ALIGNING EMBODIED AND INSTRUMENTAL EXPERIENCES TO FOSTER MATHEMATICS TEACHING AND LEARNING

Eleonora Faggiano

University of Bari Aldo Moro, Italy; eleonora.faggiano@uniba.it

According to the "embodied instrumentation" approach, recently described by Paul Drijvers in his plenary at CERME11, the aim of this paper is to focus on the idea that embodied and instrumental experiences can be coordinated and aligned. Two examples are presented in an explorative way and the interplay between bodily and instrumented activities is analysed from students' dialogues to show how an integrative approach to tool use could foster mathematics teaching and learning. This first insight into earlier works calls for the need to deeply define the main features of the embodied instrumentation as a promising approach and attempts to sketch a possible direction for further investigations into this new, interesting and relevant topic.

Keywords: Resources and instruments, Digital technology, Embodied cognition, Instrumental approach, Embodied instrumentation

INTRODUCTION AND THEORETICAL BACKGROUND

Learning and teaching with technology still needs inspiration. Despite the spread of technological tools in everyday life and the growing interest of scholars toward the potentialities of digital technology in mathematics teaching, the effects of using these tools for learning seem overall to be modest.

Researchers in mathematics education are aware that technology alone does not solve any educational problems, especially if it is used only as an auxiliary tool, and as such it enhances students' actions without qualitatively transforming them. However, research and practice have shown that the use of technology can play an important role in helping students develop an appreciation and disposition to practice genuine mathematical inquiry, posing questions, searching for diverse types of representations, and presenting different arguments during their interaction with mathematical tasks (Monaghan, Trouche & Borwein, 2016; Faggiano, Ferrara & Montone, 2017). And the importance of these aspects in helping students to view mathematics as something more than a fixed static body of knowledge has already been recognized (see for instance the NCTM Principles and Standards of School Mathematics, 2000).

Moreover, assuming that any cognitive activity is a mediated activity (Rabardel, 1995), as researchers in math education we would like to try to better understand the nature of the mediational role of tools in the learning and teaching of mathematics. The research into the nature of tool mediation is a crucial goal that can also help teachers to understand and become aware of the affordances, the constraints, and the mediating role of technologies as educational resources. In particular, as far as the use of technological tools is concerned, we believe that the bodily engagement of the students needs to be prompted together with the conceptual development of the mathematical meaning which is the aim of the intervention. In the field of mathematics education, indeed, there have been many studies about the use of tools with regard to gestures, sensorimotor experiences and embodied cognition (e.g. Edwards, Radford & Arzarello, 2009; Nemirovsky, Kelton, & Rhodehamel, 2013; Radford, 2014; Sinclair & de Freitas, 2019). They are based on contemporary studies in cognitive science, which highlight that the body actively participates in learning processes and this is connected to the centrality of the action in knowledge processes (Caruana & Cucci, 2017): conceptual learning reveals to be a dynamic and body-centred phenomenon.

A further key view in the theoretical background of this paper is the instrumental approach to tool use (Drijvers & Trouche, 2008). It distinguishes between artefact and instrument (Vérillon & Rabardel, 1995) and focuses on the artefact becoming part of an instrument through the development of associated utilization schemes. The notion of instrumental genesis was coined to reflect the long and complex process (at the same time social and individual, connected to the limits and potential of the artefact and to the student's qualities) during which a student turns an artefact into an instrument, developing schemes that allow him to use the artefact for a well-defined purpose. In this view, on the one hand, the main educational potential of using an artefact in a given situation is firmly related to the schemes and the knowledge that may co-emerge (Artigue, 2002), thus deeply depending on the task. On the other, the instrumentation schemes that students develop depend on the tool use, hence tools are not neutral.

Motivated by the above considerations and inspired by Drijvers's plenary talk at CERME11 in Utrecht (Drijvers, in printing), I'm interested in focusing on possible coordination and alignments of embodied and instrumental experiences. For this purpose, some insights, coming from earlier activities carried out through different teaching experiments, will be presented and discussed. In particular, attention is drawn to highlight the ways in which the instrumental experience and the embodied experience can be intertwined and aligned in order foster mathematics teaching and learning activities.

Herein I will present two examples to reveal how the relationship between embodied and instrumental experiences allows students' reasoning to emerge through action and so mathematical meanings to be constructed. The first example draws from a grade four students' exploration of axial symmetry through the integrated use of manipulative and digital artefacts. It is analysed to show how the artefacts have acted as mediators between the embodied and instrumental experiences and the conceptualization. The second example involves a small group of secondary school students interacting with each other and with concrete objects to find a winning strategy for a combinatorial finite game. This example illustrates how the instrumental and the embodied experiences with the concrete objects could guide further actions when working with a technological tool and allow reasoning to emerge and meaning to be built.

The aim of this paper is not to present a research study explaining the way a certain question on embodied instrumentation can been answered. The discussion of the examples, rather, attempts to address the aim of suggesting insights into possible integrative approaches to technology use, aiming to foster the development of teaching and learning activities. It is also an opportunity to stress the need of a research agenda for further investigations aiming to shed some light and more deeply define the main features of the embodied instrumentation as an integrative approach to tool use.

EXAMPLES OF ALIGNMENT

The verb "to align" comes from the French "a", meaning "to", and "ligne", meaning "line", thus it literally means to bring something into line with something else. In this work, however, I mainly refer to its metaphorical significance. In a broader sense, indeed, the alignment can be seen as a way to change something so that it has a somehow correct or desirable relationship to something else. From this point of view, the examples of alignment I'm going to describe have to be seen as examples of situations by which I intend to show how students experiencing well designed instrumented activities can be engaged in embodied experiences that naturally tend to come into coordination with the instrumented ones, thus resulting in fostering the construction of meanings.

Combined use of different type of artefacts

The first example refers to a teaching experiment for the conceptualization of axial symmetry, conducted with a class of fourth grade students. The aim of the experiment was to investigate the didactic potentiality of a designed sequence of six activities based on the combined use of manipulative and digital artefacts (Montone, Faggiano & Mariotti, 2017). The analysis of some key episodes shows that the combined, intentional and controlled use of the two artefacts may develop a synergy, so that each activity within the sequence enhances the potential of the others (Faggiano, Montone & Mariotti, 2018). What I want here is to show how the construction of the mathematical meaning arose from the coordination and alignment of the instrumental and the embodied experiences.

The manipulative artefact consists of a sheet of paper, with a straight line drawn on it marking where to fold it, and a pin to be used to pierce the paper at the right points in order to construct their symmetrical points. The digital artefact appears as an interactive book with a sequence of activities. It is embedded in a Dynamic Geometry Environment (DGE) and includes tools that allow the construction of some geometric objects (point, straight line, segment, middle point, perpendicular line, intersection point), the “Symmetry”, the “Compass” and the “Trace” tools. As in every DGE, a fundamental role is played by the drag function that, boosted by the tracing tool, allows to observe the invariance of the properties characterizing the figures.

For the purpose of this paper, I will briefly describe here only the first two activities of the sequence. The first activity involves the manipulative artefact and aims at introducing: the meaning of axial symmetry as punctual correspondence and the dependence of the symmetric figure on the folding line. Students are required: to construct a symmetric figure with respect to a given line by folding and making punctures with a pin; to construct the symmetric figure of the first figure with respect to a new given line; to explain what looks the same and what looks different about the two obtained symmetric figures. The second activity involves the digital artefact and aims at focusing on the dual dependence of a symmetric point from the point of origin and from the axis. Students are asked: to build the symmetric point of a point A with respect to a given line, using the button/tool “Symmetry” and call it C; to activate the “Trace” on point A and point C, drag A, drag C, and drag the line; to see, in each cases, what moves and what doesn’t, and explain why.

In what follows I will present and discuss some excerpts taken from the discussion held with the class at the end of the second activity. The first excerpt refers to the very beginning of the discussion when G. was summarising what they have done with the digital artefact.

G. : we clicked on the point and the line and what comes out was the point symmetric to the point A and we called it C

The expression “comes out” seems to reveal that the result has been obtained as the product of the action students have done, namely clicking on the button “Symmetry”. It is the starting point of a semiotic chain which will bring to the meaning of axial symmetry as a point-to-point correspondence and to the fact that the symmetric point depends on the point of origin. And it is followed by V.’s detailed description of what they asked to the artefact to do:

V. : clicking first on the point and then on the line you’re telling the computer: make the symmetric point of this point with respect to this line

They have already experienced the construction of the axial symmetry directly acting on the folded paper through the use of the pin and here they are at the beginning of their instrumental experience with the technology.

During the development of the discussion it becomes clear that what students are performing is a combined embodied and instrumental experience. When the teacher focuses on why, when we move A, the symmetric point C moves too, M. refers to the dynamic process visualized with the digital artefact:

M. : If you move point A only, point C has to move with point A because they must be symmetrical [Fig. 1a]… like, if you move point A higher [Fig. 1b]... point C moves lower... so it is the same [Fig. 1c]… because there must be... the same space... between the two points [Fig. 1d]

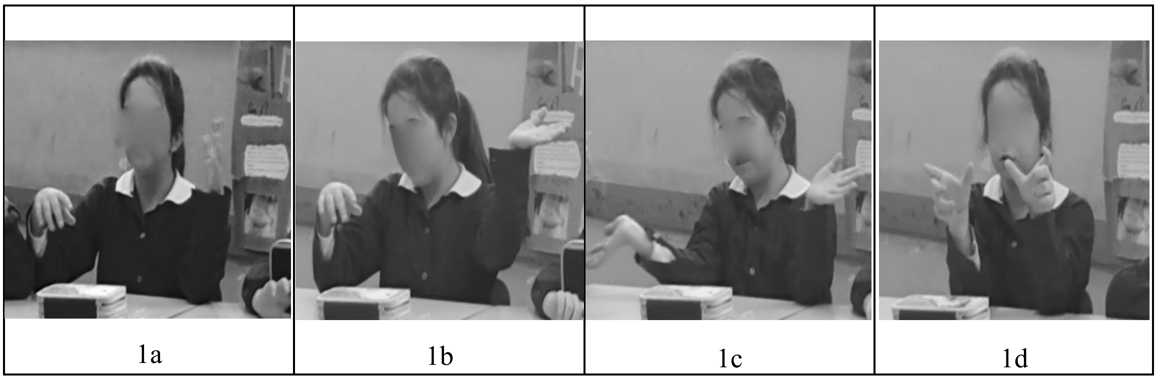


Figure 1. Gestures which matches M. speech (Faggiano, Montone & Rossi, 2017)

The pupil's body learns while acting: when M. refers to the digital artefact she describes and simulates the actions that she performs with her own hands. She identifies herself with what she observed and, moving her arms as lines and her hands as points, simulates drawing in the air the movements of the objects as seen on the screen. She is mentally moving the objects: the implicit dynamism of thinking mathematical objects is made explicit thanks to the didactical functionality of the dragging function, together with the tracing. But it is when the teacher asks how they know that the distance is always the same that the alignment between the embodied and instrumental experiences become more visible: V. asks to and receives from the teacher a sheet of paper and a pin.

V. : If we have a paper that can be folded… we draw a point… I fold the paper and I pierce... I pierce on the point, I make a hole where it is the point and on the other side it comes out a hole… This hole is the symmetric figure of this point…

Teacher. : And what about the distance? The space, as we said before, is the same?

V. : Yes, it is the same!... there [on the screen] you can move the point so I can understand more easily that if I drag the point… the given figure… there is the same distance because by moving it's clear, especially when we move away a lot the point from the line, that also point C moves away… thus there is always the same distance. But I got it even on the paper.

And the role of the coordination and alignment of the instrumental and embodied experiences emerges again at the end when the teacher asks to figure out the reason why, if you move point A, point C moves also but the line does not move [1].

M.: It is the same as on the paper [Fig. 2a]… the folded paper… This for example is the line [Fig. 2b]. If I pierce [Fig. 2c] the paper on point A… [Fig. 2d] and eventually I move also the point A [Fig. 2e], the line is always there [Fig. 2f].

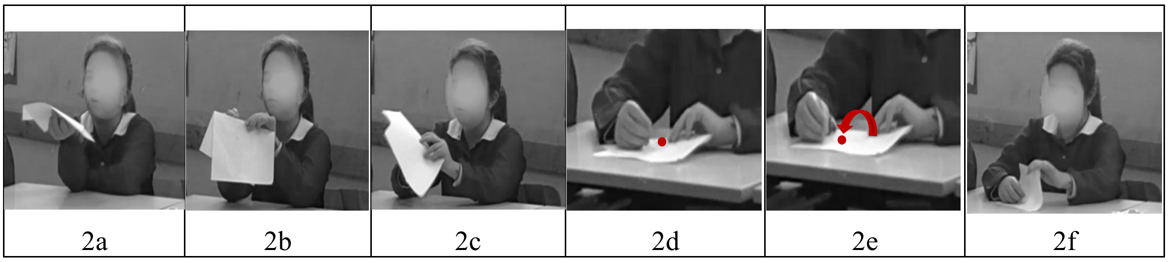


Figure 2. Gestures which matches M. speech

M. here is connecting the instrumental experience with the two artefacts: she underlines that the situation is the same; when she says "and eventually I move" she is linking the act of dragging to the consideration of another point and the act of doing another hole. The schemes related to the two different artefacts have been associated and the role of the embodied experience appears to be fundamental. The coordination and alignment of the experiences allow her to conclude that the line does not move in the digital artefact because it is the same on the paper.

Mediated investigations

The second example refers to a group of four tenth grade students attempting to find a winning strategy to the following combinatorial finite game, which is a version of the well known game called Nim:

> Start with 15 tokens. Two players take turns to remove a whole number from 1 to 3 to the running total tokens on the desk. The player who removes the last token wins the game.

The activity is part of a learning trajectory which aims at involving students in inquiry-based learning. The main idea is creating challenging situations by varying some aspects of a phenomenon while keeping the others invariant (Soldano, Luz, Arzarello & Yerushalmy, 2019). In this way students are guided to grasp the intended object of learning, drawing their attention to critical aspects and fostering inquiry processes: they are engaged in exploring various aspects of the same phenomena, asking questions, raising conjectures, carrying out experimentations and providing explanations.

Students were initially required to play the game with their classmate and to figure out a winning strategy. They were equipped with the 15 tokens and started to play some matches. They quite easily understood that if you leave to your opponent 4 tokens on the desk you win. At a certain point one of the students whispered to his friend:

A.: We can leave them 4 if there are 5, 6 or 7 tokens when it is our turn

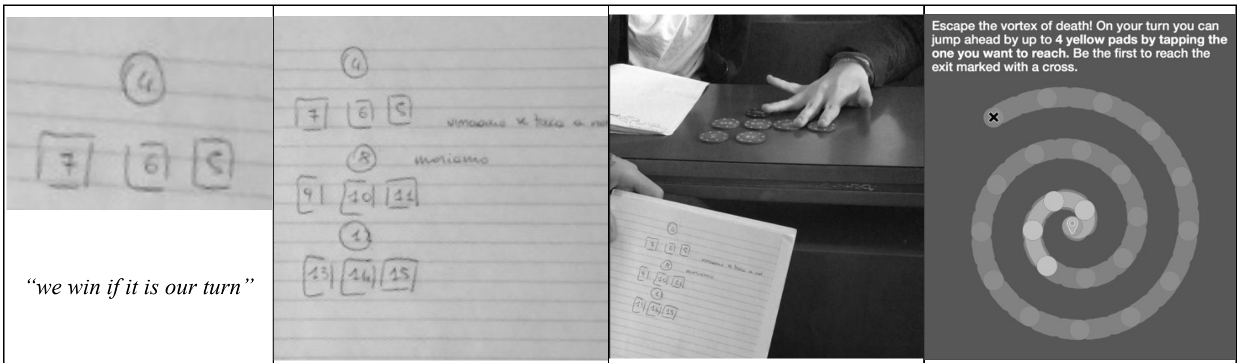
and they started to draw on their notebook a graphical representation of the winning stance and the possibilities to reach it (Fig. 3a). After a while the other student added:

B.: If we leave them 8, as they can remove 1, 2 or 3, in any case there will be 5, 6 or 7 tokens when it is our turn, so we can leave them 4… and we win!

This scheme was than represented as in Figure 3b and became the instrument they used to get control of their moves during the next matches. At the same time, they started to arrange the tokens on the desk so that they were getting in line 4 in a row (Fig. 3c).

The students' behaviour turned out to be important in the next step of the trajectory when, according to the logic of inquiry (Hintikka, 1999), they were asked to explore if the winning strategy they found is still the same when changing the total number of tokens or the highest number of tokens that can be removed at each turn. They decided to play with 17 tokens, removing from 1 to 3 tokens each turn, and in a further step also to vary the highest catch (number of tokens that can be removed), rising it up to 4. The alignment between the embodied and the instrumental experience brought students to arrange the tokens getting in line a number in a row which is one more the highest catch.

But it is mainly during the class discussion orchestrated by the teacher that the alignment between the embodied and the instrumental experiences allows them to focus on the remaining tokens and to figure out that the general winning strategy for the first player consists of removing the number of tokens corresponding to the remainder of the division between the total number of tokens and the highest catch.



3a 3b 3c 3d

Figure 3. Elements of a mediated investigation activity

The final step of the trajectory consists of getting students involved in another version of the game in which the aim is to reach the target of 23, taking turns to add a whole number from 1 to 4 to the running total (Fig. 3d). Students are required to play against the computer (https://nrich.maths.org/397) in order to find out a strategy for beating it. A final discussion, orchestrated by the teacher, aimed at the co-emergence of the entangled instrumental and embodied schemes for the related mathematical knowledge to be revealed. Students were able to tackle with this slightly different problem: winning the game became a matter of conceptual understanding. A detailed analysis of the students' video and transcript is still under study but for the purpose of this paper some interesting elements concerning the intertwinement between the instrumental and embodied experience have already emerged. Further analyses will be devoted to focus on the role of the digital tools in this trajectory: results would be important to inform a new way to design tools and tasks taking into account the opportunity to exploit the role of body in fostering mathematical conceptual development.

CONCLUDING REMARKS

Through the examples I discussed above, I aimed to highlight how the interplay between embodied and instrumental views can be coordinated and aligned in order to foster mathematics teaching and learning. Mathematical cognition could emerge throughout a bodily-based instrumental genesis grounded on embodied experience and on the related development of both cognitive and sensorimotor schemes.

The first example shown as the synergic use of the two different artefacts seems to have strengthened both the mathematical and didactical potentialities of the digital artefact. The instrumental experience with the use of the DGE was amplified by the embodied experience students had with the manipulative artefact. The second example highlighted the emergence and evolution of the alignment between instrumental and embodied schemes in tackling with investigative challenges.

In both the examples, however, it is important to stress that the alignment of the experiences was mainly fostered by the type of task (open and investigative, finalised to the discovery of the properties and the construction of the mathematical meaning) and by the teacher's behaviour during the class discussion. In this sense the discussion of this example aims at inspiring teaching in order to design and develop activities suited to foster meaningful learning trajectories.

Through an integration of many modalities including sensing, perceiving, acting and observing, we can say that higher cognitive structures emerged from recurrent patterns of perceptually guided actions. Although the arguments presented in this paper need to be further investigated, they seem to resonate well with the embodied instrumentation approach introduced by Drijvers and with the following thought:

> The language of things puts back together the physicality of the world through the impalpable but revealing net of gestures, utterances, and formulas (Arzarello's adaptation from I. Calvino, American Lectures, Exactness, 1988, in occasion of the Turin Workshop on Semiotic, April 2017).

Further investigations and empirical evidences are required to shed some light and to more deeply define the main features of the embodied instrumentation as an integrative approach to tool use.

NOTES

1. The data come from a research project presented in 2017 during the XXXIV AIRDM Italian Seminar on Research in Mathematics Education (https://www.airdm.org/sem_naz_2017_29.html) by E. Faggiano, A. Montone and P. G. Rossi and developed also in collaboration with M.A. Mariotti.

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