the investigation on the relationship between meta-instrumental genesis and instrumental genesis.

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