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Key Points:
- The aim of this paper is to contribute to understand the trend of effusion rate with respect to time
- A model is proposed to explain the evolution of effusion rate in a basaltic eruption taking into account the erosion of the volcanic conduit

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Role of mechanical erosion in controlling the effusion rate of basaltic eruptions

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Abstract In many basaltic eruptions, observations show that the effusion rate of magma has a typical dependence on time: the effusion rate curves show first a period of increasing and later a decreasing phase by a maximum value. We present a model to explain this behavior by the emptying of a magma reservoir through a vertical cylindrical conduit with elliptical cross section, coupled with the its widening due to mechanical erosion, produced by the magma flow. The model can reproduce the observed dependence on time of effusion rate in basaltic eruptions. Eruption duration and the maximum value of effusion rate depend on the size of magma chamber, on lava viscosity and strongly on erosion rate per unit traction.

1. Introduction

The simulation of lava flow emplacement is a very important topic for the risk mitigation in volcanic areas that has been dealt with different approaches, ranging from the Cellular Automata method [e.g., *Crisci et al.*, 2004; *Miyamoto and Papp*, 2004; *Del Negro et al.*, 2007], regarding lava as a non-Newtonian fluid [e.g., *Young and Wadge*, 1990], describing the flow dynamic taking into account the heat loss, the crystallization, and the thermorheological constitutive equations [e.g., *Harris et al.*, 2007], by using the Smoothed-Particle Hydrodynamics (SPH) method [*Herault et al.*, 2011]. Two fundamental ingredients of all these methods are an accurate digital elevation model of the terrain, usually available with high resolution in volcanic areas [e.g., *Favalli et al.*, 1999], and a reliable evaluation on time of the magma effusion rate.

Observations show that the effusion rate of magma has a typical dependence on time in basaltic eruptions [e.g., *Wadge*, 1981]. In many of these, the effusion rate curves show first a period of increasing and later a decreasing phase from a maximum value. The effusion rate increases from initially low values during a time of minutes or hours and decreases gradually toward the end of the eruption.

The conversion of spectral radiance to lava mass or volume flow rate has been investigated by several authors in the last six decades [e.g., *Yokoyama*, 1957; *Pieri and Baloga*, 1986; *Harris et al.*, 1997; *Dragoni and Tallarico*, 2009; *Harris and Baloga*, 2009; *Garel et al.*, 2012]. *Vicari et al.* [2011] employed infrared satellite data (Moderate Resolution Imaging Spectroradiometer, Advanced Very High Resolution Radiometer, and Spinning Enhanced Visible and Infrared Imager) to estimate lava eruption rates at Mount Etna. They used duration and lava volume data of about 130 episodes [*Behncke et al.*, 2006; *Andronico et al.*, 2008] and defined six eruptive classes representing the lava volume in relation to the entire eruption duration. In each class the effusion rate starts from a low value, reaches its maximum value after at about quarter of the entire eruption duration, and slowly decreases until the end of the eruption is reached. The volume flow rate ranges between 10 and 140 m^3/s, the emitted lava volume ranges between 0.03 and 0.20 km^3, and the eruption duration is less than 90 days.

Thermorheological models for lava flows have been developed both with an analytical and a numerical approach [e.g., *Danes*, 1972; *Hulme*, 1974; *Park and Iversen*, 1984; *Dragoni et al.*, 1992; *Dragoni and Tallarico*, 1994; *Dragoni et al.*, 1995; *Harris et al.*, 1998; *Filippucci et al.*, 2013; *Valerio et al.*, 2011]. Recently, *Harris et al.* [2016] provided an overview of risk evaluation during effusive eruption disasters.

In association with lava flows, erosion has been often mentioned and is well documented in tube-fed and channel-fed flows in basaltic lavas. *Greeley et al.* [1998] studied the role of the conduit erosion (thermal and mechanical) in active lava flows on Earth and other planets. *Hulme* [1982] for Kilauean lava flows proposed erosion by melting and estimated an erosion rate in the order of 2.5 m/month. *Peterson and Swanson* [1974] observed that to initiate melting at least a few days of continuous flow are required and that erosion rates are about 2 m/month. On Hawaii *Kauahikaua et al.* [1998] measured thermal and mechanical erosion between 1.5 and 3.0 m/month. On Etna *Calvari and Pinkerton* [1999] found evidence only of mechanical erosion inside lava tubes and channels.

Dragoni and Tallarico [2008] developed a thermal model of lava flow in a cylindrical tube with elliptical cross section. *Dragoni and Santini* [2007], assuming that the erosion rate of the wall of the tube is proportional to shear traction, calculated the erosion of the wall as a function of time and found that the shear stress produced by the flow makes elliptical tubes closer to the circular shape.

We present a model to show the role of mechanical erosion in controlling the effusion rate of basaltic eruptions and, in particular, the increase in effusion rate that is commonly observed in these eruptions.

We consider the emptying of a magma chamber through a vertical cylindrical conduit with elliptical cross section, and we investigate the possibility that the increase in flow rate is due to a widening of the volcanic conduit due to erosion of the conduit wall produced by the magma flow.

2. The Model

We represent a magma chamber as a spherical cavity of radius R placed in a homogeneous and isotropic, elastic medium with density ρ_1 and rigidity μ. The cavity is filled with magma that, as a consequence of the high temperature, can be regarded as an incompressible Newtonian fluid [e.g., *Tallarico and Dragoni*, 1999; *Filippucci et al.*, 2013] having density ρ_2 and viscosity η.

We assume that at time $t=0$ the pressure in the magma chamber exceeds the lithostatic pressure by an amount p_0. At the same instant of time, a vertical tensile fracture of length h, connecting the top of the magma chamber with the Earth's surface, is produced.

Let $p(t)$ be the overpressure in the magma chamber. If we solve the problem of the quasi-static elastic deformation of the medium, we can easily find that the inflow or outflow of a mass dm of magma from the cavity produces a pressure change

$$\frac{\mathrm{d}p}{\mathrm{d}m} = \frac{\mu}{\pi \rho_2 R^3} \tag{1}$$

This relation holds for an unbounded elastic medium (a good approximation if $R \ll h$).

First, we exclude the effects of the mechanical erosion, and we represent the fracture as a cylindrical conduit with elliptical cross section. Let a and b be the semimajor and the semiminor axis of the ellipse, respectively. Since the conduit originates from a fracture, it is reasonable to assume $a \gg b$. If the flow is laminar (small Reynolds number), the volume flow rate in the conduit is [*Dragoni and Santini*, 2007]

$$Q = \frac{\pi}{4} \frac{\gamma}{\eta} \frac{a^3 b^3}{a^2 + b^2} \tag{2}$$

where γ is the component of the body force along the conduit direction.

If we take into account the buoyancy force, a pressure gradient

$$\gamma(t) = \frac{p(t)}{h} \tag{3}$$

drives the magma flow with

$$p(0) = p_0 + \left(\rho_1 - \rho_2\right) gh \tag{4}$$

and g is the gravity acceleration.

The pressure change with time can be written as

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}m} \frac{\mathrm{d}m}{\mathrm{d}t} \tag{5}$$

where

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\rho_2 Q \tag{6}$$

We obtain the differential equation

$$\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{p}{\tau_0} = 0 \tag{7}$$

where

$$\tau_0 = 4\frac{\eta}{\mu}R^3 h \frac{a^2 + b^2}{a^3 b^3} \tag{8}$$

The solution is

$$p(t) = \left[p_0 + \left(\rho_1 - \rho_2\right) gh\right] e^{-\frac{t}{\tau_0}} \tag{9}$$

Hence, from (2) and (3)

$$Q(t) = \tilde{Q} e^{-\frac{t}{\tau_0}} \tag{10}$$

where

$$\tilde{Q} = \frac{\pi}{4} \frac{p_0 + \left(\rho_1 - \rho_2\right) gh}{\eta h} \frac{a^3 b^3}{a^2 + b^2} \tag{11}$$

This simple model does not provide the initial increase in effusion rate that is commonly observed in eruptions. We investigate the possibility that the increase in Q is due to a widening of the volcanic conduit due to erosion of the conduit wall produced by the magma flow.

3. Effect of Mechanical Erosion

Due to mechanical erosion, according to *Dragoni and Santini* [2007], the wall of conduit maintains an elliptical shape and its semiaxes $A(t)$ and $B(t)$ change as

$$\begin{cases} \dot{A}(t) = k\gamma(t) \frac{A(t)B^2(t)}{A^2(t)+B^2(t)} \\ \dot{B}(t) = k\gamma(t) \frac{A^2(t)B(t)}{A^2(t)+B^2(t)} \end{cases} \tag{12}$$

where the dots indicate derivative with respect to time and k is the erosion rate per unit traction.

We consider also the effect of pressure gradient (3), so the system of simultaneous equations becomes

$$\begin{cases} \dot{A}(t) = \frac{k}{h} \left[p(t) + \left(\rho_1 - \rho_2\right) gh\right] \frac{A(t)B^2(t)}{A^2(t)+B^2(t)} \\ \dot{B}(t) = \frac{k}{h} \left[p(t) + \left(\rho_1 - \rho_2\right) gh\right] \frac{A^2(t)B(t)}{A^2(t)+B^2(t)} \\ \dot{p}(t) = -\frac{1}{4\eta h} \frac{\mu}{R^3} \left[p(t) + \left(\rho_1 - \rho_2\right) gh\right] \frac{A^3(t)B^3(t)}{A^2(t)+B^2(t)} \end{cases} \tag{13}$$

with the initial conditions

$$\begin{cases} A(0) = a \\ B(0) = b \\ p(0) = p_0 + \left(\rho_1 - \rho_2\right) gh \end{cases} \tag{14}$$

Volume flow rate (2) becomes

$$Q(t) = \frac{\pi}{4\eta h} \frac{A^3(t)B^3(t)}{A^2(t) + B^2(t)} \left[p(t) + \left(\rho_1 - \rho_2\right) gh\right] \tag{15}$$

Since the conduit originates from a volcanic fissure, we assume

$$\begin{cases} A(t) \gg B(t) \\ A(t) \simeq a \end{cases} \tag{16}$$

The approximation on $A(t)$, $B(t)$, $p(t)$, and so $Q(t)$ is less than 1%.

The system of simultaneous equations reduces to

$$\begin{cases} \dot{B}(t) \simeq \frac{k}{h}\left[p(t) + \left(\rho_1 - \rho_2\right)gh\right]B(t) \\ \dot{p}(t) \simeq -\frac{1}{4\eta h}\frac{\mu}{R^3}a\left[p(t) + \left(\rho_1 - \rho_2\right)gh\right]B^3(t) \end{cases} \tag{17}$$

with the initial conditions

$$\begin{cases} B(0) = b \\ p(0) = p_0 + (\rho_1 - \rho_2)gh \end{cases} \tag{18}$$

and

$$Q(t) \simeq \frac{\pi}{4\eta h}a\left[p(t) + \left(\rho_1 - \rho_2\right)gh\right]B^3(t) \tag{19}$$

The solution is

$$\begin{cases} B(t) = b\left[\frac{1+\alpha}{e^{(1+\alpha)\frac{t}{\tau}}+\alpha}e^{(1+\alpha)\frac{t}{\tau}}\right]^{\frac{1}{3}} \\ p(t) = \left[p_0 + (\rho_1 - \rho_2)gh\right]\frac{1+\alpha}{e^{(1+\alpha)\frac{t}{\tau}}+\alpha} \end{cases} \tag{20}$$

where

$$\begin{cases} \alpha = 12\frac{p_0+(\rho_1-\rho_2)gh}{\mu}\frac{R^3}{b^3}\frac{\eta k}{a} \\ \tau = 4\frac{\eta}{\mu}\frac{h}{a}\frac{R^3}{b^3} \end{cases} \tag{21}$$

The volume flow rate is given by

$$Q(t) = Q_0(1+\alpha)^2\frac{e^{(1+\alpha)\frac{t}{\tau}}}{\left[e^{(1+\alpha)\frac{t}{\tau}}+\alpha\right]^2} \tag{22}$$

where

$$Q_0 = \frac{\pi}{4\eta h}ab^3\left[p_0 + \left(\rho_1 - \rho_2\right)gh\right] \tag{23}$$

We note that the solution (20) tends to (9) as $\alpha \to 0$, i.e., $k \to 0$; this is if we exclude the erosion process ($\tau \simeq \tau_0$ for $a \gg b$).

The function $Q(t)$ has a maximum given by

$$Q_{\max} = \frac{Q_0}{4}\frac{(1+\alpha)^2}{\alpha} \tag{24}$$

at $t = t_{\max}$ with

$$t_{\max} = \frac{\tau}{1+\alpha}\log\alpha \tag{25}$$

The volume flow rate has a maximum if $t_{\max} \geq 0$, i.e., if $\alpha \geq 1$. α depends in particular on lava viscosity, magma chamber dimension (or cumulative volume of emitted lava) and on the erosion rate k (21)

$$\alpha = \frac{12}{\pi}\frac{\eta k}{ab^3}V_f \tag{26}$$

where V_f is the cumulative volume of emitted lava. α results greater than 1 for the parameter values typical of basaltic eruptions.

Table 1. A Choice of the Values of Parameters[a]

Case	R (km)	η (Pa s)	k (m Pa^{-1} s^{-1})
1	1.0	500	10^{-9}
2	1.5	100	10^{-10}
3	1.5	1000	10^{-10}
4	2.0	100	10^{-10}
5	2.0	1000	10^{-10}
6	2.0	1000	$3\ 10^{-10}$

[a] $\rho_1 = 3000$ kg m^{-3}, $\rho_2 = 2500$ kg m^{-3}, $g = 9.8$ m s^{-2}, $h = 10$ km, $p_0 = 1$ MPa, $\mu = 10^{10}$ Pa, $a = 50.0$ m, and $b = 0.2$ m.

From (22) and (20) we can calculate the average velocity of lava flow as

$$v(t) = \frac{Q(t)}{\pi A(t)B(t)} \tag{27}$$

and the cumulative volume of emitted lava as

$$V(t) = \int_0^t Q(\omega)\mathrm{d}\omega = \frac{\pi R^3}{\mu}\left[p_0 + \left(\rho_1 - \rho_2\right) gh - p(t)\right] \tag{28}$$

When the eruption ends (i.e., $t \to +\infty$) the values of B, p, Q, v, and V are

$$\begin{cases} B \to B_f = b(1+\alpha)^{\frac{1}{3}} \\ p \to 0 \\ Q \to 0 \\ v \to 0 \\ V \to V_f = Q_0\tau = \frac{\pi R^3}{\mu}\left[p_0 + \left(\rho_1 - \rho_2\right) gh\right] \end{cases} \tag{29}$$

where V_f is the total volume of emitted lava and B_f is the final value of minor semiaxis of the conduit.

4. Discussion and Conclusions

Some parameters of the model can differ by some orders of magnitude, while others can be assumed to be constant. Since we are interested in the role of mechanical erosion in controlling the effusion rate, we fix the values of the lava density ρ_2, the medium rigidity and density (μ and ρ_1), the initial overpressure in the magma

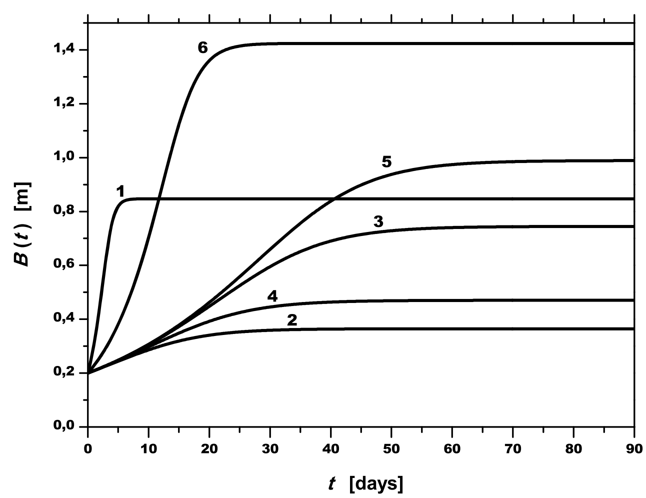

Figure 1. Evolution of the semiminor axis of a volcanic conduit $B(t)$ for a choice of values of parameters (Table 1).

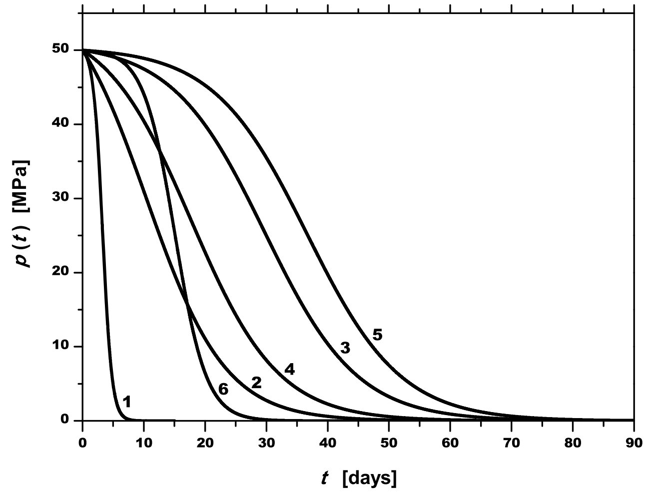

Figure 2. Evolution of the overpressure $p(t)$ for a choice of the values of parameters (Table 1).

chamber p_0 with respect to the lithostatic pressure, the length h of volcanic conduit, and the initial values of semiaxes of this conduit (a and b). However, the parameters more connected to lava dynamics can vary, i.e., the magma chamber radius R, the lava viscosity η and the erosion rate per unit traction k. In Table 1 we show a choice of values of parameters of the model.

Figures 1–4 show the effects of mechanical erosion on the conduit width, expressed by the semiaxis $B(t)$, the overpressure $p(t)$, the volume flow rate $Q(t)$, and the cumulative volume of emitted lava $V(t)$. The curves of $B(t)$ show that the erosion rate is in the order of field observations (Figure 1). In Figure 3 we recognize the typical dependence on time of $Q(t)$ in basaltic eruptions, and we conclude that mechanical erosion can explain this dependence. In particular, the model can reproduce the shape of the curves found by *Vicari et al.* [2011] for Mount Etna, the maximum values of $Q(t)$, eruption durations, and emitted lava volumes. The choice of values of parameters assures that the total lava volumes are comparable to field observations (Figure 4).

We note different values of eruption duration (Figures 2 and 3), which depends not only on magma chamber radius but also on the parameters η and k. The maximum values Q_{max} are reached for higher values of k, but at smaller time t_{max}, for the same values of R and η (curves 5 and 6). For the same values of R and k, we note that t_{max} increases with the viscosity, but Q_{max} is constant (curves 2–3 and 4–5). For the same values of η and k, Q_{max}, t_{max}, and obviously eruption duration increase with R (curves 2–4 and 3–5).

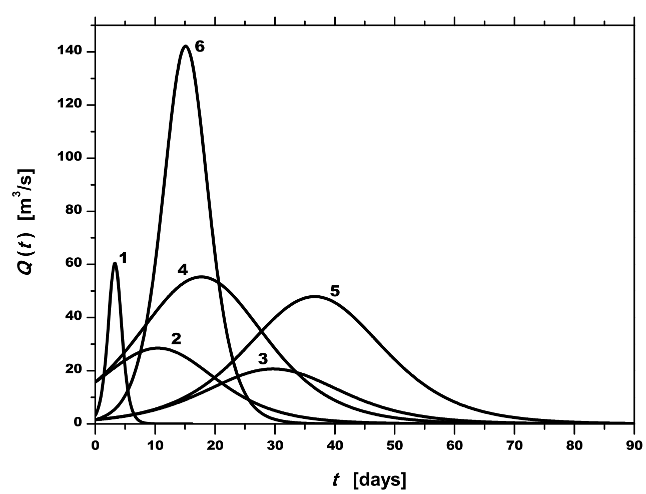

Figure 3. The volume flow rate $Q(t)$ for a choice of the values of parameters (Table 1).

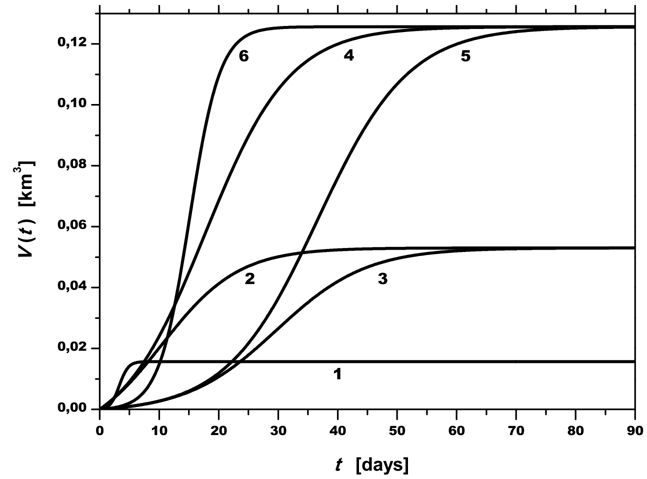

Figure 4. The cumulative volume of emitted lava $V(t)$ for a choice of the values of parameters (Table 1).

We note that in general, the value of k has a great influence on the magma flow, whereas the radius of magna chamber R and viscosity η have a smaller effect: in particular, as expected the eruption duration increases if k decreases (curves 5–6).

In order to check more completely the model predictions, systematic measurements of the effusion rate as a function of time should be performed for specific lava flows, together with measurements of lava viscosity and vent geometry.

Of course, processes other than mechanical erosion may contribute to modulate the flow rate in the volcanic conduit, such as thermal erosion, magma vesiculation, degassing, and crystallization.

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