An Experimental Study on Sequential Auctions with Privately Known Capacities

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Abstract

We experimentally study sequential procurement auctions where bidders' capacity constraints are private information. Our experiment involves two first-price auctions with a belief elicitation stage at the end of the first. Our results show that (i) observed behavior in the second auction is overall consistent with sequential rationality; (ii) first auction bids are decreasing in the capacity of the bidder, but (iii) stated beliefs are inconsistent with the actual play. Hence, subjects seem to be aware of the opportunity cost of early bids (which leads capacity constrained bidders to bid more cautiously than unconstrained ones); on the other hand, since they do not recognize the informative content of bids, the potential signaling cost associated with early bids does not come into play.

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1 Introduction

Several goods and services are procured or sold through auctions that are run in sequence. The most prominent example is represented by electricity markets: in these markets, the delivery of electricity is usually procured by means of auctions, the day-ahead market and the intra-day market, where sellers commit to deliver a certain amount of power in a specific time interval the next day or in the same day, followed by a real-time auction, the balancing market, meant to secure real time balance between actual demand and supply. Other examples of sequential auctions include the sale of spectrum rights, oil and gas leases, greenhouse gas emission permits and treasury bonds.

The distinguishing feature of sequential auctions is that the outcome of one auction may alter the setting in which the following auction takes place and/or may convey additional, though imperfect, information on some relevant elements of the environment. This introduces a strategic linkage between the auctions, as rational bidders should anticipate that their behavior in one auction will, directly or indirectly, affect their payoffs in the next.

In procurement contexts, this strategic linkage is certainly relevant when sellers have capacity constraints. This has been documented empirically by Jofre-Bonet and Pesendorfer (2000, 2003) and De Silva (2005), who show that, in auctions for road construction contracts, firms that won previous auctions typically participate less and bid less aggressively in later auctions. The idea is that, given that completing a project requires several months whereas new contracts are auctioned off at high frequency, a firm that is awarded a contract, having more committed capacity, may not have the necessary resources to carry out future projects, or can obtain them only at a relatively high cost. In other words, when firms have capacity constraints, winning an early auction entails an *opportunity cost*, as the firm will lose the opportunity to effectively compete in the next, where market conditions may possibly be more favorable. Inspired by these findings, Brosig and Reiß (2007) and Saini and Suter (2015) have then tested in the lab whether subjects do indeed properly account for this opportunity cost of early bids. However, these papers assume that bidders' capacities are common knowledge, thus potentially missing other concurring strategic effects. In particular, when bidders may have limited capacities – but this information is privately held – the opportunity cost of winning an early auction for capacity constrained bidders interplays with a *signaling cost*: bidders that are far from their capacity limits should anticipate that their bids might signal their actual capacity, thereby affecting the intensity of competition in future auctions. This adds complexity to the bidders' tasks and makes the analysis of observed bids more challenging.

To investigate how bidders react to these strategic forces, we design an experiment with two sequential first-price auctions, each involving a single unit, and two sellers, who may have one or two units to sell: hence, the winner of the first auction will bid in the second only if her initial capacity was two units. Whereas the information on capacity is privately held, costs are common knowledge and, for simplicity, normalized to zero. At the end of the first auction, the outcome and the winning bid are disclosed, and bidders reveal – through an incentive compatible mechanism – their beliefs on the opponent's (initial) capacity. To match real-world situations where firms are unsure about the demand or even the occurrence of future auctions, we also introduce (exogenous) uncertainty about the realization of the second auction. We consider four treatments, which differ in the ex ante probability distribution of bidders' capacities and in the degree of uncertainty about the second auction's implementation.

Our preliminary theoretical analysis shows that, while the opportunity cost of early bids pushes toward a separation of bids in the early auction – all else equal, bidders with only one unit to sell have an incentive to bid more cautiously than those with a two-unit capacity – the signaling cost is so strong that, in equilibrium, bids are, at least partially, pooled: a bidder with a capacity of two units prefers to significantly reduce her chance of winning the first auction than to fully reveal her type.

Our experimental results can be summarized as follows. Bidding behavior in the second auction matches the main predictions associated with the Perfect Bayesian Equilibrium of the game: in particular, a subject who lost the early auction tends to (rationally) bid the more aggressively, the higher the likelihood she attaches to her opponent having an initial capacity of two units. This implies that, for a bidder with two units to sell, revealing her own capacity would be costly, as this would intensify competition in the later auction. However, the joint analysis of the behavior in the first auction and the belief elicitation phase, shows that this signaling cost is only potential, as bidders seem to be unable to appreciate the informative content of bids. In fact, although first auction bids are significantly and negatively affected by capacity, beliefs do not respond accordingly. In particular, beliefs, net of a partial reversion to the 50/50 odds, are, on average, aligned with prior probabilities. Therefore, our evidence suggests that, while subjects seem to properly account for the opportunity cost of early bids – capacity constrained bidders bid more cautiously than those who are not – the signaling cost of early bids, which should countervail the opportunity cost, is essentially absent.

In an attempt to shed light on the behavioral reasons behind the imperfect belief updating process by subjects, we also show that winners of the first auction (who are not informed of their opponent's bid) seem to follow an irrational gambler's fallacy heuristic. On the other hand, losers, who observe the opponent's (winning) bid, are not affected by the gambler's fallacy; yet, they do not seem to recognize the link between capacity and bids, as if the information conveyed by the opponent's bid were not strong and clear enough to prompt a belief revision.

We believe, that, beyond its theoretical interest, investigating the consequences on behavior of private information about capacity constraints has also practical relevance. Hortaçsu and Puller (2008) claim that, in electricity markets, it is realistic to assume that generation costs are common knowledge across firms. Instead, it is less likely that firms have accurate information regarding each other's available capacities at the time of bidding. This is because generating firms that participate in the auctions usually also trade electricity through bilateral forward contracts with electricity users: it is unrealistic to believe that firms perfectly know the exact contract positions of their rivals at the time of bidding. Similar considerations are likely to be applicable also in other procurement markets, especially those in which firms, beyond competing for contracts tendered by public authorities, also operate in the private sector. In these contexts, our paper suggests that the information that is endogenously generated in the market may trigger significant behavioral effects. In this sense, we offer novel insights into the possible determinants of the deviations from equilibrium bidding that have been documented by the empirical literature on electricity and recurring procurement auctions (see Section 2 below).

Notice, in addition, that the strategic incentives faced by sellers in our setting are similar to those confronted by financially constrained buyers in a standard sequential auction: on the one hand, a bidder with small budget cannot effectively bid in later auctions if she depletes her budget early on (this is the opportunity cost); on the other hand, a bidder with large budget should anticipate that, should the other bidders learned that her budget is indeed large, competition will be fiercer later on (this is the signaling cost). Hence, the insights from our paper easily extend to those auction markets where buyers' budgets are limited and privately known, an assumption that is realistic in many circumstances, as argued by several authors (see, e.g., Salant, 1997; Ghosh, 2013; and Ghosh and Liu, 2019).

The rest of the article is organized as follows. Section 2 reviews the related literature. Section 3 presents the experimental design and the theoretical predictions under standard bidders' preferences and equilibrium behavior. Section 4 analyzes the experimental results. Section 5 discusses the results, focusing on the belief updating process by subjects. Finally, Section 6 concludes.

2 Related Literature

In the benchmark model of sequential auctions (see Milgrom and Weber, 2000), all bidders are assumed to have unit demand and private valuations.[1] This model highlights that, when a bidder has limited demand, the following trade-off emerges: a bidder active in a certain round knows that, if she does not win the current auction, there will be one less opportunity to get the unit on sale; on the other hand, in the next round, she will face fewer and weaker (i.e. with lower valuations) competitors. The remarkable result, which follows from an arbitrage argument, is that these two forces perfectly offset, leading to the *law of one price*: expected prices are equal across different rounds.

Inspired by the observation that, in the real world, when homogeneous goods are sold sequentially, awarding prices seem rather to decrease over selling rounds (see, e.g., Ashenfelter, 1989), the subsequent literature on sequential auctions has mainly concentrated on finding theoretical explanations to this *declining price anomaly*. These contributions provide conditions on bidders' preferences or on the market setting such that this price path can arise in equilibrium.[2]

Keser and Olson (1996) are among the first to document the decreasing price anomaly in the lab. They argue that bidders' risk aversion can only partially explain the observed pattern of bids. Neugebauer and Pezanis-Christou (2007) run an experiment involving a sequence of first-price auctions where, in each round, bidders know that the probability that there will be another auction is smaller than one. Their results provide support for the declining price path (they find that the larger the uncertainty over supply, the higher the decline in average purchasing prices); however, they record quantitative deviations from the theoretical predictions, which they attribute to a distorted perception by bidders in the likelihood that another auction will take place later on.

That capacity constraints/limited demand may constitute a crucial determinant of bidders' behavior in sequential auctions became clear after the appearance of a few empirical papers. De Silva (2005), using data from repeated procurement auctions of road construction

[1]It is worth remarking here that, whereas in standard auctions the auctioneer is the seller and the bidders are the buyers, in procurement auctions it is the opposite. A procurement auction with capacity constrained bidders corresponds to a standard auction with limited demand bidders. A bidder's cost in a procurement auction plays the same role as a bidder's valuation in a standard auction.

[2]Deltas and Kosmopoulou (2004) provide a summary of these explanations. Interestingly, based on their empirical exercise involving rare book sequential auctions, they argue that none of the proposed equilibrium theory can fully account for the price trend they observe and that nonstrategic (i.e. behavioral) factors are likely to matter. In particular, they attribute the observed bidding pattern (which displays increasing prices and decreasing probability of sale) also to a limited attention effect related to the order in which the items are presented in the catalogue.

contracts in Oklahoma, shows that firms with more committed capacity have lower probability of winning. Similarly, with reference to the construction market in California, Jofre-Bonet and Pesendorfer (2000, 2003) show that firms with higher backlogs (measured as the dollar value of the amount of work that is left to do from previously won projects) are less likely to submit a bid, and when they do, they make higher bids. They also fit a structural equilibrium model to their data in order to estimate firms' production costs. The model assumes that backlogs are always common knowledge and that an increase in a firm's backlog (resulting from winning one auction) shifts her cost distribution, whose actual realization is then privately observed. Estimates confirm that the effect of backlog on costs is positive and substantial: hence, winning the current auction entails an opportunity cost in terms of lower future profits, that firms (should) rationally take into account by bidding well above their production costs. Finally, they show that the model fits pretty well for fringe (i.e. small) firms, but there is a small though significant difference between observed and predicted bids for regular (i.e. large) firms which, as they argue, can be attributed to behavioral factors. Hortaçsu and Puller (2008) adopt a structural approach to study bidding behavior by generators in the Texas electricity market. In their model, generation costs are assumed to be common knowledge, whereas, upon bidding in the balancing market, available capacities are private information, as a firm does not observe how much power other firms have already committed through bilateral forward contracts. They derive and estimate equilibrium bid schedules in the balancing market (firms decisions regarding bilateral contracts and in the day-ahead market are considered exogenous), which are then compared to data. Their main result is that, while large firms' behavior is broadly consistent with equilibrium, small firms' observed bid schedules are steeper than equilibrium ones, so that these firms are not called to supply much power even if it were optimal to do so. They hypothesize that this difference may be due to some sophistication cost, i.e. the cost associated with establishing a complicated trading operation. Interestingly, the observed deviations from theoretical benchmarks generate relevant efficiency losses.

The empirical evidence described above suggests that firms' capacity constraints significantly affect competition in auctions: in particular, a higher committed capacity, possibly as a result of winning an earlier auction, may prevent the firm from competing effectively in later auctions.[3] A few subsequent papers have then tested experimentally whether this opportunity cost is properly accounted for by individuals. In particular, Brosig and Reiß (2007) consider a sequence of two procurement auctions where each bidder has a capacity of one unit, and this is common knowledge; bidders' production costs are private information, but they are in general different in the two auctioned projects. In their model, a bidder, knowing her costs for the two projects in advance, may want to skip the current auction if her cost for the second project is (sufficiently) lower.[4] Upon bidding in the second auction, bidders are informed of the entry decision of the other bidder (i.e. they know whether they will face an opponent or not). The results of their experiment show that a majority of the subjects makes the correct entry decision in the first auction, thus broadly supporting the idea that they are aware of the opportunity cost associated with first auction bids. On the other hand, bidding strategies in the second auction significantly depart from the theoretical prediction:

[3]Based on evidence on sequential water auctions run in Spain, Donna and Espín-Sánchez (2018) argue that the limited demand of water by farmers, especially in rainy seasons, explains part of the observed pattern of bids.

[4]In fact, if a bidder participates in the first auction, she may unwillingly win it even if she bids the reserve price.

instead, they closely track those associated with the equilibrium of a one-shot auction, as if they neglected the signaling effect conveyed by the decision of the opponent to skip the first auction. However, having no information on the subjects' actual beliefs (there is no belief elicitation in their experiment), the authors are neither able to confirm this hypothesis, nor assess whether subjects anticipate this lack of updating in their entry decisions.

Saini and Suter (2015) implement in the lab a stylized version of the model by Jofre-Bonet and Pesendorfer (2000, 2003): sellers are not literally capacity constrained, but the winner of the current auction will experience a (probabilistic) cost increase in the next. Since the identity of the winner is communicated at the end of each round, the loser (winner) of the previous auction knows she is going to face an opponent with stochastically higher (lower) cost in the current one. In other words, no issue related to the signaling effect of previous bids is present. While their main focus is on collusive behavior in sequential auctions with potentially infinite horizon, they also run one treatment with a sequence of two auctions. Results from this latter treatment show that, although bidders appear to be aware of the opportunity cost associated with winning the first auction, they seem to underestimate its real magnitude, as their bids in the first auction are lower than predicted by theory.[5]

All the experimental literature reviewed above has focused on situations where the private information involves the cost/value parameter specific to each bidder. Instead, each bidder's capacity/demand has always been assumed to be common knowledge. The novelty in our experiment is that we assume that capacities are private information. This introduces an additional strategic element in the game, in that a bidder that is not capacity constrained has to take into account a different opportunity cost: her bid in the first auction may reveal some information about her actual capacity, and this may dramatically affect the intensity of competition in the second. To investigate the impact of this signaling cost, we explicitly require subjects to elicit their beliefs at the end of the first auction. This allows us to deeply scrutinize the determinants of their behavior and to identify in the belief updating process the crucial elements of departure from the theory.

In this last respect, our work provides a deeper understanding of the belief updating process in sequential auctions. This element has been largely overlooked in the received experimental literature. In fact, in most of these papers, the signaling issues were either fully avoided by the choice of the experimental design (this is the case in Saini and Suter, 2015, and Otamendi et al., 2018, where the type of each bidder is re-drawn in every round, so a bid in one auction does not convey any relevant information; see also Leufkens et al., 2012) or, when present, were not deeply investigated (such as in Brosig and Reiß, 2007). One relevant exception in this sense is the article by Cason et al. (2011): in their experiment involving two procurement auctions with unconstrained sellers and privately known costs, they envisage a belief elicitation phase at the end of the first auction. The descriptive analysis of these results seems to indicate that the belief updating process by bidders is qualitatively correct, though quantitatively imperfect.

More generally, our article contributes to the literature on belief updating in dynamic games with incomplete information. In particular, the fact that the behavior we observe in the second auction is (essentially) sequentially rational is consistent with several other works that suggest that subjects use their stated beliefs as the basis of their choices (see, e.g., Schotter

[5]In the context of standard sequential auctions, Février et al. (2007) and Otamendi et al. (2018) reached a similar conclusion that, in general, bidders tend to underestimate the opportunity cost of winning an early auction.

and Trevino, 2014). On the other hand, we observe a clear inconsistency between behavior and stated beliefs, a result that, starting from the classical work by Tversky and Kahneman (1974), has been repeatedly documented in the experimental literature. In particular, our results seem to support two commonly observed heuristics: the gambler's fallacy, i.e. the tendency of people to believe that, when independent draws from a binary distribution are made, a certain draw (say, head) is likely to be followed by one different draw (say, tail); and conservatism, according to which subjects tend to insufficiently adjust their ex post beliefs, especially when the newly arrived information does not clearly favor one alternative over the others (in this respect, see Mullainathan, 2002, and Henckel et al., 2018). Our evidence suggests that the use of one heuristic or the other may be related to the amount of information received by the individual, as, in our experiment, Winners, that are less informed, seem to follow the former, whereas Losers, that have richer information, seem to follow the latter.

3 Experimental Design and Theory

3.1 Baseline Game and Treatments

Our experimental setup consists of two sequential first-price auctions with incomplete information. Two sellers participate in two consecutive auctions, $A1$ and $A2$: in each auction, they compete to sell one unit of a homogeneous good to a hypothetical buyer, and the buyer buys the unit from the seller posting the lowest price, provided this price does not exceed a commonly known reserve price R, that was set equal to 120. Sellers do not bear any cost for producing the units they sell. However, sellers may or may not be capacity constrained: specifically, before $A1$ starts, each seller is randomly assigned one or two units of the good. Let c denote a seller's capacity (i.e. c is the seller's *type*) and p be the ex ante probability that $c = 2$.[6] Clearly, if her capacity is $c = 2$, the seller will participate in $A2$, whatever the outcome of $A1$ is. Instead, if her capacity is $c = 1$, the seller will bid in $A2$ only if she did not win $A1$. In other words, if the (initial) capacity of the seller who won $A1$ was $c = 1$, the seller who lost $A1$ will surely win $A2$, regardless of her bid. Each seller knows her own but not the opponent's capacity, whereas p is common knowledge. At the end of $A1$, and before $A2$ begins, sellers are informed on whether they won $A1$ and on the winning bid. Finally, bidders know from the outset that the outcome of $A2$ will be implemented (the winner will sell the good and will be paid her bid) only with probability q; instead, with the remaining probability $(1 - q)$, $A2$ will be revoked, and no bidder will receive anything.[7] Whether the second auction is implemented or revoked is communicated to bidders at the end of it.

We consider four treatments: $T1$, $T2$, $T3$ and $T4$. Treatments differ in the parameters p and q. Specifically, p, the prior probability of having capacity $c = 2$, is equal to 0.5 in $T1$ and $T3$, to 0.75 in $T2$ and $T4$; q, the probability that $A2$ is implemented, is equal to 0.5 in $T1$ and $T4$, to 1 in $T2$, to 0.25 in $T3$. These values have been chosen to produce pairwise equal

[6]Throughout the article, when we talk about a seller's capacity, we always intend her *initial* capacity. When, instead, we want to refer to the remaining capacity of a seller after the end of the first auction, we will explicitly talk about *residual* capacity.

[7]This element is introduced to match real-world situations: in recurring procurement auctions, it is reasonable to believe that firms may expect another auction to take place in the near future, but are not certain on whether and when; in electricity markets, the demand for electricity in the balancing market is not known a priori; this uncertainty has recently been exacerbated by the increased penetration of intermittent renewable sources. Moreover, the assumption of uncertainty regarding future auctions is also made in other related experiments (see, e.g., Neugebauer and Pezanis-Christou, 2007, Saini and Suter, 2015).

pooling equilibrium bids in the two auctions: specifically, predicted bids in $A1$ are the same in $T1$ and $T2$, as they are in $T3$ and $T4$; predicted bids in $A2$ are the same in $T1$ and $T3$, as they are in $T2$ and $T4$. Treatments' parameters and the theoretical pooling equilibrium bids are reported in Table 1. Notice that, in all treatments, predicted bids are much higher in $A2$ than in $A1$.

(Table 1 about here)

3.2 Equilibrium Predictions

Our benchmark model is based on the hypothesis of risk neutral bidders and equilibrium behavior. For each auction, given that production costs are null, a bidder's payoff is simply given by the winning price in case of win, whereas it is equal to zero if she does not win the auction or, for $A2$, if she wins but the auction is revoked. A bidder's total payoff is simply given by the (undiscounted) sum of the payoffs obtained in the two auctions. Being a dynamic game with incomplete information, our equilibrium concept will be that of Perfect Bayesian Equilibrium (PBE). We will use the letters a and b to denote bids in $A1$ and $A2$, respectively; and the Greek letters α and β to denote mixed strategies.

Following a backward induction logic, we begin our analysis with the second auction. We will refer to the bidder who won $A1$ as the *Winner*, and to the bidder who lost the first auction as the *Loser*. Notice that $A2$ is a two-bidder or a one-bidder auction depending on whether the capacity of the Winner was two units or one unit. Notice also that, when the Winner bids in $A2$ (i.e. her capacity was $c = 2$), she knows that she will surely face an opponent; on the other hand, the Loser should rationally anticipate that this will be the case only if the Winner's capacity was $c = 2$.

The concept of PBE requires that bidders' behavior in $A2$ must be sequentially rational: *given their beliefs*, bids in $A2$ maximize bidders' expected payoffs. The proposition below characterizes it.[8]

PROPOSITION 1. *Let μ_L (μ_W) denote the probability that the Loser (the Winner) assigns to the fact that the capacity of the Winner (the Loser) was two units. In a Perfect Bayesian Equilibrium, bids in $A2$ are the following:*

(i) For $\mu_L \in (0, 1]$, the (mixed) strategy of the Winner (i.e. a bidder with capacity $c = 2$ who won the first auction) is given by the following distribution function:

$$\beta_W(b) = \begin{cases} 0 & \text{if} \quad b < (1-\mu_L)R \\ \frac{b-(1-\mu_L)R}{\mu_L \cdot b} & \text{if} \quad (1-\mu_L)R \leq b \leq R \end{cases}.$$

The Loser bids $b_L = R$ with probability $1 - \mu_L$; conditional on not bidding R, the Loser bids according to the mixed strategy $\beta_L(b|b \neq R) = \beta_W(b)$.

(ii) For $\mu_L = 0$, the Loser bids $b_L = R$ and the Winner bids $b_W = R - \varepsilon$.

[8]Notice that, to allow us to use calculus, the theoretical analysis in this section is carried out under the hypothesis that bids can be any real number between 0 and R. This is quite an accurate approximation, given that, in the experiment, any integer bid was allowed. The proofs of the propositions are in the Appendix.

Proposition 1 shows that sequentially rational bidding strategies depend on the first auction's outcome in two respects: first, in general, the strategies of the Winner and the Loser are different; second, they depend on the Loser's belief μ_L.

In particular, the expected bid of the Loser is strictly decreasing in μ_L. In fact, for $\mu_L \in [0,1)$, the expected bid of the Loser is $\mathbb{E}[\beta_L(b)] = (1-\mu_L)R[1-\ln(1-\mu_L)]$, with first derivative (with respect to μ_L) equal to $R \times \ln(1-\mu_L) < 0$. This is easily understandable once one considers that μ_L – the Loser's belief about the probability that the capacity of the other bidder was $c = 2$ – coincides with the likelihood that the Loser will indeed face an opponent in $A2$. Clearly, for strategic response, also the Winner's bid depends (negatively) on μ_L: for $\mu_L \in (0,1)$, the expected bid of the Winner is $\mathbb{E}[\beta_W(b)] = -\frac{(1-\mu_L)R}{\mu_L}\ln(1-\mu_L)$, with first derivative (with respect to μ_L) equal to $R \times \frac{\mu_L+\ln(1-\mu_L)}{\mu_L^2} < 0$. It is worth remarking that the (negative) response of bids to the Loser's belief represents the signaling cost: if a bidder with capacity $c = 2$ wins the first auction but reveals her type, at least partially, then competition in the second auction would be fiercer, pushing the expected winning price down.

On the other hand, μ_W does not enter anywhere in the bidding functions. This is obvious as, conditional on participating, the Winner knows that she will face an opponent for sure, because the Loser will certainly have strictly positive residual capacity. Hence, whether the Loser's initial capacity was one or two units is totally irrelevant to the Winner.

Notice also that, except for the case in which $\mu_L = 1$, the Loser's bid is, on average, strictly larger than the Winner's bid, i.e. $\mathbb{E}[\beta_L(b)] > \mathbb{E}[\beta_W(b)]$, for all $\mu_L \in [0,1)$. This is due to the fact that the Loser bids R with strictly positive probability, whereas the Winner never does so (R is outside the support of $\beta_W(b)$). By bidding R, the Loser wins only if she does not face any opponent (the capacity of the Winner was equal to 1), but, if this happens, her payoff is the largest.

Finally, neither p nor q enter anywhere in the bidding functions: this means that the treatment variables p and q may affect bidding behavior in $A2$ only through the beliefs.

We summarize the above considerations in the following testable predictions:

A2-(a) *Neither p nor q have a direct impact on bids: hence,* for given beliefs*, bids are the same in the four treatments.*

A2-(b) *The Loser's bid negatively depends on her belief; the Winner's bid is unaffected by her belief.*

A2-(c) *The expected bid of the Loser is higher than the expected bid of the Winner. This difference is due to the fact that the Loser bids R with strictly positive probability.*

After characterizing rational bidders' behavior in $A2$ *for given beliefs*, let us now consider bidding in the first auction and how bidders update their beliefs at the end of the auction. We will denote by a_c and α_c, a pure and a mixed strategy of a bidder with capacity c. The following proposition characterizes the PBE bidding strategies in $A1$ and the corresponding beliefs.

Proposition 2.

(i) There is no separating equilibrium, either in pure or in mixed strategies.

(ii) The following is the unique pooling equilibrium: $a_1 = a_2 = a^* = q(1-p)R$*, with Loser's belief:* $\mu_L(a^*) = p$*,* $\mu_L(a) \geq (1+p)/2 - (a^* - a)/(qR)$ *for* $a < a^*$*,* $\mu_L(a) \in [0,1]$ *for* $a > a^*$*.*

(iii) Suppose that (α_1, α_2) *is a hybrid equilibrium (necessarily in mixed strategies). Let* A_1 *and* A_2 *be the supports of* α_1 *and* α_2*, respectively, and let* $A = A_1 \cap A_2$*. Then: (a)* $A_2 = A$*; (b) either* $A_1 \setminus A_2 = \emptyset$*, or, for all* $a' \in A_1 \setminus A_2$ *and all* $a'' \in A_2$*, it must be* $a' > a''$*; (c) for all* $a', a'' \in A$*, if* $a' < a''$*, then* $\mu_L(a') > \mu_L(a'')$*; (d)* $\inf A \geq q(1-p)R$*.*

The intuition behind the absence of a separating equilibrium (point *(i)* of Proposition 2) is straightforward: for a bidder with capacity $c = 2$, revealing her type (as it would happen in a separating equilibrium) is extremely costly because this would imply a null payoff in $A2$. Hence, a bidder with capacity $c = 2$ is willing to reveal her type only if, by doing so, she expects to get a sufficiently large payoff from the first auction. But if this is the case, a bidder with initial capacity $c = 1$, who is totally uninterested in $A2$ in case of winning, would rather mimic a bidder with capacity $c = 2$.

To see why the one described in point *(ii)* of Proposition 2 is an equilibrium, consider first a bidder with capacity $c = 1$. This bidder knows that, if she pools with the other type (i.e. a bidder with capacity $c = 2$) at some bid $\hat{a}$, she may win the auction and obtain payoff equal to $\hat{a}$ or may lose it: in the latter case, she will enjoy the expected payoff from $A2$, equal to $q(1-p)R$. Hence, if $\hat{a} < q(1-p)R$, she will strictly prefer to lose $A1$ (bidding anything above $\hat{a}$); if, on the other hand, $\hat{a} > q(1-p)R$, she will strictly prefer to win $A1$ (bidding slightly below $\hat{a}$); only if $\hat{a} = a^* = q(1-p)R$, this bidder does not want to deviate from the pooling bid.[9] Consider now a bidder with capacity $c = 2$. Clearly, relative to a bidder with capacity $c = 1$, this bidder has stronger incentive to win the first auction. She will abstain from undercutting a^* (winning $A1$ for sure) only if such a lower bid significantly reduces her expected payoff from $A2$, i.e. significantly increases the Loser's belief.

The third part of Proposition 2 characterizes a fundamental property that any hybrid equilibrium, if it exists, must satisfy in our context: the Loser's belief must be strictly decreasing in bids.[10] Here is an intuitive argument: consider two bids a', $a'' \in A = A_1 \cap A_2$, with $a' < a''$.[11] Clearly, both types of bidders must be indifferent between bidding a' and a'', i.e. $\pi_1(a') = \pi_1(a'')$ and $\pi_2(a') = \pi_2(a'')$. But then, we have $\pi_2(a') - \pi_1(a') = \pi_2(a'') - \pi_1(a'')$: the difference between type-2 and type-1 expected payoffs must be constant as well. Now, for given bid, the difference between the expected payoff of a type-2 bidder and the expected payoff of a type-1 bidder is solely due to the fact that, in case of winning $A1$, a type-2 bidder will also take part in $A2$, where she expects to get an additional payoff equal to $q(1-\mu_L(a))R$. Hence, for given bid, we have $\pi_2(a) - \pi_1(a) = q(1-\mu_L(a))R \times \mathrm{PW}(a)$, where $\mathrm{PW}(a)$ is the probability of winning $A1$ with a bid equal to a. Now, consider what happens when a type-2 bidder decreases her bid from a'' to a': the probability of winning $A1$ clearly increases. Thus, to keep the payoff difference constant, the expected payoff obtainable from $A2$ ($q(1-\mu_L)R$) has to decrease, i.e. μ_L must increase.

[9] This argument implies that there can be no pooling equilibrium in mixed strategies as well. It also implies that no bid below $q(1-p)R$ can be part of an equilibrium strategy in $A1$.

[10] More precisely, the Loser's belief is strictly decreasing on A (the intersection of the supports of the mixed strategies). As stated in Proposition 2, the mixed strategy of a bidder with capacity $c = 1$ may also include bids greater than A; for these bids, the Loser's belief must be equal to 0.

[11] Notice that, in a hybrid equilibrium, the intersection of the supports must be nonempty.

An immediate corollary of part *(iii)* of Proposition 2 is that the expected bid of a bidder with capacity $c = 2$ must be strictly lower than the expected bid of a bidder with capacity $c = 1$.

The previous analysis sheds light on the relative importance of the two strategic forces at play in our setting: the opportunity cost and the signaling cost of early bids. Everything else equal, the opportunity cost pushes bidders with capacity $c = 2$ to bid more aggressively than those with capacity $c = 1$, as the former have a stronger pressure to sell immediately (they have two units to sell). On the other hand, the signaling cost of early bids pushes bidders with capacity $c = 2$ to mimic those with capacity $c = 1$ in order to avoid revealing their true capacity. Equilibrium predictions imply that the signaling cost is strong enough that a bidder with capacity $c = 2$ prefers to significantly reduce her chance of winning the first auction than to fully reveal her capacity.

To sum up, equilibrium behavior in $A1$ produces the following testable predictions:

A1-(a) [Pooling Equilibrium] *Either bidders make the same bid $a^* = q(1-p)R$, regardless of their capacities: in this case, the Loser's belief must be equal to the prior probability;*

A1-(b) [Hybrid Equilibrium] *or, the expected bid of a bidder with capacity $c = 2$ is strictly lower than the expected bid of a bidder with capacity $c = 1$, and both are strictly larger than $q(1-p)R$: in this case, the Loser's belief must be strictly decreasing in the winning bid.*

Notice that, in these predictions, the treatment parameters, p and q, only affect the bid levels. Therefore, the qualitative features of the equilibria hold for all treatments.[12]

3.3 Procedures

Upon their arrival, subjects were randomly assigned to a computer terminal. In all sessions, instructions were distributed at the beginning of the experiment and read aloud. Before the experiment started, subjects were asked to answer a number of control questions to make sure they understood the instructions as well as the consequences of their choices. When necessary, answers to these questions were privately checked and explained. The experiment could not start until all subjects correctly answered the control questions. At the beginning of the experiment, the computer randomly formed four rematching groups of six subjects each. The composition of the rematching groups was kept constant throughout the session. At the beginning of every period, subjects were randomly and anonymously divided into pairs. Pairs were randomly formed in every period within rematching groups. Subjects were told that pairs were randomly formed in a way that they would never interact with the same opponent in two consecutive periods.[13]

[12]Notice, in particular, that a change in q (the probability that the second auction will be implemented) affects the opportunity cost and the signaling cost of early bids in the same direction: a lower q reduces the incentive for a bidder with capacity $c = 2$ to mimic a bidder with capacity $c = 1$, but also reduces the incentive of the latter to bid higher. As a result, the equilibrium (qualitatively) does not change.

[13]Our rematching protocol implies that, given the size of the subgroups (six subjects), on average subjects interacted with the same opponent every five periods. Although this does not represent perfect stranger protocol, it leaves very little room for developing punishment-reward strategies over multiple periods. The rematching protocol was intended to increase the number of independent observations and improve precision and reliability of both nonparametric and parametric results.

In every period, subjects participated in three consecutive phases: (i) $A1$, (ii) a belief elicitation procedure, and (iii) $A2$. In $A1$, after being informed about their own capacities, subjects chose simultaneously their bids. Bids were restricted to be integer numbers between 1 and the reserve price that was set to 120. Then, once choices were posted, each subject received feedback about the winning bid and her corresponding payoff. In the second phase, subjects were asked to elicit their beliefs on the capacity of the opponent. To this end, we relied on the *Binary Lottery Procedure* (McKelvey and Page, 1990; Schlag and van der Weele, 2013; Hossain and Okui, 2013; Harrison et al., 2014) as a proper incentive compatible belief elicitation mechanism. In particular, subjects were presented two boxes on the screen: in the first box, they had to indicate the probability that the opponent's capacity was $c = 1$; in the second, the probability that the opponent's capacity was $c = 2$. Both probabilities were restricted to be integers between 0 and 100 and to sum up to 100. Let η and μ denote the subjective probabilities attached by a subject to an opponent's capacity of one and two units, respectively. Each subject was informed that, at the end of the period, she would have participated in either of two possible lotteries: the first implemented if the opponent's actual capacity was $c = 1$; the second if it was $c = 2$. The number of tickets assigned to the subject in each of the two lotteries depended on the reported probabilities. In particular, conditional on the actual capacity of the opponent, the numbers of lottery tickets were computed by using the following two quadratic rules:

$$tickets(1u) = 10000[1 - (1 - \eta/100)^2]$$

$$tickets(2u) = 10000[1 - (1 - \mu/100)^2]$$

Thus, from the previous expressions and depending on her stated probabilities, each subject received a number of tickets between 0 and 10000 for each lottery. For simplicity, the tickets were numbered in ascending order, starting from 0 to the total number of tickets assigned to the subject. At the end of the period, after being informed about the actual capacity of the opponent, each subject participated in the corresponding lottery. In particular, the computer randomly selected one of the 10000 tickets, numbered from 1 to 10000. If the subject possessed the selected ticket, then she received 20 points to be added to her overall earnings in the period.

Although the Binary Lottery Procedure represents one of the best experimental tasks to elicit probabilistic beliefs on a dichotomous variable, it can be considered somewhat difficult to understand for experimental subjects. In order to limit the risk of confusion, instructions included an example describing in details how stated beliefs determined payoffs under the Binary Lottery Procedure. Moreover, one of the control questions administered before playing the experimental game explicitly referred to the Binary Lottery Procedure and in case of necessity, subjects were privately assisted by experimenters. Finally, as it will become clear in the next section, we do find evidence that subjects reported meaningful beliefs, as they "used" them in a rational way upon placing their bids in the second auction.

Finally, in the third phase, subjects whose residual capacity at the end of $A1$ was not null, competed in $A2$. Again, subjects simultaneously chose their bids and were then informed about the winning bid as well as their payoffs. Of course, in case a subject did not have any unit left after the first auction, she could not place any bid in the second auction.[14]

[14]It might be argued that our payment scheme induces risk-averse subjects to hedge with their stated beliefs against adverse outcomes in the two auctions. However, there are at least two reasons to believe that

Two features of the experimental design were specifically intended to facilitate learning and make personal history easily accessible. First, at the beginning of the experiment subjects were endowed with a hard copy record sheet containing a table with 15 lines, one for each period, and a number of columns that they could use to take note of bids, payoffs, and stated beliefs. Second, upon making their decisions in the second and third phase of the experiment, their choices and information about the previous tasks were visualized on the screen. At the end of every period, the results of the two auctions were summarized on the screen, together with the decision on the annulment of the second auction, with the outcome of the belief lottery and with their overall earnings in the period.

For each of the 4 treatments, we ran 2 sessions, each involving 24 subjects, thus generating 8 independent observations at the rematching group level. The experiment took place at the Bocconi Experimental Laboratory for the Social Sciences (BELSS) of Bocconi University, Milan, between November and December 2016. Participants were mainly undergraduate students, recruited by using the SONA recruitment system (http://www.sona-systems.com/default.aspx). The experiment was computerized using the *z-Tree* software (Fischbacher, 2007). At the end of the experiment, the number of points obtained by a subject during the experiment was converted with an exchange rate of 0.02 euro per 1 point and monetary earnings were paid in cash privately. On average, each subject earned 16.67 euro, and each session lasted 75 minutes, including the time for instructions and payments. Before leaving the laboratory, subjects completed a short questionnaire containing questions on their socio-demographics and their perception of the experimental task.

4 Experimental Results

In line with the structure of the theoretical part, we will first analyze bids in $A2$, studying how they depend on the outcome of the first auction and on the beliefs elicited between the two auctions. We will then focus our attention on behavior in $A1$, identifying its main determinants. Lastly, we will examine beliefs and how they respond to the information provided at the end of the first auction. In the next section, we will comment on the results, suggesting a possible explanation of the observed departure from the theory and providing insights into the adjustment process of bids in $A1$.

The nonparametric tests presented below are based on eight independent observations (at the rematching group level) per treatment. Similarly, in the parametric analysis, we either cluster standard errors or introduce random effects at the rematching group level to control for dependency of observations over repetitions. All regressions pool data from the four treatments and use $T3$ as baseline. Unless otherwise noted, all the statistic tests are $\chi^2(1)$, for which we only report the p-values.

4.1 Bids in $A2$

In order to analyze bids in $A2$, we only consider subjects with strictly positive residual capacity at the end of $A1$, namely Winners with (initial) capacity $c = 2$ and Losers. Figure

the (potential) hedging problem plays a marginal role in our setting. First, there is experimental evidence suggesting that hedging is not a major problem in strategic interaction settings, unless hedging opportunities are very prominent (Blanco et al, 2010). Second, in order to avoid confusion-driven pseudo-hedging, subjects were explicitly instructed that, by stating their beliefs truthfully, they would have minimized the penalization due to errors and maximized the corresponding gains.

1 shows, for each treatment, the distribution of bids in $A2$ and its evolution over periods, by separately considering Winners (top panel) and Losers (bottom panel). Table 2 reports summary statistics.

(Figure 1 about here)

(Table 2 about here)

The preliminary descriptive analysis highlights several interesting facts. First, bids in $T1$ and $T3$ are, on average, larger than those in $T2$ and $T4$, for both Winners and Losers of $A1$. Second, Losers bid higher than Winners. Third, the difference between Losers' and Winners' bids seems to widen over time as, at least in most treatments, the former tend to increase, while the latter tend to decrease across periods.

In order to test the statistical validity of these preliminary empirical observations, Table 3 collects parametric results on the determinants of bids in $A2$.

(Table 3 about here)

Estimates reported in column (1) provide a first look at differences in (overall) bids across treatments. Bids are higher in $T1$ than in $T2$ ($p < 0.001$) and $T4$ ($p = 0.003$), whereas the difference between $T1$ and $T3$, though positive, is not significant ($p = 0.247$). No significant differences are found between $T2$ and $T4$ ($p = 0.272$). We document higher bids in $T3$ than in $T2$ ($p = 0.004$) and (marginally) $T4$ ($p = 0.076$).[15] Overall, this evidence is broadly consistent with the theoretical pooling equilibrium (see Table 1). However, in order to assess whether bidders' behavior is sequentially rational, we need to analyze it along with the beliefs, which may not be in line with the equilibrium ones.

To this end, in column (2), we add the bidder's stated belief about the probability that the opponent's capacity is $c = 2$. According to the theoretical prediction *A2-(a)*, any observed difference in bids across treatments should be solely due to different beliefs. This prediction is empirically validated by the parametric analysis: after controlling for the stated beliefs, all pairwise comparisons between average bids become nonsignificant (between $T1$ and $T2$, $p = 0.182$; between $T1$ and $T3$, $p = 0.810$; between $T1$ and $T4$, $p = 0.923$; between $T2$ and $T3$, $p = 0.274$; between $T2$ and $T4$, $p = 0.246$; between $T3$ and $T4$, $p = 0.899$). In all treatments, we also detect a negative and significant effect of beliefs on bids (in $T1$, $p = 0.013$; in $T2$, $T3$, and $T4$, $p < 0.001$).

Still controlling for the beliefs, column (3) investigates the effects of the outcome of the first auction. Consistent with prediction *A2-(c)*, we find that winning in $A1$ significantly reduces bids in $A2$ in all treatments ($p < 0.001$ for all treatments).

Coherently with the previous result, we also detect a large difference between Winners and Losers in the frequency of bids that are equal to 120 (the reserve price). Indeed, whereas the proportion of Winners bidding the reserve price is 1.9% in $T1$, 3.7% in $T2$, 2.7% in $T3$, and 0.4% in $T4$, it is substantially larger when considering Losers: 19.4% in $T1$, 16.1% in $T2$, 20.6% in $T3$, and 15.0% in $T4$. These differences are highly significant in all treatments (according to a two-sided proportion test, $p < 0.001$ for all treatments).

[15] Parametric results are generally confirmed by nonparametric tests. According to a (two-sided) Mann-Whitney rank-sum test, we find significant differences in bids between $T1$ and $T2$ ($p = 0.006$), $T1$ and $T4$ ($p = 0.036$), $T2$ and $T3$ ($p = 0.021$) and (marginally) $T3$ and $T4$ ($p = 0.093$). In all other cases, the differences are not statistically significant (between $T1$ and $T3$, $p = 0.208$; between $T2$ and $T4$, $p = 0.345$).

Column (4) adds a treatment-specific time trend to the empirical model considered in column (3). Results show that the time trend is not significant in $T1$ ($p = 0.308$) and in $T3$ ($p = 0.978$), it is negative and marginally significant in $T2$ ($p = 0.091$), and, finally, it is negative and significant in $T4$ ($p = 0.033$).

To assess whether and how the relation between beliefs and bids in $A2$ depends on the outcome of $A1$, columns (5) and (6) replicate the parametric specification presented in column (4) on the two subsamples of Winners and Losers, separately. As shown in the theoretical section (prediction *A2-(b)*), if bidders were sequentially rational, the effect of beliefs should be strong and negative for Losers and negligible for Winners. The experimental results generally confirm this prediction: looking at column (5), the effect of Winners' beliefs on their own bids in $A2$ is not significant in $T1$ ($p = 0.945$), $T3$ ($p = 0.602$) and $T4$ ($p = 0.232$); only in $T2$ it is negative and significant ($p = 0.041$). On the contrary, column (6) shows that this effect is always negative and significant for Losers (in all treatments, $p < 0.001$).[16]

Results in columns (5) and (6) also confirm the preliminary observation that the difference between Losers' and Winners' bids tend to widen over periods, at least in most treatments. As a matter of fact, for Losers, we find a positively sloped linear trend in $T1$ ($p = 0.010$), $T2$ ($p = 0.044$), and $T3$ ($p = 0.027$), while no significant time pattern is observed in $T4$ ($p = 0.382$). Instead, for Winners, we find a negatively sloped linear trend in $T2$ ($p < 0.001$), $T3$ ($p < 0.001$), and $T4$ ($p < 0.001$), while no significant time pattern is present in $T1$ ($p = 0.161$).

We collect the main empirical findings regarding bidding behavior in $A2$ in the following statements.

R1. **Differences in bids in $A2$ across treatments.** *Differences in bids across treatments disappear once stated beliefs are controlled for.*

R2. **Bids in $A2$ and outcome of $A1$.** *In all treatments, Losers make substantially higher bids than Winners. Moreover, Losers are more likely to bid the reserve price.*

R3. **Bids in $A2$ and stated beliefs.** *In all treatments, stated beliefs reduce bids: this effect is particularly strong for Losers and negligible for Winners.*

Together, *R1*, *R2*, and *R3* suggest that subjects' choices in $A2$ are qualitatively coherent with the sequentially rational behavior summarized in predictions *A2-(a)*, *A2-(b)* and *A2-(c)*. First, treatment parameters have no direct effect on bids, as bidders rationally condition their behavior only on their beliefs, which summarize the whole relevant information to them. Second, on average, Losers rationally bid more than Winners. This reflects the fact that, upon choosing their bids in $A2$, Winners know that they will compete against an opponent, whereas Losers are in general uncertain about that. In other words, Winners expect more competition than Losers: as a consequence, they make lower bids on average. Third, bidders make a rational use of their beliefs: whereas Winners' bids are unaffected by their beliefs, which are

[16] We also investigated whether the Loser's capacity affects her bid in the second auction. This would be the case if, for instance, a bidder endowed with two units to sell, upon losing the first auction, acted more aggressively in the second to avoid ending up empty-handed. To test this conjecture, we re-estimate column (6) of Table 3 including as a regressor a subject's capacity in the period and the corresponding interactions with the treatment dummies. We find that, in all treatments, the Loser's capacity does not exert any effect on her bids in $A2$ (in $T1$, $p = 0.700$; in $T2$, $p = 0.382$; in $T3$, $p = 0.635$; in $T4$, $p = 0.845$). Results are available upon request.

totally useless, Losers bid the more aggressively the more they believe that the opponent will indeed participate in $A2$ (i.e. her capacity is $c = 2$).

Overall, this evidence suggests that signaling is potentially costly for a bidder with capacity $c = 2$: in fact, should she win the first auction, competition will be fiercer in the second, the more the opponent is able to infer her actual capacity.

It has to be noticed that, while bidding behavior in $A2$ is *qualitatively* in line with equilibrium, we quantitatively detect overbidding: observed bids tend to be more aggressive (i.e. lower) than those predicted by equilibrium under risk neutral bidders. To see this, Table 2 reports, for both Winners and Losers, the predicted (expected) bids: these are obtained by plugging into the expression for the theoretical (expected) bid of a subject conditional on her beliefs, the average beliefs stated in the experiment. Comparing predicted and observed bids, we see that, in all treatments and for both Winners and Losers, the latter are well below the former.[17] Overbidding is a very common phenomenon in first-price experimental auctions and several behavioral explanations have been proposed, mostly in terms of non-standard preferences. The presence of overbidding in $A2$ has no particular consequences on the rest of our analysis, so we will not investigate it further. Our main interest is on bidders' behavior in the early auction, and, in particular, whether they account for the opportunity and signaling cost that early bids exert on their future payoffs. Hence, as long as actual behavior in $A2$ makes it potentially costly for a bidder to signal her type, bid levels are of secondary importance.

4.2 Bids in $A1$ and Beliefs

Figure 2 shows, for each treatment, the distribution of bids in $A1$ and their evolution over periods. Descriptive statistics are reported in the first column of Table 4. We also present the same data split by capacity (see Figure 3 and the columns with headings a_1 and a_2 in Table 4). Notice that bids in $A1$ are lower than those observed in $A2$, though the difference is smaller than what is predicted by the (pooling) equilibrium.[18]

(Figure 2 about here)

(Figure 3 about here)

From a first look at the data, we can make the following remarks. First, at least in early periods, there are no remarkable differences across treatments. Second, in all treatments and regardless of their capacities, subjects bid substantially more than what is predicted by the theoretical pooling equilibrium. Third, in all treatments, bids made by subjects with capacity $c = 2$ are lower than those placed by subjects with capacity $c = 1$. Fourth, in all treatments, bids decline over repetitions, though the negative trend seems to disappear in the last few periods.

(Table 4 about here)

[17] According to a Wilcoxon signed-rank test, bids of Winners are lower than the predicted ones in all treatments (in $T1$, $p = 0.049$; in $T2$, $p = 0.017$; in $T3$, $p = 0.012$; in $T4$, $p = 0.012$). For Losers, the difference is negative in all treatments, and it is significant in $T3$ ($p = 0.012$) and $T4$ ($p = 0.025$), marginally significant in $T2$ ($p = 0.093$) and not significant in $T1$ ($p = 0.434$).

[18] Given that an increasing pattern of bids in a procurement auction corresponds to a decreasing one in a standard auction, our model is consistent with the decreasing price anomaly, both theoretically and empirically.

In order to confirm these preliminary observations and to provide further insights into the determinants of bids, Table 5 reports parametric results by pooling data from all treatments.

(Table 5 about here)

Column (1) shows that differences in bids across treatments are very small. Indeed, only the difference between $T1$ and $T3$ is marginally significant ($p = 0.086$). All other differences are not statistically significant (between $T1$ and $T2$, $p = 0.135$; between $T1$ and $T4$, $p = 0.233$; between $T2$ and $T3$, $p = 0.824$; between $T2$ and $T4$, $p = 0.763$; between $T3$ and $T4$, $p = 0.599$).[19]

Results in column (1) also support the observation that subjects bid above the pooling equilibrium levels reported in Table 1 ($p < 0.001$ for all treatments).[20]

Column (2) documents what we call "capacity effect": subjects make lower bids when their capacity is $c = 2$ than when it is $c = 1$ (for $T1$, $T2$, and $T4$: $p < 0.001$; for $T3$: $p = 0.039$). Nevertheless, in all treatments, bids are significantly above the predicted levels associated with the pooling equilibrium also for subjects with capacity $c = 2$ ($p < 0.001$ in all treatments).

Finally, column (3) documents a strong declining pattern of bids over time: considering all periods, we find a negative and highly significant linear time trend in all treatments ($p < 0.001$). This decreasing trend may question the significance of the capacity effect and the conclusions regarding differences across treatments: indeed, in the absence of a formal theory to model the convergence process of bids in $A1$, these results can be substantially altered by standard issues of serial correlation and heteroscedasticity. In order to mitigate the problem and give robustness to the previous parametric results, we perform a convergence analysis similar to the one made by Noussair at al. (1995) in the context of experimental market data. In particular, to assess whether differences in bids in $A1$ persist asymptotically, we estimate the following dynamic model:

$$a_{ij\tau t} = \frac{1}{t}\gamma_{j\tau} + \frac{t-1}{t}\lambda_\tau + \varepsilon_{ij\tau t}, \tag{1}$$

where i, j, τ and t represent identifiers for subject, subgroup, treatment and period, respectively; $\gamma_{j\tau}$ captures the origin of a possible convergence process undertaken by the subgroup; λ_τ is the asymptote of bids in treatment τ; and $\varepsilon_{ij\tau t}$ is the random error term that is distributed normally with zero mean. Notice that, when $t = 1$, the average bid made by subjects belonging to subgroup j of treatment τ is given by $\gamma_{j\tau}$: thus, the model allows different subgroups to have different starting points for their convergence processes. As t gets larger, the weight attached to $\gamma_{j\tau}$ gets smaller and smaller, whereas the weight attached to λ_τ increases toward 1, thus providing an estimate of the treatment-specific asymptotic mean of the bid in $A1$.[21] The same empirical strategy can be used to assess the capacity effect within each of the

[19] Nonparametric tests produce similar conclusions. A (two-sided) Mann-Whitney rank-sum test detects a weakly significant difference in bids only between $T1$ and $T3$ ($p = 0.093$). In all other pairwise comparisons, instead, the difference is not statistically significant (between $T1$ and $T2$, $p = 0.156$; between $T1$ and $T4$, $p = 0.345$; between $T2$ and $T3$, $p = 0.916$; between $T2$ and $T4$, $p = 0.834$; between $T3$ and $T4$, $p = 0.674$).

[20] Again, these results are confirmed by nonparametric tests: according to a (two-sided) Wilcoxon signed-rank test performed in each treatment, the difference between average bids and predicted levels is significant ($p = 0.012$).

[21] The underlying assumption behind this methodology is that bids in $A1$ do converge to their asymptotic value prior to the last round of the experiment. In order to provide evidence in support of this assumption, we

four treatments. In this case, we run separate regressions, one for each treatment $\tau = 1, \ldots, 4$, on the basis of the following model:

$$a_{ij\tau t} = \left(\frac{1}{t}\gamma^{(1)}_{j\tau} + \frac{t-1}{t}\lambda^{(1)}_{\tau}\right)\mathbb{1}_{\{c_{ij\tau t}=1\}} + \left(\frac{1}{t}\gamma^{(2)}_{j\tau} + \frac{t-1}{t}\lambda^{(2)}_{\tau}\right)\mathbb{1}_{\{c_{ij\tau t}=2\}} + \varepsilon_{ij\tau t}, \qquad (2)$$

where $\mathbb{1}_{\{c_{ijt}=1\}}$ ($\mathbb{1}_{\{c_{ijt}=2\}}$) takes value 1 when subject i of subgroup j was endowed with 1 (2) unit(s) in period t. In Eq. (2), $\gamma^{(c)}_{j\tau}$ and $\lambda^{(c)}_{\tau}$, $c = 1, 2$, are capacity-specific versions of $\gamma_{j\tau}$ and λ_τ from Eq. (1). Therefore, the new model is aimed at estimating different asymptotic bids in $A1$ for subjects with different capacities.

(Table 6 about here)

Table 6 reports estimates of the asymptotic bids from models (1) (top part) and (2) (bottom part). In both specifications, we allow for heteroscedasticity and first-order autocorrelation.

Not surprisingly, in all treatments, the asymptotic value is lower than the (raw) mean reported in Table 4. Moreover, differences across treatments are aligned with those obtained from the standard specification (see Table 5, column (1)): bids in $T1$ are higher than in any other treatment (between $T1$ and any of the remaining three treatments, $p < 0.001$); all other pairwise comparisons between treatments yield nonsignificant results (between $T2$ and $T3$, $p = 0.890$; between $T2$ and $T4$, $p = 0.408$; between $T3$ and $T4$, $p = 0.362$).[22]

Results in the bottom part of Table 6 confirm that the capacity effect is present and strong also asymptotically: indeed, in all treatments, bids placed by subjects with capacity $c = 2$ are significantly lower than those made by subjects with capacity $c = 1$ (for the difference between $\lambda^{(1)}$ and $\lambda^{(2)}$ in $T1$ and $T2$, $p < 0.001$; in $T3$, $p = 0.027$; in $T4$, $p = 0.006$).[23]

The following three statements summarize the main findings about bids in $A1$.

R4. **Differences in bids in $A1$ across treatments.** *We detect no remarkable differences in bids across treatments.*

run a simple pooled regression with bids in the last three periods of the experiment as the dependent variable and the constant, the treatment dummies (with $T3$ as baseline), the linear time trend starting from 0 and the interaction terms between the linear time trend and the treatment dummies, as covariates. Results (available upon request) show that bids are substantially more stable in the last three periods of the experiment as the linear time trend is not significant in $T1$ ($p = 0.968$), $T2$ ($p = 0.765$) and $T3$ ($p = 0.510$), whereas it reaches only marginal significance in $T4$ ($p = 0.074$).

[22] Convergence of bids is further validated by parametric tests. In particular, we replicate the same specification used in column (1) of Table 5 to estimate the mean bids in the four treatments in two time intervals: the first twelve periods and the last three periods. Then, for each treatment, we conduct parametric t-tests on the difference between these estimated mean bids and the asymptotic values. For the first twelve periods, with the only exception of $T1$, the difference between mean bids and asymptotic values is always highly significant (in $T1$, $p = 0.106$; in $T2$, $T3$, and $T4$, $p < 0.01$). Instead, when focusing on the last three periods, this difference becomes nonsignificant (in $T1$, $p = 0.705$; in $T2$, $p = 0.327$; in $T3$, $p = 0.387$; in $T4$, $p = 0.359$). Results are available upon request.

[23] We parametrically test convergence of bids to their asymptotic values in each treatment and, separately, for subjects with different capacities. To this end, we replicate the same specification used in column (2) of Table 5 to estimate the mean bids by capacity in the four treatments in two time intervals: the first twelve periods and the last three periods. The t-tests show that, in the first twelve periods, the difference between mean bids and asymptotic values is generally significant at the 5% level, with the only exceptions of $T1$ (for subjects with a capacity of one unit: $p = 0.100$; for subjects with a capacity of two units: $p = 0.087$), whereas, in the last three periods, the difference is never significant (in all treatments and for any capacity level, $p > 0.1$). Results are available upon request.

R5. **Bid levels in $A1$.** *In all treatments and regardless of a subject's capacity, bids are larger than the one associated with the pooling equilibrium.*

R6. **Bids in $A1$ and capacity.** *In all treatments, subjects with capacity $c = 2$ make lower bids than those with capacity $c = 1$.*

Overall, results *R4-R6* are clearly at odds with the characteristics of the pooling equilibrium (prediction *A1-(a)*) but are compatible with those of the hybrid equilibrium (prediction *A1-(b)*). However, to corroborate the validity of the hybrid equilibrium in explaining our evidence, bids must be looked at in connection with beliefs. Figure 4 shows, for each treatment, the kernel densities of the beliefs stated by subjects about the probability that the opponent's capacity was $c = 2$, both overall and after splitting subjects between Winners and Losers of $A1$. Table 7 reports the corresponding descriptive statistics.

(Figure 4 about here)

(Table 7 about here)

Looking at the data, three facts emerge. First, in line with the ordering in the prior probabilities, average beliefs in $T1$ and $T3$ are quite close to one another, as they are in $T2$ and $T4$; moreover, beliefs in the former treatments are lower than those in the latter. Second, in $T2$, $T3$, and $T4$, both Winners' and Losers' stated beliefs are, on average, lower than the prior probabilities (50% in $T3$, 75% in $T2$ and $T4$); in $T1$, instead, whereas the average beliefs of Winners are lower than the prior probability (50%), the opposite occurs for Losers (overall, average beliefs are below the prior probability). Third, whereas in $T1$ and $T2$ Winners report, on average, lower beliefs than Losers, the opposite occurs in $T3$ and $T4$. We can assess the relevance of these preliminary observations in the first three columns of Table 8.

(Table 8 about here)

Column (1) confirms that the ordering in the stated beliefs across treatments follows the one in the priors. Beliefs in $T1$ and $T3$ are generally lower than those in $T2$ and $T4$: when pooling subjects, we find significant differences between $T1$ and $T2$, between $T1$ and $T4$, between $T2$ and $T3$, and between $T3$ and $T4$ (for each pairwise comparison, $p < 0.001$). The remaining pairwise comparisons are not significant (between $T1$ and $T3$, $p = 0.381$; between $T2$ and $T4$: $p = 0.440$).[24]

Although they follow the same ordering, beliefs and prior probabilities do quantitatively differ, at least in most of the cases (see column (2)). In particular, Losers' beliefs are significantly lower than the prior probability in $T2$ ($p < 0.001$), $T3$ ($p = 0.007$) and $T4$ ($p < 0.001$), whereas the difference is not significant in $T1$ ($p = 0.215$). For Winners, we detect a negative and significant difference between beliefs and the prior probability in $T2$ and $T4$ (in both cases, $p < 0.001$). The difference is positive and significant in $T1$ ($p = 0.012$), whereas it is not significant in $T3$ ($p = 0.401$).[25]

[24] These results are confirmed by nonparametric tests. A (two-sided) Mann-Whitney rank-sum test detects a significant difference in stated beliefs between $T2$ and $T1$, $T4$ and $T1$, $T2$ and $T3$ and $T4$ and $T3$ (in all cases, $p < 0.001$). All other differences are not significant (between $T1$ and $T3$, $p = 0.208$; between $T2$ and $T4$, $p = 0.462$).

[25] Nonparametric tests generally confirm these results. According to a (two-sided) Wilcoxon signed-rank test, the difference between Winners' stated beliefs and prior probabilities is significant in $T2$ and $T4$ (in both

Column (2) also shows that the effect on beliefs of winning in $A1$ is heterogeneous across treatments. Indeed, the difference between Winners' and Losers' beliefs is positive and significant in $T3$ ($p = 0.034$), negative and significant in $T1$ and $T2$ (in both cases, $p < 0.001$), whereas it is positive but not significant in $T4$ ($p = 0.258$).

Column (3) adds the subject's own capacity as an additional regressor. While, theoretically, no effect should be observed, we find that, in all treatments, being endowed with a two-unit capacity significantly reduces a subject's beliefs (in $T1$ and $T4$, $p < 0.001$; in $T2$, $p = 0.072$; in $T3$, $p = 0.004$).

To control for potential learning effects, column (4) includes treatment-specific time trends to the previous parametric specification. The coefficient attached to the linear trend is positive and significant in $T1$ ($p = 0.017$), positive and marginally significant in $T2$ ($p = 0.077$) and $T4$ ($p = 0.082$), whereas it is not significant in $T3$ ($p = 0.261$).

Together, the previous results suggest that subjects' belief updating process is far from perfect. On the one hand, given the (strong) capacity effect, a subject who wins $A1$ should rationally understand that the opponent is relatively more likely to have capacity $c = 1$: hence, the effect of winning on beliefs should be negative. Instead, the relation we observe is heterogeneous across treatments, presenting the predicted sign only in $T1$ and $T2$. On the other hand, we document a "gambler's fallacy" effect:[26] in all treatments, and in spite of the independent, random procedure used in the experiment, a subject with a certain capacity believes that, everything else equal, the opponent is more likely to have a different capacity.[27]

Although stated beliefs exhibit a less clear time pattern than bids in $A1$, one may wonder whether the previous results continue to hold asymptotically. We therefore replicate the previous convergence analysis on beliefs. Table 9 reports the estimates of the asymptotic beliefs in the four treatments, both overall (top part) and split between Winners and Losers (bottom part).

(Table 9 about here)

In all treatments, the asymptotic beliefs are pretty close to the mean values reported in Table 7: this confirms that stated beliefs are definitely more stable than bids in $A1$. Moreover, the results of the convergence analysis are aligned with those presented in Table 8: first, no significant differences in asymptotic beliefs are detected between $T1$ and $T3$ ($p = 0.111$), and

cases, $p = 0.012$); it is not significant in $T1$ ($p = 0.208$) and $T3$ ($p = 0.401$). For Losers, the difference is significant in $T2$, $T4$ (in both cases, $p = 0.012$) and (marginally) $T3$ ($p = 0.093$); it is not significant in $T1$ ($p = 0.484$).

[26]The gambler's fallacy – the failure of people to understand that, when draws from a certain (known) distribution are independent, previous realizations do not affect the probability distribution of the current draw – has long been documented by psychologists and economists. See, e.g., Tversky and Kahneman (1974) and Rabin (2002). The gambler's fallacy can be seen as a manifestation of the more general representativeness heuristic: people tend to believe that even a small sample should resemble (i.e. be representative of) the underlying distribution. In this sense, when the toss of a coin yields "head", people believe that a "tail" is more likely to occur in the next toss, because the sequence "head-tail" is more representative of the probability distribution than the sequence "head-head".

[27]One could also argue that this may be simply the result of a lack of understanding of the instructions. Although the instructions stated explicitly that "the number of objects assigned to each seller is independent of [...] the number of objects assigned to the other seller", subjects may have been incorrectly believing that, within each session/group, exactly a fraction p of subjects was assigned capacity $c = 2$, and the remaining fraction a capacity $c = 1$. In this case, a subject with capacity $c = 1$ would then expect that the opponent has a greater than p chance of having capacity $c = 2$. We thank an anonymous referee for suggesting this alternative, observationally equivalent, interpretation.

between $T2$ and $T4$ ($p = 0.388$); second, asymptotic beliefs in $T1$ and $T3$ are lower than in $T2$ and $T4$ (for all the pairwise comparisons, $p < 0.001$); third, while asymptotic beliefs in $T1$ and $T3$ are aligned with the prior probabilities (in $T1$, $p = 0.595$; in $T3$, $p = 0.056$), they are significantly lower in $T2$ and $T4$ (in both treatments, $p < 0.001$).

Finally, all the results concerning the effect of winning $A1$ on the beliefs are confirmed asymptotically: the difference between Winners' and Losers' beliefs is negative and significant in $T1$ ($p = 0.002$) and $T2$ ($p < 0.001$), while it is positive but not significant in $T3$ ($p = 0.383$) and $T4$ ($p = 0.168$).

We now analyze in more detail the determinants of beliefs, by separately considering Losers and Winners of $A1$.[28] The effect of the winning bid on Losers' beliefs, which, theoretically, should be negative (see prediction *A1-(b)*), is parametrically investigated in the first two columns of Table 10. We employ two different empirical strategies: in column (1), we assess the direct effect of the winning bid; in column (2), we use the difference between the Loser's bid and the (observed) winning bid as the main regressor. Both regressions also control for the time trend and for the subject's own capacity.

(Table 10 about here)

In general, the information revealed by the winning bid in $A1$ exerts limited effects on the Losers' beliefs: the level of the winning bid reduces the Loser's stated beliefs only in $T1$, ($p = 0.003$), whereas this effect is not significant in the other three treatments (in $T2$, $p = 0.307$; in $T3$, $p = 0.161$; in $T4$, $p = 0.140$). Similarly, when we use the difference between the Loser's bid and the observed winning bid as a regressor, we find a negative and significant effect in $T4$ only ($p = 0.012$) and no significant effect in the other treatments (in $T1$, $p = 0.132$; in $T2$, $p = 0.665$; in $T3$, $p = 0.219$).

Moreover, Losers' beliefs do not depend on own capacity. Indeed, in both specifications, the coefficient of the capacity dummy is never significant (from column (1): $p = 0.146$ in $T1$; $p = 0.954$ in $T2$; $p = 0.771$ in $T3$; $p = 0.297$ in $T4$; from column (2): $p = 0.286$ in $T1$; $p = 0.890$ in $T2$; $p = 0.840$ in $T3$; $p = 0.144$ in $T4$).

Column (3) focuses on Winners. Results suggest that the gambler's fallacy effect detected in Table 8 is mainly driven by Winners' beliefs: indeed, we find a negative and significant coefficient of the capacity dummy in all treatments (in $T1$ and $T4$, $p < 0.001$; in $T2$, $p = 0.021$; in $T3$, $p = 0.003$).

We summarize our findings on stated beliefs in the following statements.

R7. **Differences in beliefs across treatments.** *In line with the ordering in the prior probabilities, there is no statistical difference in beliefs between $T1$ and $T3$ and between $T2$ and $T4$. Moreover, beliefs in $T1$ and $T3$ are lower than those in $T2$ and $T4$.*

R8. **Beliefs and prior probabilities.** *In $T2$ and $T4$, beliefs of both Winners and Losers of $A1$ are below the prior probabilities. In $T1$, Losers' beliefs are aligned with the prior probability, whereas Winners report lower beliefs. Finally, in $T3$, Winners' beliefs are aligned with the prior probability, whereas Losers report lower beliefs.*

R9. **Beliefs and outcome of $A1$.** *Winning in $A1$ substantially reduces beliefs in $T1$ and $T2$. Instead, this effect is positive in $T3$ and $T4$, although limited in magnitude.*

[28] It is worth recalling that, upon stating their beliefs on the (initial) capacity of the opponent, Winners and Losers possess different information on the outcome of $A1$: whereas Losers are informed of the (winning) bid of the opponent, Winners only know that the opponent's bid was larger than (or maybe equal to) their own.

R10. **Losers' and Winners' beliefs.** *In most treatments, Losers' beliefs are insensitive to the winning bid. In all treatments, Winners' beliefs negatively and significantly depend on their own capacity.*

Results *R7-R10*, when combined with results *R4-R6*, suggest that the belief updating process by bidders is imperfect. To see this, notice first that, in $T2$ and $T4$ (where the prior probability is equal to 75%), average beliefs are significantly biased downward (result *R8*); in particular, they lie around the midpoint between 50% and the prior probability (75%). Most importantly, it is the observed relation with the bidding behavior in $A1$ that clearly points toward an inconsistency of the beliefs. Although subjects with capacity $c = 2$ tend to make lower bids than those with capacity $c = 1$ (result *R6*), this is not reflected in the beliefs: Losers do not properly adjust their beliefs when they observe a lower winning bid (result *R10*); moreover, in two treatments out of four ($T3$ and $T4$), Winners' beliefs are larger than Losers' (result *R9*), whereas result *R6* would imply the opposite. Finally, Winners' beliefs are affected by the gambler's fallacy. Taken together, these considerations greatly weaken the empirical validity of the hybrid equilibrium (prediction *A1-(b)*).

This conclusion is further confirmed when we analyze the degree at which subjects are able to formulate accurate beliefs. To this end, we construct a (conservative) indicator of belief correctness: we classify a belief as correct if it is strictly larger (smaller) than 50 and the opponent has indeed a capacity $c = 2$ ($c = 1$). Notice, first, that, in general, in spite of the conservativeness of our indicator, the proportion of incorrect beliefs is relatively large: 47.80% in $T1$, 35.23% in $T2$, 54.68% in $T3$, and 38.10% in $T4$, respectively. Second, even though Losers have more precise information than Winners on the actual bid placed by the opponent, this advantage does not translate into more accurate beliefs. Indeed, the proportions of Losers formulating correct beliefs (53.58% in $T1$, 66.67% in $T2$, 47.63% in $T3$, 61.85% in $T4$, respectively) are very similar to those expressed by Winners (50.79% in $T1$, 62.84% in $T2$, 42.90% in $T3$, 61.96% in $T4$, respectively), leading to nonsignificant differences (according to a two-sided proportion test: $p = 0.481$ in $T1$; $p = 0.301$ in $T2$; $p = 0.237$ in $T3$; $p = 0.976$ in $T4$).

We summarize the evidence on belief correctness in the following statement.

R11. **Correctness of beliefs.** *In all treatments, the proportion of incorrect beliefs is high and not different between Winners and Losers.*

5 Discussion

A direct comparison between our experimental evidence, summarized in results *R1-R11* in Section 4, and the predictions implied by the equilibrium theory (presented in Section 3.2), leads us to draw mixed conclusions: whereas bidding behavior in $A2$ basically conforms with sequential rationality, behavior in $A1$ departs from equilibrium requirements in that beliefs are clearly inconsistent with actual play.[29] In particular, although we detect a strong and persistent capacity effect – bidders with capacity $c = 2$ make significantly lower bids than those with capacity $c = 1$ –, subjects do not seem able to appreciate the informative content

[29]We can be pretty confident that the beliefs stated by subjects in the belief elicitation phase genuinely reflect their thoughts. In fact, subjects were rewarded for the accuracy of their beliefs. Most importantly, the analysis of $A2$ clearly shows that Losers' bids strongly depend on their own beliefs and they do so in the correct direction, suggesting that they truly reflect what they were thinking.

revealed by the outcome of the auction (observed by all) and by the winning bid (observed by the Loser).

Notice that the presence of the capacity effect together with a lack of proper belief updating suggests that, while subjects seem to properly account for the opportunity cost of early bids (which pushes bidders with capacity $c = 1$ to bid more cautiously, i.e. higher, than those with capacity $c = 2$), the signaling cost of early bids is essentially absent. Theoretically, the signaling cost should countervail the opportunity cost of early bids: a bidder with capacity $c = 2$ should mimic, at least partially, one with capacity $c = 1$, in order to avoid revealing her type, as this would dramatically increase competition in the second auction. Clearly, absent this signaling cost, but given the opportunity cost of early bids, bidders with different capacities rationally place different bids.

Interestingly, the response by subjects to the information revealed at the end of the first auction is heterogeneous across Winners and Losers. Winners, who do not observe their opponents' bids, seem to be affected by an irrational sort of the gambler's fallacy: they tend to attach a higher probability to their opponents having a different capacity than their own (whereas, if they were rational, and given the capacity effect, they should realize that the opponent, being a Loser, is more likely to have capacity $c = 1$).

On the other hand, Losers, who do observe their opponents' bids, do not seem to be affected by the gambler's fallacy. Still, they do not realize that, because of the capacity effect, the mere fact of losing the first auction and the observation of a lower winning bid should lead them to adjust their beliefs upward. In fact, in most treatments, Losers' beliefs are unaffected by the level of the winning bid; moreover, in treatments $T1$ and $T3$ (where the prior probability that $c = 2$ is 50%), there is, on average, very little adjustment of beliefs, whereas in treatments $T2$ and $T4$ (where the prior probability that $c = 2$ is 75%), average beliefs are adjusted downward, as if they were attracted toward the 50/50 odds.

Therefore, it seems that, though both Winners and Losers do not correctly update their beliefs, whether or not the opponent's bid is revealed at the end of the first auction prompts different responses. The absence of exact information on the bid placed by the opponent seems to induce Winners to rely on a simple, though irrational, heuristic: the gambler's fallacy. The observation of the opponent's bid, instead, leads Losers to abandon the gambler's fallacy heuristic, as if they realized that the bid may have some informative content regarding her opponent's capacity. However, they eventually do not significantly adjust their beliefs, as if the information incorporated in the bid were not informative enough. This hypothesis has already been explored in the literature. Mullainathan (2002), starting from the idea that people use coarse categories to make inferences, argues that, rather than updating continuously, individuals change categories only when they see enough data to suggest that an alternative category better fits the data. In the same vein, Henckel et al. (2018) speak about sticky belief adjustment to explain their experimental evidence: subjects stick to their initial beliefs until enough evidence has accumulated to pass a threshold of statistical significance; at this point, they revise their beliefs, but this adjustment is typically insufficient. Clearly, the consequence of this type of cognitive bias is that individuals' beliefs will tend to stick too much on the priors, leading to what is sometimes called conservatism.

The idea that, despite the strong capacity effect, Losers may find it hard to infer the relationship between the capacity and the bid of the opponent has some supporting evidence in our experiment. Notice, first, that, despite the fact that the distributions of the bids placed by subjects with capacity $c = 2$ and $c = 1$ significantly differ, they have very similar lower bounds. The right part of Table 4 reports, for each treatment, the 5th and the 95th percentile of the conditional bid distributions for subjects with capacity $c = 1$ and $c = 2$. In all treatments, the difference in the 5th percentile is very small: according to a (two-sided) Wilcoxon signed-rank test, the difference reaches marginal significance only in $T3$ ($p = 0.089$), whereas it is nonsignificant in the remaining three treatments (in $T1$, $p = 0.140$; in $T2$, $p = 0.778$; in $T4$, $p = 0.566$). This means that also bidders with capacity $c = 1$ sometimes make very low bids. And, as a matter of fact, they often outbid bidders with capacity $c = 2$: when a subject with capacity $c = 1$ is matched with one with capacity $c = 2$, the former wins 38.97% of the times in $T1$, 43.94% in $T2$, 45.70% in $T3$ and 47.90% in $T4$. Therefore, losing the first auction and observing a low winning bid is far from being clear-cut evidence in favor of the opponent having capacity $c = 2$.

In addition, if we look at subjects' behaviors in $A1$ dynamically, we can see that capacity is not the unique determinant of bids. To this end, Table 11 studies the relationship between bids in $A1$ in two consecutive periods ($(t - 1)$ and t).

(Table 11 about here)

The specifications in columns (1) and (2) include a dummy variable that captures the outcome (winning or losing) of $A1$ in the previous period of the experiment. They show that, after controlling for a subject's capacity, winning in one period does not significantly affect bids in the subsequent period (in $T1$, $p = 0.603$; in $T2$, $p = 0.376$; in $T3$, $p = 0.131$; in $T4$, $p = 0.359$). On the other hand, the negative effect of capacity on bids (result *R6*) remains highly significant (in $T1$, $T2$, and $T4$, $p < 0.001$; in $T3$, $p = 0.016$).

Column (3) presents the results of a regression in which the dependent variable is the difference between the bids in $A1$ placed by the same bidder in two consecutive periods. Columns (4) and (5) replicate the same regression after splitting the sample between Winners and Losers of $A1$ in the previous period.[30] Notice that, in all treatments, Winners (column(4)) tend to increase their bids in the next period ($p < 0.001$ in all treatments), whereas Losers (column (5)) adjust their bids in the opposite direction ($p < 0.001$ in all treatments). This explains why, between subjects, and after controlling for bidders' capacities, there is no significant difference between Winners' and Losers' bids (columns (1) and (2)). Columns (4) and (5) show that the negative effect of capacity on bids is still present for the split samples (for Winners: $p < 0.001$ in $T1$ and $T2$; $p = 0.294$ in $T3$; $p = 0.067$ in $T4$; for Losers: $p < 0.001$ in $T1$; $p = 0.034$ in $T2$; $p = 0.032$ in $T3$; $p = 0.237$ in $T4$). Notice, however, that the downward adjustment in Losers' bids is, on average, larger than the upward adjustment in Winners' bids (the constant is 8.401 in column (4) and -13.370 in column (5)), so that the net effect is negative. These results indicate that subjects, beyond bidding more aggressively when they have two units to sell, adjust bids over periods according to a Learning Direction Theory style (see, e.g., Selten, 2004), bidding more (less) aggressively if they lost (won) the previous period's first auction. The presence of this dynamic determinant of bidding behavior may obfuscate the link between capacity and bids, adding complexity to the task of inferring the actual capacity of the opponent.

[30]It is worth remarking that, whereas in the analysis of the beliefs and of the bids in $A2$, the terms Winners and Losers referred to the outcome of $A1$ *in that period*, here, in the analysis of first auction bids, the terms Winners and Losers refer to the outcome of $A1$ *in the previous period*.

6 Conclusion

This article reported the results of an experiment involving two sequential first-price procurement auctions, where sellers may be capacity constrained, and this information is privately held.

That capacity constraints may crucially affect behavior in repeated procurement auctions is a well documented empirical fact. However, we are not aware of other experimental papers that study sequential auctions where bidders' capacities are private information.

We designed four treatments that differ in the ex ante probability distribution of sellers' capacities and in the (exogenous) likelihood that the second auction will actually be carried out. This last assumption was made to match real world situations, such as electricity markets, where the quantity that will be demanded in later auctions is not known a priori, depending on exogenous, institutional and environmental factors. In all treatments, at the end of the first auction, bidders were informed of the outcome and of the winning bid, and their beliefs regarding the initial capacity of the opponent were elicited through an incentive compatible procedure.

The results of the experiment show that: (i) observed behavior in the second auction is overall consistent with sequential rationality, (ii) first auction bids are decreasing in the capacity of the bidder, but (iii) stated beliefs are inconsistent with actual play.[31] Hence, subjects seem to be aware of the opportunity cost of early bids (which leads capacity constrained bidders to bid more cautiously than unconstrained ones); on the other hand, since they do not recognize the informative content of bids, the potential signaling cost associated with early bids does not come into play.

Our evidence suggests that the main source of departure from the theoretical predictions rests in the failure of subjects to properly adjust their beliefs from the observation of others' behavior. This result does not come as a surprise. On the one hand, that individuals find it hard to appreciate the relation between behavior and private information is at the heart of the well-known phenomenon of winner's curse in common value auctions. On the other hand, there is large evidence in the experimental literature that most individuals are, at best, imperfect Bayesian updaters. In most of the experiments showing this, the task is relatively simple: subjects are asked to assign probabilities to some unknown event after receiving an exogenously generated signal. In our case, instead, the signal is represented by the actual play of the opponent (who, by the way, varies every period). Hence, forming a belief on the opponent's private information requires, first, forming a belief on her strategy of play and how this strategy depends on her private information. This, in turn, involves asking how and whether the opponent is rational, what she believes and so on. In brief, bidding in the first auction and, even more so, correctly guessing the opponent's capacity from her bid is, admittedly, a fairly complicated task, much harder than bidding in the second auction.[32]

In conclusion, our paper confirms that predictions based on the assumption that individuals update correctly are likely to be belied by the facts. In the specific context of procurement

[31] Interestingly, the result that bids in the second auction are "closer" to the theoretical prediction than those in the first is also found by Février et al. (2007), Saini and Suter (2015) and Otamendi et al. (2018). Notice that in these experiments signaling issues are absent. On the other hand, Brosig and Reiß (2007), where, instead, first auction bids have signaling content, find a significant departure from equilibrium bids in the second auction. However, given that, in their experiment, beliefs are not elicited, they cannot assess whether these bids are consistent with sequential rationality.

[32] By the way, at the end of the experiment, more than one subject confided us that, while it was clear to her or him what strategy to follow in the second auction, it was much less clear what to do in the first.

markets, a failure by firms to appreciate the relationship between other competitors' past bidding behavior and their private information translates into a misperception of the actual level of competitiveness in future auctions. This may produce significant discrepancies between expected and actual awarding prices; moreover, in situations where firms also differ in their production costs, a wrong perception of the competitive environment may eventually lead to an inefficient allocation of the contract. In this light, our paper suggests that the differences between observed and predicted behavior – and their welfare and distributional consequences – highlighted by the empirical literature on sequential auctions (e.g., Jofre-Bonet and Pesendorfer, 2000, 2003, and Hortaçsu and Puller, 2008) may be related to a misinterpretation of the information transmitted endogenously across subsequent auctions.

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A Proofs

A.1 Proof of Proposition 1

(i) Suppose $\mu_L \in (0,1]$. Let the cumulative distribution functions $\beta_W(b)$ and $\beta_L(b)$ denote the equilibrium bidding strategies of the Winner and the Loser, respectively. Let S_W and S_L denote the corresponding supports.

Notice, first, that bidding $b < (1-\mu_L)R$ is strictly dominated for the Loser, as the Loser can guarantee herself an expected payoff equal to (at least) $(1-\mu_L)R$ by bidding exactly R. This immediately implies that, in equilibrium, the Winner will not bid below $(1-\mu_L)R$ either. Moreover, for the Winner bidding R cannot be optimal because either it gives her a zero probability of winning or, if R is a mass point of $\beta_L(b)$, bidding slightly less than R is a profitable deviation. Hence, we have that, necessarily, $S_L \subseteq [(1-\mu_L)R, R]$, $S_W \subseteq [(1-\mu_L)R, R)$.

Second, equilibrium bidding strategies are necessarily continuous in $[(1-\mu_L)R, R)$, i.e. there are no mass points in that interval. To see this, suppose, to the contrary, that the strategy of, say, the Loser attaches strictly positive probability to some $b \in [(1-\mu_L)R, R)$. Then, for sufficiently small ε, the Winner will never make a bid in $[b, b+\varepsilon]$, as these bids are certainly worse than bidding just below b. But if the Winner never makes a bid in $[b, b+\varepsilon]$, then bidding b for the Loser cannot be optimal.

Third, the Winner will not place a bid in an interval in which the Loser never places any bid. This is also true for the Loser, unless the Loser bids exactly R. This means that $S_W = S_L \setminus \{R\}$.

Fourth, $S_W = [(1-\mu_L)R, R)$, $S_L = S_W \cup \{R\}$ and the Loser bids R with strictly positive probability. To see this, suppose that R is not played with strictly positive probability by the Loser: then a bid close to $\sup(S_W)$ cannot be optimal for the Winner, as it would give her (essentially) a null probability of winning, and bidding $(1-\mu_L)R$ would be a profitable deviation. Given that $R \in S_L$, the payoff of the Loser must be equal to $(1-\mu_L)R$ for any bid in S_L, but this implies that $\inf(S_W) = \inf(S_L) = (1-\mu_L)R$: in fact, if $\inf(S_W) > (1-\mu_L)R$, then bidding just below $\inf(S_W)$ would guarantee the Loser a payoff that is strictly larger than $(1-\mu_L)R$. Moreover, $\sup(S_W) = \max(S_L) = R$: if $\sup(S_W) < R$, then bidding $\sup(S_W)$ would give the Loser the same probability of winning as bidding R, but with a strictly lower payoff in case of winning. Finally, there is no interval in $[(1-\mu_L)R, R)$ to which equilibrium bidding strategies attach null probability: if there were such an interval (x, y), then any bid $b \in (x, y)$ would give strictly lower payoff than any bid larger than b, still within (x, y).

To sum up, the equilibrium bidding strategy of the Winner, $\beta_W(b)$, has no mass points and it is strictly increasing over $[(1-\mu_L)R, R)$; the equilibrium bidding strategy of the Loser, $\beta_L(b)$, has a mass point at R and it is strictly increasing over $[(1-\mu_L)R, R)$.

Now, the expected payoff in $A2$ of the Loser when she bids $b \in [(1-\mu_L)R, R)$ and the Winner bids according to $\beta_W(b)$ is

$$\pi_L^2(b \in [(1-\mu_L)R, R)) = [(1-\mu_L) + \mu_L (1-\beta_W(b))]\, b = [1-\mu_L \beta_W(b)]\, b.$$

The expected payoff in $A2$ of the Winner when she bids $b \in [(1-\mu_L)R, R)$ and the Loser bids according to $\beta_L(b)$ is

$$\pi_W^2(b \in [(1-\mu_L)R, R)) = [\gamma + (1-\gamma)(1-\beta_L(b))]\, b = [1-(1-\gamma)\beta_L(b)]\, b,$$

where $\gamma > 0$ is the probability with which the Loser bids R. In a mixed strategy equilibrium, the bidder's expected payoff must be constant over its support. In particular, given that we know that $\pi_L^2(b = (1-\mu_L)R) = \pi_W^2(b = (1-\mu_L)R) = (1-\mu_L)R)$, it must be that, for all $b \in ((1-\mu_L)R, R)$:

$$[1 - \mu_L \beta_W(b)]\, b = (1-\mu_L)R,$$

and

$$[1 - (1-\gamma)\beta_L(b)]\, b = (1-\mu_L)R.$$

The above system admits a unique solution, which is the one stated in the proposition. In particular, $\gamma = (1-\mu_L)$. This completes the proof for the case in which $\mu_L \in (0,1]$.

(ii) When, instead, $\mu_L = 0$ (the Loser believes that the Winner is not going to bid in the second auction), the Loser will certainly bid R. The Winner will thus best respond by bidding just below R.

□

A.2 Proof of Proposition 2

Let A_i ($i = 1, 2$) denote the support of the equilibrium bidding strategy of a bidder with capacity i.

Notice, first, that, in equilibrium, the expected payoff of a bidder with capacity $c = 2$ must be strictly larger than the expected payoff of a bidder with capacity $c = 1$, i.e. $\pi_2(a_2 \in A_2) > \pi_1(a_1 \in A_1)$.[33] To see this, notice that the difference between the expected payoff of a bidder with capacity $c = 2$ and the expected payoff of a bidder with capacity $c = 1$ is due to the fact that, in case of winning the first auction, a bidder of type $c = 2$ will also take part in $A2$, where she can possibly sell a second good. In symbols, we have that, for all a,

$$\pi_2(a) - \pi_1(a) = \mathrm{PW}(a) \times q(1 - \mu_L(a))R \geq 0,$$

where $\mathrm{PW}(a)$ is the probability of winning the first auction with a bid equal to a and $(1 - \mu_L(a))R$ is the payoff in $A2$ that a bidder with capacity $c = 2$ can expect to obtain if she wins the first with a bid equal to a (see Proposition 1). Now, for all $a' \in A_1$ and $a'' \in A_2$ (i.e. a' and a'' are equilibrium bids for type $c = 1$ and $c = 2$, respectively), it must be:

$$\pi_1(a') \geq \pi_1(a''), \qquad \pi_2(a') \leq \pi_2(a'').$$

Given that, for all a, $\pi_2(a) - \pi_1(a) \geq 0$, we have the following chain of inequalities:

$$\pi_1(a'') \leq \pi_1(a') \leq \pi_2(a') \leq \pi_2(a'').$$

Now, suppose, by contradiction that, in equilibrium, expected payoffs of types $c = 2$ and $c = 1$ are equal, i.e. $\pi_1(a') = \pi_2(a'')$. Then it must also be that $\pi_1(a') = \pi_2(a')$, or $\mathrm{PW}(a') \times q(1 - \mu_L(a'))R = 0$. But this inequality cannot be satisfied for all $a' \in A_1$: in fact, if the equilibrium bidding strategy of a type-1 bidder attaches strictly positive probability to some

[33] For the sake of clarity, here we suppress the dependence of a bidder's expected payoff from the other bidder's strategy.

a', then $\mathrm{PW}(a') > 0$ and $\mu_L(a') < 1$; if, instead, no element of A_1 is played with strictly positive probability, then, necessarily, $\mathrm{PW}(a') > 0$ for all $a' < \sup(A_1)$ and $\mu_L(a')$ cannot be equal to 1 for all of them. Hence, it must necessarily be $\pi_1(a') < \pi_2(a'')$, and the above chain of inequalities reduces to

$$\pi_1(a'') \leq \pi_1(a') < \pi_2(a') \leq \pi_2(a''). \tag{3}$$

(i) **Separating Equilibrium.** Suppose that there is a separating equilibrium. As before, let $a' \in A_1$ and $a'' \in A_2$ be equilibrium bids for type $c = 1$ and $c = 2$, respectively (notice that, in a separating equilibrium, A_1 and A_2 are disjoint sets). Notice that, because $a'' \in A_2$ but $a'' \notin A_1$, it must be $\mu_L(a'') = 1$. Therefore, we have

$$\pi_2(a'') - \pi_1(a'') = \mathrm{PW}(a'') \times q(1 - \mu_L(a''))R = 0,$$

i.e. $\pi_2(a'') = \pi_1(a'')$. But then (3) would be violated. We conclude that there is no separating equilibrium.[34]

(ii) **Pooling Equilibrium.** In a pooling equilibrium, $A_1 = A_2 = A$ and $\mu_L(a) = p$ for all $a \in A$. Consider a bidder with capacity $c = 1$. This bidder can always guarantee herself a payoff equal to $q(1-p)R$ (by bidding R in the first auction, *de facto* skipping it). This implies that $\inf(A) \geq q(1-p)R$. Moreover, $\max(A) = q(1-p)R$. To see this, notice first that $\sup(A) \in A$ (therefore $\sup(A) = \max(A)$): in fact, $\pi_2(a) - \pi_1(a)$ must be strictly positive and constant for all $a \in A$; if $\sup(A) \notin A$, you can always find $\hat{a}$ sufficiently close to $\sup(A)$ such that $\pi_2(\hat{a}) - \pi_1(\hat{a})$ is lower than any strictly positive constant. Second, A must attach strictly positive probability to $\max(A)$: if this were not true, the condition $\pi_1(a) < \pi_2(a)$, that must always hold in equilibrium, would be violated for $a = \max(A)$. Third, if $\max(A) > q(1-p)R$, then a bidder with capacity $c = 1$ would rather slightly reduce her bid. We conclude that $A = \{q(1-p)R\}$. Now, given that a bidder with capacity $c = 1$ can always guarantee herself a payoff equal to $q(1-p)R$ by bidding R, such a bidder is certainly willing to play $a^* = q(1-p)R$ in the first auction, if the other bidder does so. For a bidder with capacity $c = 2$, the expected payoff by bidding $a^* = q(1-p)R$ (when the other bidder does so) is equal to

$$\pi_2(a^*) = \frac{3}{2}a^*.$$

Bidding more than a^* is certainly not profitable. Bidding $a < a^*$ is not profitable either if

$$\frac{3}{2}a^* \geq a + q(1 - \mu_L(a))R,$$

or

$$\mu_L(a) \geq \frac{1+p}{2} - \frac{a^* - a}{qR}.$$

Hence, if the Loser's beliefs satisfy the above condition for all $a < a^*$, the pure strategy a^* constitutes a pooling equilibrium (and no other pooling equilibrium is possible).

[34] This shows that, more generally, there is no equilibrium in which type $c = 2$ fully reveals her type with strictly positive probability.

(iii) **Hybrid Equilibrium.** (a) In a hybrid equilibrium, $A = A_1 \cap A_2$ is nonempty. It can be directly observed that $A_2 \setminus A_1 = \emptyset$, i.e. $A_2 = A$ (see point *(i)* of this proof).

(b) Moreover, if $A_1 \setminus A_2 \neq \emptyset$, then, for all $a' \in A_1 \setminus A_2$ and all $a'' \in A_2$, it must be $a' > a''$. To see this, observe that

$$\pi_1(a') = \pi_1(a''), \qquad \pi_2(a') \leq \pi_2(a''),$$

which implies

$$\pi_2(a') - \pi_1(a') \leq \pi_2(a'') - \pi_1(a''),$$

or

$$\text{PW}(a') \times qR \leq \text{PW}(a'') \times q(1 - \mu_L(a''))R,$$

where, in the last expression, we used the fact that $\mu_L(a') = 0$. Notice that the RHS of the inequality above must be strictly positive; hence, to satisfy it, it must be that, for any $a' \in A_1 \setminus A_2$, either $\text{PW}(a') = 0$ or $\text{PW}(a') < \text{PW}(a'')$. In both cases, the implication is the same: any $a' \in A_1 \setminus A_2$ must be larger than any $a'' \in A_2 = A$.

(c) Consider now any two bids $a', a'' \in A$, with $a' < a''$, Since they both belong to A, it must be

$$\pi_1(a') = \pi_1(a''), \qquad \pi_2(a') = \pi_2(a''),$$

which implies

$$\pi_2(a') - \pi_1(a') = \pi_2(a'') - \pi_1(a''),$$

or

$$\text{PW}(a') \times q(1 - \mu_L(a'))R = \text{PW}(a'') \times q(1 - \mu_L(a''))R.$$

Notice that it is not possible that, in equilibrium, $\text{PW}(a') = \text{PW}(a'')$ (which would imply $\mu_L(a') = \mu_L(a'')$): if this were the case, bidding a' would not be optimal, as a bid equal to a'' would guarantee the same probability of winning the first auction, with no impacts on beliefs, but a larger first auction payoff in case of winning. Hence, it must be $\text{PW}(a') > \text{PW}(a'')$, but then, to satisfy the equality above, we necessarily have $\mu_L(a') > \mu_L(a'')$, i.e. μ_L is strictly decreasing over A.

(d) See point *(ii)* of this proof.

□

A.3 Proof of Proposition 3

From Proposition 1, we know that a bidder's expected payoff from the second auction is $q(1 - \mu_L)R$. Under the assumption that bidders' beliefs coincide with the prior probability p, a bidder's expected payoff from the second auction is $\omega = q(1 - p)R$. Given this, the total expected payoff of a bidder with capacity $c = 2$ that bids a in $A1$ is

$$\pi_2(a) = (a + \omega)\,\text{PW}_1(a) + \omega\,(1 - \text{PW}_1(a)) = a\,\text{PW}_1(a) + \omega,$$

whereas the total expected payoff of a bidder with capacity $c = 1$ that bids a in $A1$ is

$$\pi_1(a) = a\,\text{PW}_1(a) + \omega\,(1 - \text{PW}_1(a)),$$

where $\text{PW}_1(a)$ is the probability of winning $A1$ with a bid equal to a.

Consider a level-k bidder: this bidder believes that the opponent is a level-$(k-1)$ bidder who, depending on her capacity, bids a_2^{k-1} or a_1^{k-1}. The only relevant bids to be considered are the following:

- $a > a_1^{k-1}$: with such bid, the bidder will not win $A1$;
- $a = a_1^{k-1}$: with such bid, the bidder will win $A1$ only if the opponent has capacity $c = 1$ and the tie is resolved in her favor;
- $a = a_1^{k-1} - 1$: with such bid, the bidder will win $A1$ only if the opponent has capacity $c = 1$ (notice that, under the assumptions of the Proposition, $a_1^{k-1} - a_2^{k-1} > 1$, i.e. $a_1^{k-1} - 1 > a_2^{k-1}$);
- $a = a_2^{k-1}$: with such bid, the bidder will win $A1$ if the opponent has capacity $c = 1$ or the opponent has capacity $c = 1$ and the tie is resolved in her favor;
- $a = a_2^{k-1} - 1$: with such bid, the bidder will win $A1$ for sure.

All other bids are certainly suboptimal.

Now, consider a level-k bidder with capacity $c = 2$. Her expected payoff associated with each of the bids specified above is:

- $\pi_2^k(a > a_1^{k-1}) = \omega$;
- $\pi_2^k(a = a_1^{k-1}) = a_1^{k-1}\,(1-p)/2 + \omega$;
- $\pi_2^k(a = a_1^{k-1} - 1) = (a_1^{k-1} - 1)\,(1-p) + \omega$;
- $\pi_2^k(a = a_2^{k-1}) = a_2^{k-1}\,(1-p/2) + \omega$;
- $\pi_2^k(a = a_2^{k-1} - 1) = a_2^{k-1} - 1 + \omega$.

It is immediate to see that a bid $a > a_1^{k-1}$ cannot be optimal. We then have that π_2^k is maximized at $a = a_2^{k-1} - 1$ if the following conditions simultaneously hold:

(a) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_1^{k-1}) \iff a_1^{k-1} - a_2^{k-1} \leq \frac{1+p}{2} a_1^{k-1} - 1$;

(b) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_1^{k-1} - 1) \iff a_1^{k-1} - a_2^{k-1} \leq p(a_1^{k-1} - 1)$;

(c) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_2^{k-1}) \iff a_2^{k-1} \geq \frac{2}{p}$.

Now, consider a level-k bidder with capacity $c = 1$. Her expected payoff associated with each of the bids specified above is:

- $\pi_1^k(a > a_1^{k-1}) = \omega$;
- $\pi_1^k(a = a_1^{k-1}) = a_1^{k-1}\,(1-p)/2 + \omega\,(1+p)/2$;
- $\pi_1^k(a = a_1^{k-1} - 1) = (a_1^{k-1} - 1)\,(1-p) + \omega\, p$;
- $\pi_1^k(a = a_2^{k-1}) = a_2^{k-1}\,(1-p/2) + \omega\, p/2$;
- $\pi_1^k(a = a_2^{k-1} - 1) = a_2^{k-1} - 1$.

π_1^k is maximized at $a = a_1^{k-1} - 1$ if the following conditions simultaneously hold:

(d) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a > a_1^{k-1}) \iff a_1^{k-1} \geq \omega + 1$;

(e) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_1^{k-1}) \iff a_1^{k-1} \geq \omega + 2$;

(f) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_2^{k-1}) \iff a_1^{k-1} - a_2^{k-1} \geq \frac{p\,(a_1^{k-1} - \omega - 1)}{2-p} + 1$;

(g) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_2^{k-1} - 1) \iff a_1^{k-1} - a_2^{k-1} \geq p\,(a_1^{k-1} - \omega - 1)$.

Hence, if conditions (a)-(g) simultaneously hold, a level-k bidder will find it optimal to bid $a_2^{k-1} - 1$ when her capacity is $c = 2$, and $a_1^{k-1} - 1$ when her capacity is $c = 1$. After noticing that (d) is implied by (e) and (a) is implied by (b) and (e), we are left with conditions (b), (c), (e), (f) and (g), i.e. $a_1^{k-1} \geq \omega + 2$, $a_2^{k-1} \geq 2/p$, and

$$\max \left[p\,(a_1^{k-1} - \omega - 1); \frac{p\,(a_1^{k-1} - \omega - 1)}{2-p} + 1 \right] \leq a_1^{k-1} - a_2^{k-1} \leq p\,(a_1^{k-1} - 1).$$

□

B Experimental Instructions

[Instructions were originally written in Italian. The following instructions refer to treatment $T1$, where p (the probability of being assigned a capacity $c = 2$) and q (the probability of implementing the second auction) were both set to 50%.]

Instructions

Welcome! Thanks for participating in this experiment. By following these rules carefully, you can earn an amount of money that will be paid cash at the end of the experiment.

There are 24 subjects participating in this experiment. Both the identities and the final payments of the subjects will remain anonymous throughout the experiment.

During the experiment you are not allowed to communicate with the other participants. If you have questions, raise your hand and one of the assistants will come to your seat and assist you. The following instructions are the same for all the participants.

General rules

The experiment will consist of 15 periods, and in every period you will be presented with the same economic situation that is described in what follows.

At the beginning of each period, you will be randomly and anonymously assigned to a new group of two subjects. This means that the composition of the group will change in every period and you will never interact with the same participant in two consecutive periods.

During the experiment, your earnings will be expressed in points. At the end of the experiment, the total number of points earned in the 15 periods will be exchanged in euro at the following exchange rate: 1 point = 2 euro cents.

How earnings are determined in each period

In each period of the experiment, you and the other subject in your group will compete as sellers of some (identical) objects to a buyer. In particular, you and the other subject in your group will participate in two sequential auctions, each involving a single object. The rules for determining the winner in each auction are very simple: the winner is the seller who offers the lowest selling price for the object.

The maximum price that the buyer is willing to pay for each object is 120 points. This means that you and the other subject cannot post a price for an object that is higher than 120 points.

At the beginning of the period, the computer will randomly assign an endowment of objects to each subject in the group. The endowment represents the maximum number of objects that the subject can sell to the buyer in that period. Specifically, each seller has a 50% probability of receiving an endowment of 2 objects and a 50% probability of receiving an endowment of 1 object. The number of objects assigned to each seller is independent of both the number of objects assigned to the other seller and the number of objects assigned in previous periods. You will receive information about your endowment at the beginning of each period and before participating in the first auction. However, you will receive no information about the endowment assigned to the other subject in your group.

The first auction

You and the other seller in your group will choose the price for the first object simultaneously and anonymously. Then, the computer will compare the two prices: the seller who offers the lowest price wins the first auction. If sellers offer the same price, the computer randomly selects the winner (each seller has a 50% probability of being selected). The winner's earnings are equal to the winning price and his or her endowment is reduced by 1 object. The loser's earnings are equal to zero, and his or her endowment remains unchanged.

The second auction

Only sellers that, at the end of the first auction, have a nonzero endowment of objects make an offer in the second auction. If your endowment is empty, you will not make any offer in the second auction. To determine the winner of the second auction, the computer will use the same rule of the first auction: the seller who chose the lowest price wins the second auction and ties are broken by the computer randomly and with equal probability. If only one seller makes an offer in the second auction, he or she wins regardless of his or her choice.

There is the possibility that the second auction will be revoked. If the auction is not revoked, the winner's earnings are equal to the winning price and are added to those obtained in the first auction, whereas the loser does not receive any additional point. If, instead, the second auction is revoked, the earnings of both subjects in that auction are equal to 0. Whether the second auction is revoked or not will be determined by the computer randomly: in particular, the probability that the auction is revoked is equal to 50%. Moreover, whether the second auction in a period is revoked or not does not depend on what happened in the previous periods. You will be informed about whether the second auction has been revoked only at the end of the period, after you and the other subject have made your choices.

Conjectures on the endowment of the other subject in the group

After being informed about the outcome of the first auction and before making your offer in the second auction, you will be asked to formulate two conjectures: the first about the probability that the other seller in your group has an endowment of 1 object and the second about the probability that he or she has an endowment of 2 objects.

Every conjecture must be a number between 0 and 100, where 0 means that you assign no chance to the fact that other subject has that specific endowment, and 100 means that you are sure that the other subject has that specific endowment. The two conjectures must sum up to 100.

Given your conjectures, at the end of the period you will take part in a lottery that, in case of success, will add 20 additional points to your earnings in that period. The lottery is designed in a way that it is in your interest to formulate your conjectures carefully: the higher the probability you expect the other seller to have a specific endowment, the larger should be your conjecture associated with that endowment.

The computer will assign a number of tickets included between 0 and 10000 to each of the two conjectures according to the following expressions:

$$\textit{tickets (endowment of 1 object)} = 10000[1 - (1 - \textit{conjecture 1 object}/100)^2],$$

$$\textit{tickets (endowment of 2 objects)} = 10000[1 - (1 - \textit{conjecture 2 objects}/100)^2].$$

Tickets assigned to each of the two conjectures will be numbered from 1 to the number determined according to the previous expressions.

At the end of the experiment, the computer will randomly draw an integer number between 1 and 10000 with equal probability. This random number will be compared with the tickets associated, according to the previous expressions, with the conjecture for the actual endowment of the other subject. If the random number is smaller than or equal to the number of tickets associated with that conjecture, you will earn 20 additional points; otherwise you will earn nothing.

Example. You expect the other subject to have an endowment of 1 object with a probability of 70 over 100 and, therefore, you expect the other subject to have an endowment of 2 objects with a probability of 30 over 100. In this case, according to the expressions above, the computer assigns 9100 tickets to the former conjecture and 5100 tickets to the latter conjecture. Note that formulating a higher conjecture for the event you expect to be more likely to occur is always convenient for you as, in case your expectation turns out to be correct, you will have a higher probability to win the lottery. Suppose that the actual endowment of the other subject is 2 objects: this means that you will take part in the lottery with 5100 tickets. Suppose that the random number drawn by the computer is 4812. Since the random number is lower than the overall number of tickets assigned to your conjecture that the other subject's endowment is 2 objects, you earn the 20 additional points.

If you attach a probability of 100 over 100 to a certain endowment, then, regardless of the random number drawn by the computer, you will earn the additional 20 points if the other subject has actually been assigned that endowment, whereas you will earn nothing in the opposite case.

Before confirming your choices, you will have the opportunity to simulate the number of tickets assigned to each of the two conjectures using the "Calculate tickets" button. You can modify your conjectures how many times you wish. To confirm your conjectures, click on the "Confirm your conjectures" button.

At the end of each period, you will be informed about the actual endowment assigned to the other seller, the result of the lottery, the winning bids in the first and in the second auction, and your earnings in that period.

C Figures and Tables

Table 1 – Treatments: parameters and pooling equilibrium predictions.

	p	q	N	a	$b_{\min}$	$\mathbb{E}[b_W]$	$\mathbb{E}[b_L]$
$T1$	0.5	0.5	48	30	60	83.2	101.6
$T2$	0.75	1	48	30	30	55.5	71.6
$T3$	0.5	0.25	48	15	60	83.2	101.6
$T4$	0.75	0.5	48	15	30	55.5	71.6

Notes: N is the number of subjects; a is the pooling equilibrium bid in $A1$; $b_{\min}$ is the lower bound of the equilibrium bidding (mixed) strategy in $A2$; $\mathbb{E}[b_W]$ and $\mathbb{E}[b_L]$ are the expected values of equilibrium bids in $A2$ placed by Winners and Losers of $A1$, respectively.

Table 2 – Bids in $A2$, by outcome of $A1$.

	b_W	$\widehat{b}_W\|\mu$	*Obs.*	b_L	$\widehat{b}_L\|\mu$	*Obs.*
$T1$	53.937	71.801	209	80.842	85.681	360
	(26.595)	(35.650)		(35.911)	(36.878)	
$T2$	39.414	57.799	273	54.172	69.785	360
	(29.743)	(29.234)		(39.964)	(34.152)	
$T3$	48.033	84.195	184	72.383	98.622	360
	(27.435)	(25.529)		(36.392)	(23.195)	
$T4$	41.582	65.863	273	64.197	78.594	360
	(23.446)	(26.775)		(37.076)	(28.835)	

Notes: This table reports the mean and standard deviation (in parentheses) of bids in the second auction of each treatment and over all periods by splitting the sample according to the outcome of the first auction (b_W for Winners and b_L for Losers). The table also reports predicted bids of both Winners and Losers ($\widehat{b}_W$ and $\widehat{b}_L$, respectively) computed by using results in Proposition 1 and subjects' stated beliefs.

Table 3 – Bids in $A2$: parametric analysis.

	b				b_W	b_L
	(1)	(2)	(3)	(4)	(5)	(6)
$T1$	6.603	1.615	9.630	8.003	1.096	11.799
	(5.704)	(6.702)	(6.783)	(7.108)	(8.182)	(8.218)
$T2$	-16.322^{***}	-8.292	-2.823	-0.672	5.641	6.503
	(5.690)	(7.575)	(7.599)	(7.842)	(9.247)	(9.386)
$T4$	-10.095^{*}	0.928	1.794	4.965	4.226	1.835
	(5.690)	(7.278)	(7.204)	(7.501)	(8.908)	(8.859)
μ		-0.240^{***}	-0.247^{***}	-0.247^{***}	0.055	-0.387^{***}
		(0.062)	(0.058)	(0.059)	(0.105)	(0.073)
$T1 \times \mu$		0.117	0.005	-0.001	-0.049	-0.048
		(0.079)	(0.075)	(0.076)	(0.132)	(0.094)
$T2 \times \mu$		-0.046	-0.121	-0.112	-0.238^{*}	-0.218^{*}
		(0.089)	(0.085)	(0.085)	(0.138)	(0.111)
$T4 \times \mu$		-0.104	-0.099	-0.093	-0.152	-0.017
		(0.085)	(0.080)	(0.080)	(0.132)	(0.101)
w_{A1}			-20.864^{***}	-20.861^{***}		
			(2.492)	(2.491)		
$T1 \times w_{A1}$			-5.222	-5.353		
			(3.538)	(3.538)		
$T2 \times w_{A1}$			3.801	3.892		
			(3.365)	(3.364)		
$T4 \times w_{A1}$			2.125	2.164		
			(3.343)	(3.341)		
$Period$				-0.007	-1.342^{***}	0.715^{**}
				(0.266)	(0.378)	(0.324)
$T1 \times Period$				0.273	0.856^{*}	0.119
				(0.372)	(0.513)	(0.458)
$T2 \times Period$				-0.406	-0.325	-0.063
				(0.361)	(0.475)	(0.458)
$T4 \times Period$				-0.513	-0.387	-0.435
				(0.361)	(0.479)	(0.455)
$cons$	64.208^{***}	75.044^{***}	82.480^{***}	82.519^{***}	56.569^{***}	83.629^{***}
	(4.038)	(4.859)	(4.828)	(5.046)	(6.225)	(5.870)
lrl	-11536.58	-11506.98	-11355.97	-11353.57	-4283.03	-6906.19
$Wald - \chi^2$	19.42	95.66	411.16	420.40	102.14	196.21
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
$Obs.$	2379	2379	2379	2379	939	1440

Notes: this table reports estimates (with clustered standard errors in parentheses) from two-way linear random effect models accounting for both potential individual dependency over repetitions and dependency within re-matching group. The dependent variable is the bid in $A2$. The table reports results by pooling observations (columns 1-4), and by splitting the sample between Winners and Losers of $A1$ (columns 5-6). $T1$, $T2$ and $T4$ are treatment dummies. μ is a subject's stated belief that the opponent's capacity was two units. w_{A1} is a dummy whose value is 1 if that subject won the first auction. $Period$ is a linear trend that takes value 0 in the first period. $T1 \times \mu$, $T2 \times \mu$, $T4 \times \mu$, $T1 \times w_{A1}$, $T2 \times w_{A1}$, $T4 \times w_{A1}$, $T1 \times Period$, $T2 \times Period$, $T4 \times Period$, are interaction terms. Significance levels are denoted as follows: $^{*} : p < 0.1$; $^{**} : p < 0.05$; $^{***} : p < 0.01$.

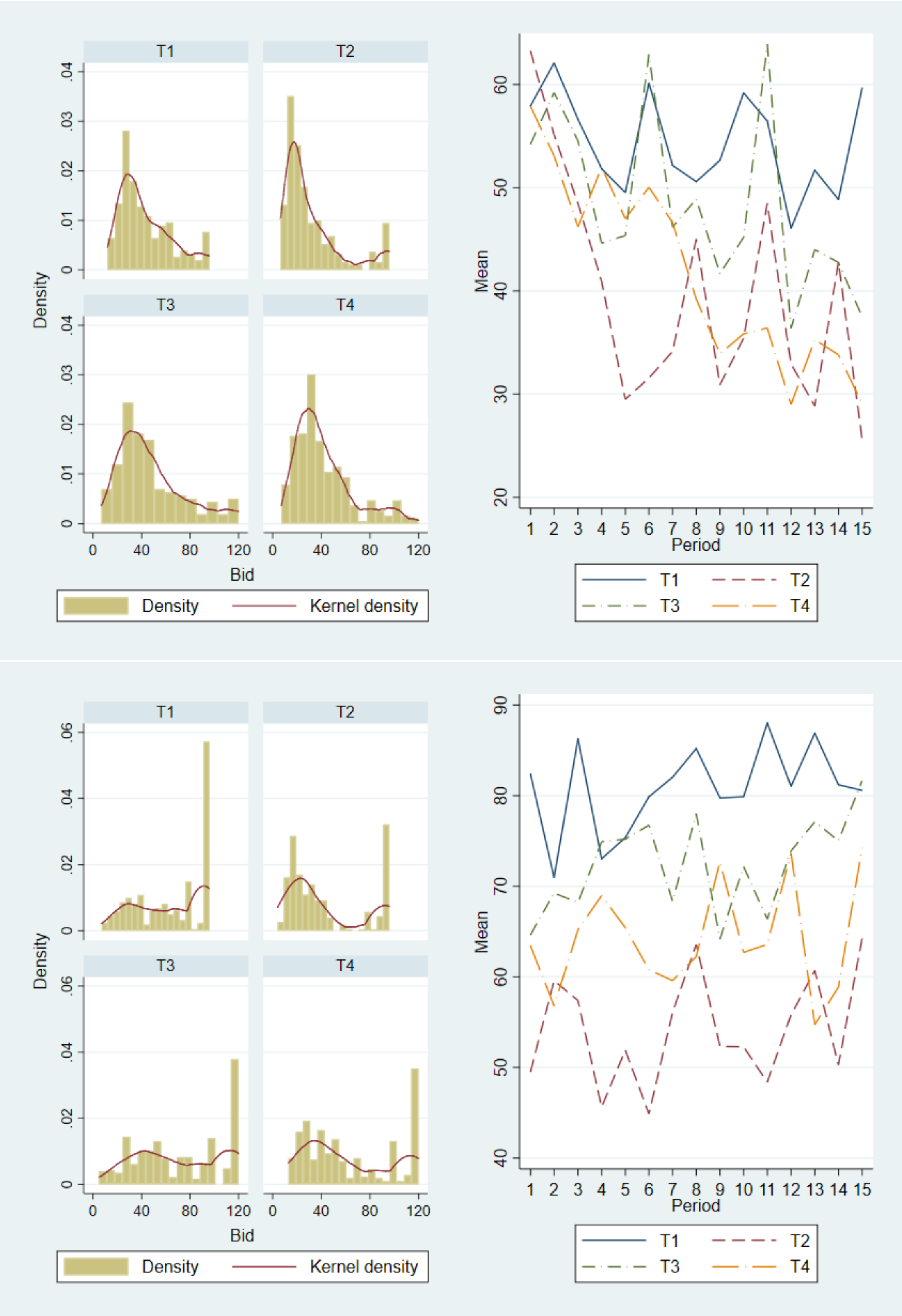

Figure 1 – Bids in $A2$: Winners (upper panel) and Losers (lower panel) of $A1$.

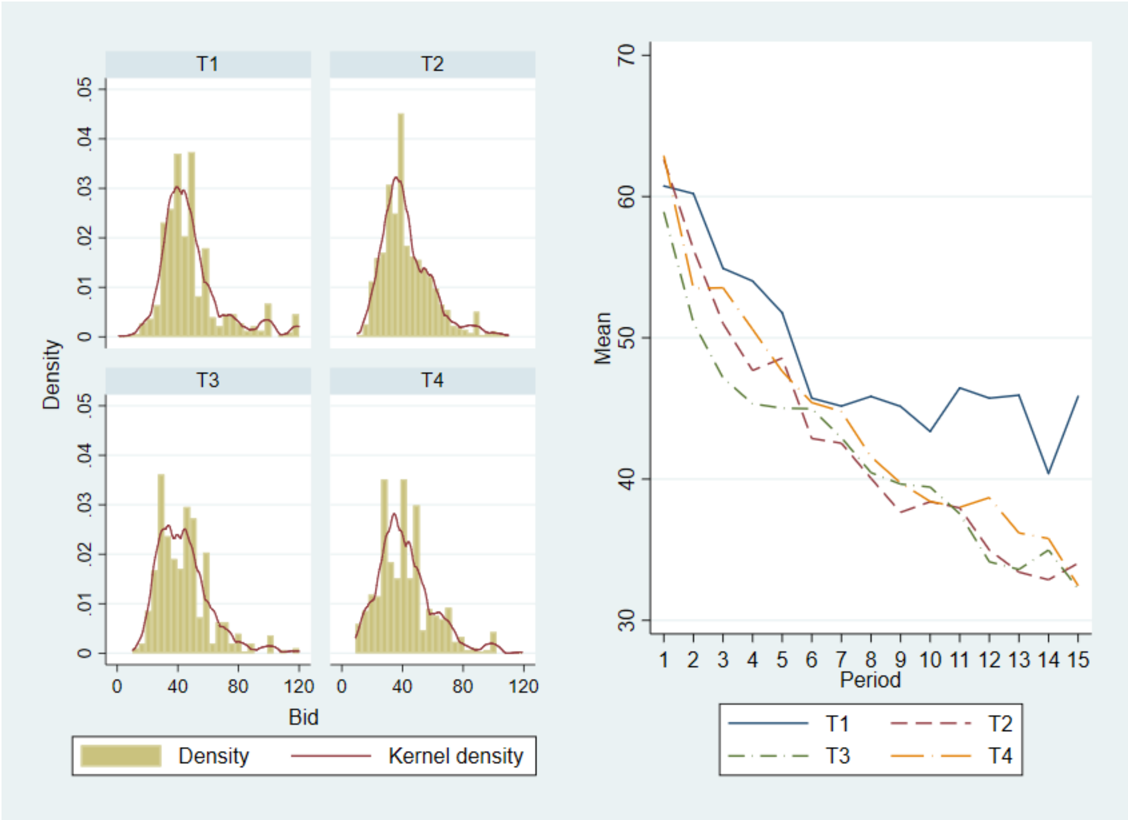

Figure 2 – Bids in $A1$: full sample.

Table 4 – Bids in $A1$.

	a	*Obs.*	a_1	*Obs.*	a_2	*Obs.*	a_1 *5th perc.*	a_1 *95th perc.*	a_2 *5th perc.*	a_2 *95th perc.*
$T1$	48.754	720	51.638	345	46.101	375	29.625	88.875	27.500	81.750
	(20.425)		(21.906)		(18.599)		(7.945)	(20.925)	(8.619)	(19.426)
$T2$	42.731	720	46.189	190	41.491	530	27.000	83.875	26.875	69.000
	(16.308)		(19.480)		(14.835)		(5.855)	(11.357)	(6.600)	(8.896)
$T3$	41.832	720	42.957	368	40.656	352	26.625	74.750	23.813	70.813
	(18.021)		(18.390)		(17.576)		(9.855)	(17.710)	(8.229)	(11.563)
$T4$	43.949	720	47.173	179	42.882	541	27.625	81.375	26.875	72.750
	(17.344)		(18.279)		(16.906)		(8.070)	(23.317)	(7.453)	(17.726)

Notes: the left part of the table reports the mean and standard deviations (in parentheses) of bids in the first auction for each treatment and over all periods. Descriptives are presented both by pooling subjects (a), and by splitting the sample according to the subjects capacities (a_1 for one unit, a_2 for two units). The right part of the table reports, for each treatment, the bids associated with the 5th and the 95th percentiles of the bid distributions in the first auction, by subgroup and over all periods.

Figure 3 – Bids in $A1$ by subjects' capacity: 1 unit (upper panel) and 2 units (lower panel).

Table 5 – Bids in $A1$: parametric analysis.

	(1)	(2)	(3)
$T1$	6.922^{*}	8.762^{**}	6.040
	(4.031)	(4.107)	(4.235)
$T2$	0.899	4.110	5.950
	(4.031)	(4.171)	(4.278)
$T4$	2.117	4.367	5.421
	(4.031)	(4.181)	(4.283)
$2u$		-2.340^{**}	-2.311^{**}
		(1.135)	(0.994)
$T1 \times 2u$		-3.389^{**}	-3.446^{**}
		(1.601)	(1.401)
$T2 \times 2u$		-3.577^{**}	-3.103^{**}
		(1.706)	(1.494)
$T4 \times 2u$		-2.178	-1.440
		(1.729)	(1.514)
$Period$			-1.545^{***}
			(0.111)
$T1 \times Period$			0.393^{**}
			(0.157)
$T2 \times Period$			-0.314^{**}
			(0.157)
$T4 \times Period$			-0.231
			(0.157)
$cons$	41.832^{***}	42.976^{***}	53.775^{***}
	(2.850)	(2.901)	(2.992)
lrl	-12042.03	-12006.18	-11647.88
$Wald - \chi^2$	3.51	67.08	920.88
$p > \chi^2$	0.319	0.000	0.000
$Obs.$	2880	2880	2880

Notes: this table reports estimates (standard errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the bid placed by the subject in the first auction. $2u$ is a dummy whose value is equal to 1 if the subject has a capacity of 2 units. $T1 \times 2u$, $T2 \times 2u$, $T4 \times 2u$ are interaction terms. The other remarks of Table 3 apply.

Table 6 – Bids in $A1$: convergence analysis.

	λ_τ	$\lambda_\tau^{(1)}$	$\lambda_\tau^{(2)}$	χ^2	$p > \chi^2$	ρ	R^2	$Obs.$
$T1$	45.256***			12824.26	0.000	0.456	0.789	2880
	(1.317)							
$T2$	36.535***							
	(0.966)							
$T3$	36.333***							
	(1.094)							
$T4$	37.708***							
	(1.039)							
$T1$		47.907***	42.218***	3190.70	0.000	0.421	0.787	720
		(1.513)	(1.371)					
$T2$		40.155***	35.108***	4322.32	0.000	0.426	0.844	720
		(1.250)	(0.968)					
$T3$		37.304***	34.576***	3090.88	0.000	0.499	0.765	720
		(1.289)	(1.302)					
$T4$		40.548***	36.846***	3497.70	0.000	0.468	0.814	720
		(1.442)	(1.081)					

Notes: the first part of the table reports the asymptotes of the bids in the first auction in the four treatments. Results are based on a single regression pooling data from the four treatments. The regression includes treatments specific asymptotes and 32 terms capturing the origin of the possible convergence process for each of the 32 subgroups of the experiment. The second part of the table reports, for each treatment, the asymptotes of the bids in the first auction differentiated by capacity (a_1 when the capacity is 1 and a_2 when the capacity is 2). Results are based on four regressions, one for each treatment. Each regression includes capacity specific asymptotes and 16 terms capturing the origin of the possible convergence process for each of the 8 subgroups involved in the treatment and by splitting them by capacity. The standard errors (in parentheses) in both models are corrected for heteroscedasticity as well as first-order autocorrelation. The other remarks of Table 3 apply.

Table 7 – Stated beliefs.

	μ	$Obs.$	μ_W	$Obs.$	μ_L	$Obs.$
$T1$	48.825	720	45.544	360	52.106	360
	(28.010)		(25.164)		(30.272)	
$T2$	66.407	720	63.422	360	69.392	360
	(19.984)		(17.666)		(21.678)	
$T3$	46.765	720	48.203	360	45.328	360
	(21.127)		(19.457)		(22.610)	
$T4$	64.592	720	65.536	360	63.647	360
	(22.299)		(21.285)		(23.260)	

Notes: this table reports means and standard deviations (in parentheses) of the stated belief about the probability that the opponent's capacity was two units in each treatment and over all periods. Descriptives are presented either by pooling subjects (μ), or by splitting the sample according to the outcome of the first auction (μ_W for Winners, μ_L for Losers).

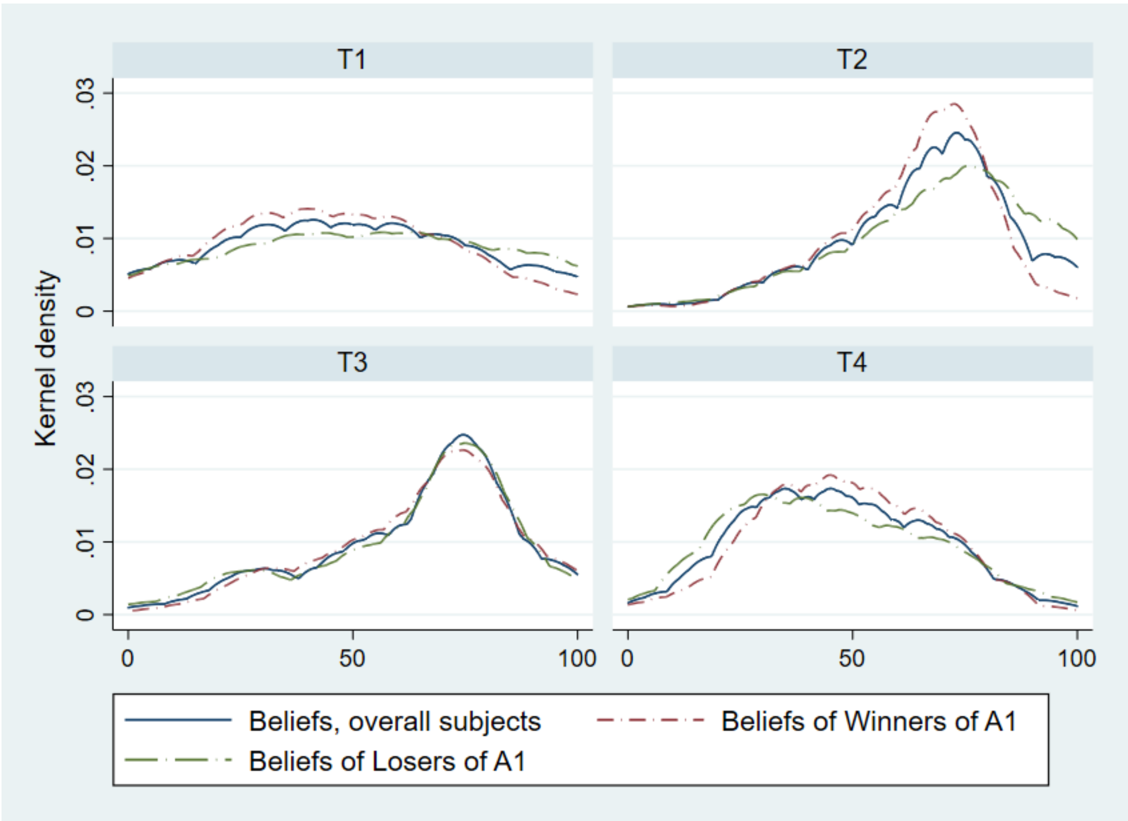

Figure 4 – Stated beliefs.

Table 8 – Stated beliefs: parametric analysis.

	(1)	(2)	(3)	(4)
$T1$	2.060	7.207***	9.247***	7.670**
	(2.352)	(2.604)	(2.795)	(3.292)
$T2$	19.642***	23.743***	23.879***	23.145***
	(2.352)	(2.603)	(2.972)	(3.429)
$T4$	17.826***	18.605***	21.127***	20.468***
	(2.352)	(2.602)	(3.013)	(3.457)
w_{A1}		3.372**	3.572**	3.573**
		(1.589)	(1.575)	(1.573)
$T1 \times w_{A1}$		−10.295***	−9.334***	−9.337***
		(2.264)	(2.253)	(2.249)
$T2 \times w_{A1}$		−8.203***	−8.187***	−8.180***
		(2.260)	(2.243)	(2.239)
$T4 \times w_{A1}$		−1.558	−1.646	−1.644
		(2.257)	(2.237)	(2.234)
$2u$			−4.489***	−4.496***
			(1.571)	(1.568)
$T1 \times 2u$			−4.563**	−4.543**
			(2.224)	(2.220)
$T2 \times 2u$			1.312	1.223
			(2.364)	(2.361)
$T4 \times 2u$			−1.729	−1.859
			(2.391)	(2.388)
$Period$				0.198
				(0.176)
$T1 \times Period$				0.224
				(0.249)
$T2 \times Period$				0.114
				(0.249)
$T4 \times Period$				0.108
				(0.249)
$cons$	46.765***	45.079***	47.174***	45.794***
	(1.663)	(1.838)	(1.977)	(2.326)
lrl	−12957.31	−12935.21	−12901.54	−12898.24
$Wald - \chi^2$	114.73	148.64	205.58	219.04
$p > \chi^2$	0.000	0.000	0.000	0.000
$Obs.$	2880	2880	2880	2880

Notes: this table report estimates (std. errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the belief that the opponent's capacity was 2. The other remarks of Table 3 apply.

Table 9 – Stated beliefs: convergence analysis.

	λ_τ	$\lambda_\tau^{(w_{A1})}$	$\lambda_\tau^{(l_{A1})}$	χ^2	$p > \chi^2$	ρ	R^2	*Obs.*
$T1$	50.918***			13591.02	0.000	0.250	0.798	2880
	(1.727)							
$T2$	66.975***							
	(1.207)							
$T3$	47.424***							
	(1.347)							
$T4$	65.426***							
	(1.328)							
$T1$		46.606***	55.056***	1941.70	0.000	0.145	0.723	720
		(1.958)	(2.118)					
$T2$		63.462***	70.421***	5210.70	0.000	0.287	0.874	720
		(1.509)	(1.575)					
$T3$		48.330***	46.536***	3126.65	0.000	0.136	0.805	720
		(1.527)	(1.573)					
$T4$		66.913***	64.050***	3750.91	0.000	0.327	0.829	720
		(1.737)	(1.789)					

Notes: the first part of the table reports the asymptotes of the stated beliefs in the four treatments. The second part of the table reports, for each treatment, the asymptotes of the stated beliefs differentiated by the outcome of the first auction (w_{A1} for the Winners and l_{A1} for the Loser). The other remarks of Table 3 and Table 6 apply.

Table 10 – The determinants of Losers and Winners' beliefs: parametric analysis.

	μ_L		μ_W
	(1)	(2)	(3)
$T1$	28.857***	7.389	−4.742
	(8.082)	(5.008)	(4.064)
$T2$	35.029***	22.957***	14.055***
	(9.177)	(5.354)	(4.252)
$T4$	20.283**	29.264***	14.784***
	(9.113)	(5.282)	(4.240)
$2u$	−0.665	−0.463	−5.579***
	(2.288)	(2.296)	(1.852)
$T1 \times 2u$	−2.677	−1.998	−3.198
	(3.244)	(3.249)	(2.627)
$T2 \times 2u$	0.519	0.822	0.675
	(3.413)	(3.457)	(2.812)
$T4 \times 2u$	−2.057	−3.400	−1.817
	(3.473)	(3.500)	(2.799)
a_W	0.165		
	(0.117)		
$T1 \times a_W$	−0.476***		
	(0.156)		
$T2 \times a_W$	−0.300*		
	(0.177)		
$T4 \times a_W$	0.027		
	(0.175)		
$(a - a_W)$		0.097	
		(0.079)	
$T1 \times (a - a_W)$		0.008	
		(0.105)	
$T2 \times (a - a_W)$		−0.055	
		(0.126)	
$T4 \times (a - a_W)$		−0.322***	
		(0.119)	
$Period$	0.602**	0.540**	−0.204
	(0.279)	(0.266)	(0.206)
$T1 \times Period$	−0.369	0.045	0.563*
	(0.387)	(0.370)	(0.292)
$T2 \times Period$	−0.231	0.064	0.165
	(0.423)	(0.375)	(0.292)
$T4 \times Period$	−0.260	−0.624*	0.653**
	(0.417)	(0.371)	(0.292)
$cons$	35.618***	40.310***	53.098***
	(5.785)	(3.568)	(2.858)
lrl	−6491.30	−6494.80	−6225.11
$Wald - \chi^2$	93.31	87.95	111.60
$p > \chi^2$	0.000	0.000	0.000
$Obs.$	1440	1440	1440

Notes: this table employs the same parametric strategy of Table 7 to study the determinants of stated beliefs of Losers (columns 1 and 2) and Winners (column 3) of the first auction. a_W is the (observed) winning bid in $A1$. $(a - a_W)$ is the difference between a subject's bid and the (observed) winning bid. $T1 \times a_W$, $T2 \times a_W$, $T4 \times a_W$, $T1 \times (a - a_W)$, $T2 \times (a - a_W)$, $T4 \times (a - a_W)$ are interaction terms. The other remarks of Table 3 and Table 8 apply.

Table 11 – Adjustment of bids in $A1$: parametric analysis.

	a		Δa	$\Delta a \vert w_{A1(t-1)}$	$\Delta a \vert l_{A1(t-1)}$
	(1)	(2)	(3)	(4)	(5)
$T1$	9.661^{**}	7.207	2.775	3.543^{*}	2.138
	(4.253)	(4.391)	(1.848)	(2.071)	(2.632)
$T2$	3.974	5.916	1.608	-0.365	2.212
	(4.313)	(4.433)	(1.979)	(2.172)	(2.851)
$T4$	4.757	5.775	0.315	-1.268	1.605
	(4.314)	(4.430)	(1.979)	(2.201)	(2.806)
$w_{A1(t-1)}$	1.583	1.706^{*}			
	(1.048)	(0.939)			
$T1 \times w_{A1(t-1)}$	-1.031	-0.987			
	(1.493)	(1.337)			
$T2 \times w_{A1(t-1)}$	-0.639	-0.606			
	(1.495)	(1.339)			
$T4 \times w_{A1(t-1)}$	-0.609	-0.623			
	(1.492)	(1.336)			
$2u$	-2.529^{**}	-2.395^{**}	-2.574^{**}	-1.231	-3.401^{**}
	(1.046)	(0.936)	(1.123)	(1.174)	(1.586)
$T1 \times 2u$	-3.457^{**}	-3.482^{***}	-4.471^{***}	-5.058^{***}	-4.105^{*}
	(1.477)	(1.322)	(1.588)	(1.663)	(2.244)
$T2 \times 2u$	-3.156^{**}	-2.827^{**}	-2.898^{*}	-4.115^{**}	-0.542
	(1.574)	(1.409)	(1.699)	(1.734)	(2.445)
$T4 \times 2u$	-2.414	-1.622	$5.04 * 10^{-4}$	-1.276	1.254
	(1.589)	(1.424)	(1.715)	(1.802)	(2.412)
$Period$		-1.335^{***}	0.251^{*}	-0.334^{**}	0.846^{***}
		(0.112)	(0.139)	(0.143)	(0.197)
$T1 \times Period$		0.325^{**}	0.060	0.085	0.048
		(0.158)	(0.197)	(0.202)	(0.278)
$T2 \times Period$		-0.298^{*}	0.138	0.282	0.029
		(0.158)	(0.197)	(0.202)	(0.278)
$T4 \times Period$		-0.219	0.012	0.159	-0.123
		(0.158)	(0.197)	(0.202)	(0.278)
$cons$	41.051^{***}	50.939^{***}	-2.535^{*}	8.401^{***}	-13.370^{***}
	(3.007)	(3.104)	(1.300)	(1.468)	(1.833)
lrl	-10912.19	-10636.73	-11003.88	-5112.40	-5491.91
$Wald - \chi^2$	87.59	738.81	87.26	71.01	107.92
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000
$Obs.$	2688	2688	2688	1344	1344

Notes: this table employs the same parametric strategy of Table 7 to study the adjustment of bids in $A1$. The dependent variable used in columns (1)-(2) is the bid placed by the subject in $A1$. The dependent variable used in columns (3)-(5) is the difference between the bids placed by the same subject in $A1$ in two consecutive periods. Regressions in columns (4) and (5) are restricted to the subsamples of Winners and Losers of $A1$ in the previous period, respectively. $w_{A1(t-1)}$ is a dummy whose value is equal to 1 if the subject won the first auction in the previous period. $T1 \times w_{A1(t-1)}$, $T2 \times w_{A1(t-1)}$, $T4 \times w_{A1(t-1)}$ are interaction terms. The other remarks of Table 5 apply.