From fixed to state-dependent duration in public-private partnerships*

Daniel Danau† Annalisa Vinella‡

Abstract

A government delegates a build-operate-transfer project to a private firm. In the contracting stage, the operating cost is unknown. The firm can increase the likelihood of facing a low cost, rather than a high cost, by exerting costly effort when building the infrastructure. Once the infrastructure is in place, the firm learns the true cost and begins to operate. Under limited commitment, either partner may renege on the contract at any moment thereafter. The novelty with respect to incentive theory is that the contractual length is stipulated in the contract in such a way that it depends on the cost realization. Our main result is that, if the break-up of the partnership is sufficiently costly to the government and/or adverse selection and moral hazard are sufficiently severe, then the efficient contract is not robust to renegotiation unless it has a longer duration when the realized cost is low. This result is at odds with the literature on flexible-term contracts, which recommends a longer duration when operating conditions are unfavorable, yet, with regards to a different setting, where the demand is uncertain and the cash-flow is exogenous.

Keywords: Public-private partnerships; state-dependent duration; flexible-term contracts; limited commitment; renegotiation.

J.E.L. Classification Numbers: D82; H57; H81

*We are indebted to an Associate Editor and two anonymous Reviewers for their useful suggestions. We are grateful to Alexander Galetovic for fruitful discussions. We also acknowledge helpful comments from Stéphane Saussier, participants in the 2015 Workshop on Economics and Management of Public-Private Partnerships (Venice) and seminar participants at Nanzan University (Nagoya) and Nihon University (Tokyo). A previous version of the paper was presented at the 2015 EEA Meeting (Mannheim), the 2014 EARIE Conference (Milan) and the 2014 IO Workshop on Theory, Empirics and Experiments (Alberobello). We thank those audiences for their various suggestions. All remaining errors are our own.

†Université de Caen Basse-Normandie - Centre de Recherche en Economie et Management, Esplanade de la Paix, 14000 Caen (France). E-mail: daniel.danau@unicaen.fr

‡Università degli Studi di Bari "Aldo Moro" - Dipartimento di Scienze economiche e metodi matematici, Largo Abbazia S. Scolastica, 53 - 70124 Bari (Italy). E-mail: annalisa.vinella@uniba.it

1 Introduction

Public-private partnerships (PPPs) in infrastructure projects include two main phases, namely construction and operation, and it is well known that incentive problems affect their performance in either phase. When building the infrastructure, the private firm may be little motivated to exert costly effort (Hart [17], Bennett and Iossa [3], Martimort and Pouyet [27], Iossa and Martimort [21]). In the operation phase, the firm is likely to observe the operating conditions privately, as agency theory suggests, and to conceal them *vis-à-vis* the government (e.g., Laffont [23], Guasch *et al.* [14] - [15], Iossa and Martimort [21], Danau and Vinella [5]). Moreover, both the firm and the government may have an interest in abjuring the PPP contract (see the report of Guasch [13] and the cases described by Estache and Wren-Lewis [12]). As a choice variable of the contract designer, the contractual length represents a powerful tool to address these incentive problems (Danau and Vinella [5]). This is because variations in the contractual length permit the contract designer some flexibility in adjusting the per-period compensation, which can be exploited to solve incentive problems arising during operation, without affecting the total compensation, which is instead used to address moral hazard arising during construction. The degree of flexibility available depends on the severity of moral hazard. The theory of incentives tells us that the more severe moral hazard is, the more uncertain the total compensation should be. However, as the firm is exposed to more risk, there is less flexibility in adjusting the per-period compensation through changes in the contractual length. Hence, it becomes more difficult to incentivize the firm during operation. The choice of a suitable contractual length is thus related to how important each of the incentive problems is.

Despite this essential link between the duration of PPP contracts and the partners' incentives during construction and operation, the choice of the optimal contractual length in PPP projects remains under-explored. Particularly, to the best of our knowledge, the literature on agency relationships has not yet considered the possibility of conditioning the duration of the contract on the state of nature as a tool to solve incentive problems. The idea of a state-dependent duration is inspired by the studies of Engel *et al.* [8] - [9] although, in their case, the duration is not stipulated in the contract, but rather determined through the mechanics of the contract. The authors focus on frameworks in which fixed-term contracts are incomplete for an exogenous reason (the market demand is uncertain) and show that incompleteness is eliminated if the contractual length is adjusted according to the realized state of nature (the level of demand) in such a way that the firm attains its reservation utility regardless of the specific state. This requires allowing the firm to operate the activity for a larger number of periods when the operating conditions are unfavorable. Contracts with this characteristic are referred to as flexible-term contracts. However, one may wonder whether the benefits of flexible-term contracts also extend to the frameworks we have in mind, in which moral hazard and adverse selection have bite and contractual frictions are due to the lack of enforcement mechanisms (as in the cases described by Estache and Wren-Lewis [12]) rather than to the parties' inability of writing a complete contract for all future contingencies.[1] As we have noted, both moral hazard and adverse selection require the firm to be exposed to some risk. This involves providing lower compensation to the firm in bad states of nature. Moreover, if a contract with a long duration is used in bad states of nature, as is the case for flexible-term contracts, then, as time passes, the firm might prefer to cease honoring its obligations, provided that its residual compensation falls below some alternative opportunity, which could be derived from another activity or from a new deal with the same partner. One then needs to understand whether a contract with a state-dependent duration would be useful in the environments we consider and, if so, how exactly it should be structured. We explore these issues in our paper.

The analysis we develop delivers one main lesson. In situations in which the firm enjoys an informational advantage early in its relationship with the government and, in addition, either partner may behave opportunistically during the operation phase, the contract that stipulates an efficient allocation may fail to be renegotiation-proof, unless its duration is conditioned on the state of nature. When that is the case, unlike in flexible-term contracts, the duration should

[1] Laffont [23], Guasch *et al.* [14] - [15], Danau and Vinella [5] are theoretical studies in which the vulnerability of the contract follows from the lack of enforcement mechanisms in the economy. Similarly to Engel *et al.* [8] - [9], Iossa and Martimort [20] also rely on an exogenous, incomplete contracting approach to model the vulnerability of PPP contracts.

be set longer in favorable states than in unfavorable states. A contract with this characteristic is more likely to be necessary the more severe moral hazard and adverse selection are and/or the more costly the break-up of the partnership would be to the government.[2]

As an additional contribution, our analysis offers a foundation to the understanding of when and how often each of the partners may want to initiate renegotiation during the execution of the contract, which is essential to draw conclusions on the optimal contractual design in PPPs. We ascertain that, should the firm initiate renegotiation in some period of operation, it will take the initiative again in each of the subsequent periods. In so doing, the firm will collect the annuity of the government's cost saving from the continuation of the partnership, leveraging itself on the damage that it can occasion to the government by reneging on the contract, rather than on the characteristics of the PPP project. On the other hand, should the government initiate renegotiation, it will do so at most once, as renegotiation is onerous to it. The government will take the initiative early in the operation phase to appropriate more of the up-front private investment under the new deal that replaces the initial contract.

This study is related to Danau and Vinella [5], which serves as the basis for the model. However, while that paper explores the financial structure of the project, here, we assume that the firm is the only investor. This simplification does not affect the general insights of our study, but it serves to focus the analysis on the use of a state-dependent duration as an incentive tool in PPP contracts. However, this simplification leads to a complication. In any period in which some partner could breach the contract, renegotiation would be Pareto-improving on the termination of the partnership. Hence, following a contractual breach, the parties would actually reach a new agreement and continue the relationship. The contract must thus be robust to the possibility of repeated renegotiation. In Danau and Vinella [5], the convenience of seeking new deals is excluded by the presence of a financial institution that can impose high debt payments on the government to destroy any surplus to be shared in renegotiation.

[2] A good example of the cost that governments fear in the event of a break-up is provided by Ehrhardt and Irwin [7] concerning the 1999 Melbourne transport franchises. The authors report that the State Government of Victoria agreed to renegotiate to escape the expenses associated with the retrieval of the activities and possible litigation to be incurred in the event of a break-up. See also Trebicock and Rosenstock [30], who acknowledge that governments face transaction costs when PPPs are broken up.

Our framework differs from the literature on flexible-term contracts in two essential respects, which pave the way for a different result. First, the compensation to the firm is endogenous and used as a tool to fine-tune incentives. Second, it is necessary to take explicit analytical consideration of the renegotiation game in which the partners engage following a contractual breach. This is not the case in the framework of Engel *et al.* [8] - [9], in which renegotiation can only be due to the parties' inability of addressing all possible future contingencies in the contract, an issue that is eliminated by making the firm's payoff independent of the realized operating conditions.

In addition to the aforementioned studies on PPPs, our analysis is more generally related to those on long-term principal-agent relationships. Baron and Besanko [2] characterize the optimal dynamic contract in a repeated adverse-selection regulatory problem. Dewatripont [6], Hart and Tirole [18], and Rey and Salanié [28] show that the parties to an incentive contract, signed in the *interim*, may want to renegotiate the initially stipulated allocation once private information is revealed. This desire arises because, under complete information, a Pareto-improving allocation is available to the contractual parties. More recent contributions recognize the importance that limits to the enforcement ability of courts of law may have in contractual design (see Levin [25], who focuses on relational contracts). Unlike in this strand of literature, in the PPP context we address, the contract specifies a single intertemporal compensation to the firm, and the choice of the contractual length involves how that compensation is diluted over multiple periods. In the same vein as Levin [25], we identify conditions under which Harris and Raviv's [16] result that efficiency is attained with *ex ante* contracting holds even if the contractual parties are unable to commit. The specificity of our analysis is that this result rests on the way in which the duration of the contract is chosen by the principal.

The reminder of the paper is organized as follows. The model is described and the efficient allocation is characterized in section 2. In section 3, we identify values of the contractual variables such that moral hazard and adverse selection are addressed without inducing distortions away from efficiency in the contractual allocation. We also highlight the flexibility gain that a state-dependent duration grants to the contract designer. In section 4, after describing the renegotiation game in which the partners would hypothetically engage, we show how a state-dependent duration can be useful for making the contract robust to renegotiation under limited commitment. In section 5, we extend the analysis to the case in which the cost of break-up to the government declines over time rather than being constant across periods. Section 6 provides further discussion. Section 7 concludes.

2 The model

A government (G) delegates a public project to a private firm (F). The project includes two tasks: the construction of an infrastructure and the provision of a good (or service) to society. F is a Special Purpose Vehicle (SPV) created by a group of private investors to perform these tasks. The contract is signed and the infrastructure is built at the beginning of period 0. Management of the infrastructure occurs in all periods $\tau \in \{0, ..., T-1\}$. At the beginning of period T, the contract ends. For simplicity, we assume that the infrastructure has an infinite life during which it does not depreciate. If T is finite, then the infrastructure is transferred to G at the end of the contract, as is typical of PPP arrangements.

Production technology and profit of F At the beginning of period 0, F incurs a sunk cost of $I > 0$ to construct the infrastructure. In each period τ, F operates the production process, incurring a cost of $\theta q + K$, where $\theta > 0$ is the marginal cost, $q \geq 0$ is the number of units of the good delivered in each period and $K > 0$ is the fixed cost. In return for supply, F receives a transfer of t from G and collects revenues $p(q)q$ from the market. This assumption encompasses a variety of real-world situations ranging from conventional infrastructure provision, where the firm only receives governmental transfers, to traditional concession, where the firm only collects market revenues. The per-period operating profit of F is $\pi = t + p(q)q - (\theta q + K)$.

Preferences for the good and return of G Consumption of q units of the good yields a gross surplus of $S(q)$, such that $S'(\cdot) > 0$, $S''(\cdot) < 0$, $S(0) = 0$, and the Inada conditions hold. Customers purchase the output produced in each period τ at a price of $p(q) \equiv S'(q)$.

In the same vein as in traditional procurement and regulation models, the per-period value of the project to G is a weighed sum of net consumer surplus and the firm's profit, namely $S(q) - p(q)q - t + \beta\pi$, where $0 \leq \beta < 1$.[3] Defining $w(q) \equiv S(q) - (\theta q + K)$ and using the expression of the profit to write $t = \pi - (p(q) - \theta)q + K$, the per-period value of the project to G is $w(q) - (1-\beta)\pi$. This expression indicates that G would prefer $w(\cdot)$ to be as high as possible, whereas it dislikes leaving a profit to F as $1 - \beta > 0$. Because this is true regardless of the value of β, for simplicity, we take $\beta = 0$ such that the per-period value of the project to G reduces to $w(q) - \pi$.[4]

Information structure In the delegation stage, the value of the marginal cost θ is unknown. Its distribution depends on some unobservable effort $a \in \{0, 1\}$ that F exerts when constructing the infrastructure. Once the infrastructure is in place and F begins to operate, the marginal cost is realized and takes one of two possible values, namely θ_l and θ_h, such that $0 < \theta_l < \theta_h$. Provided that θ is an intrinsic characteristic of the infrastructure, the value it takes remains the same throughout the life of the project. Henceforth, we denote $i \in \{l, h\}$ as the realized state of nature. F observes the state privately, which leads to an adverse selection problem at the outset of the operation phase. However, it is commonly known that the "good" state l occurs with probability ν_1, if $a = 1$, and with probability ν_0, if $a = 0$. As we refer to a PPP project, we can reasonably assume that exerting effort makes it more likely that the marginal cost will be low: $0 < \nu_0 < \nu_1 < 1$. By exerting effort, F incurs a disutility of $\psi(0) = 0 < \psi(1) = \psi$. This is the root of a moral hazard problem in construction.[5]

[3] Notice that, with distortionary taxation, spending one unit of public funds requires collecting more than one unit of money from taxpayers. To capture this circumstance formally, we could introduce some parameter $\lambda > 0$, expressing the shadow cost of public funds (see Dahlby [4], for instance). Then, a transfer of t would cost $(1+\lambda)t$ to G. As this would have no impact on results, we take $\lambda = 0$ for simplicity.

[4] The case of $\beta = 0$ well accommodates the widespread situations in which the private partner is a foreign firm, the profits of which are not included in the government's objective function.

[5] In the literature, the choice of a suitable duration is viewed as an important issue in highway concession contracts. In that context, θ could represent a combination of operation and maintenance costs, which are recognized to exhibit synergies with the investment in infrastructure. See, for instance, Engel *et al.* [11], who state: "Bundling forces investors to internalize operation and maintenance costs and generates incentives to design the project so that it minimizes life cycle costs." (p.5).

Payoffs under complete information Suppose for the time being that parties act under complete information. That is, both F and G know the effort exerted in construction and the marginal cost of production. We present the parties' payoffs in this framework, for some given value of θ realized at the outset of the operation phase.

Letting $\gamma_x \equiv \frac{1}{r}\left(1 - \frac{1}{(1+r)^x}\right)$, where $x \in \{0, ..., T-1\}$ and r is the discount rate, the value of the future stream of profits at the beginning of period τ is given by $\Pi_\tau = \pi\gamma_{T-\tau}$. The net present value of the project is:

$$\widetilde{\Pi} = \Pi_0 - (I + \psi(a)).$$

The discounted return of G from private management is given by $V_\tau = w(q)\gamma_{T-\tau} - \Pi_\tau$. Thus, the period$-0$ discounted return of G from private management amounts to:

$$V_0 = w(q)\gamma_T - \Pi_0.$$

After the PPP ends, G has no reason to leave any surplus to the firm which will manage the infrastructure thereafter, provided that the outside option of the firm is zero. Hence, in each of the subsequent periods, the return of G is $w(q^*)$, where q^* is the level of output that maximizes $w(\cdot)$, characterized by the standard marginal-cost pricing rule:

$$p(q^*) = \theta. \tag{1}$$

Henceforth, stars are appended to denote efficient values. Given θ, the payoff that G obtains from the entire life of the project is:

$$\begin{aligned} W &= V_0 + \frac{w(q^*)}{r(1+r)^T} \\ &= \frac{w(q^*)}{r} + (w(q) - w(q^*))\gamma_T - \Pi_0. \end{aligned}$$

Contract G makes a take-it-or-leave-it offer to F. The Revelation Principle applies, and G can restrict attention to direct mechanisms under which F releases information. The offer includes the menu of allocations $\{\{q_l, t_l, T_l\}, \{q_h, t_h, T_h\}\}$, where q_i is the quantity to be produced at the beginning of each period τ and t_i is the transfer to be made at the end of the period in the event that the realized state is $i \in \{l, h\}$. Furthermore, T_i is the date at which the contract will end. The fact that, in addition to the quantity and to the transfer, the contractual length is also state-dependent is the novel aspect of our model. Assuming that effort is desirable to G $(a^* = 1)$, and considering that it affects cost stochastically, conditioning the contractual allocation on the realization of the cost is a way to address not only the adverse selection problem but also the moral hazard problem.[6] Henceforth, the subscript i will be appended to all state-dependent variables.

Timing In summary, the relationship between G and F unfolds as follows.

1. G makes the take-it-or-leave-it offer to F.

2. If F accepts, then it constructs the infrastructure exerting effort a with disutility $\psi(a)$.

3. The marginal cost θ_i is realized, F observes it privately and reports it to G; accordingly, the allocation $\{q_i, t_i, T_i\}$ is selected from the contractual menu.

4. In each period $\tau \in \{0, ..., T_i - 1\}$, F produces q_i, collects revenues $p(q_i) q_i$ on the market and receives the transfer t_i, unless either G or F reneges on the contract.

2.1 Efficient allocation

Referring to $\Pi_{i,0}$, rather than to t_i, with a standard change of variable, the triplet of choice variables is $\{q_i, \Pi_{i,0}; T_i\}$, $\forall i$. Accordingly, an efficient allocation is one that solves the following

[6] Effort is desirable when $\mathbb{E}[w(q_i^*)] - \widetilde{\mathbb{E}}[w(q_i^*)] > r\psi$, where $\mathbb{E}$ and $\widetilde{\mathbb{E}}$ are the expectation operators over the two states l and h, corresponding to $a = 1$ and $a = 0$, respectively, and q_i^* is the optimal quantity when the marginal cost is θ_i.

program:

$$\underset{\{q_i,\Pi_{i,0};T_i\},\forall i}{Max} \mathbb{E}[W_i]$$

subject to

$$\mathbb{E}[\Pi_{i,0}] \geq I + \psi. \tag{2}$$

Constraint (2) is the firm's *ex ante* participation constraint under the assumption of a zero outside option. As the contract is signed *ex ante*, it suffices to ensure that the firm breaks even in expectation. At the optimum, $q_i = q_i^* \ \forall i$ and the values $\Pi_{l,0}^*$ and $\Pi_{h,0}^*$ are such that (2) is saturated. The optimized payoffs are given by

$$\begin{aligned} \mathbb{E}\left[\Pi_{i,0}^*\right] &= I + \psi \\ \mathbb{E}[W_i^*] &= \frac{\mathbb{E}[w(q_i^*)]}{r} - (I + \psi). \end{aligned}$$

Because $\mathbb{E}[W_i^*]$ is independent of T_l and T_h, the specific choice of the termination dates is irrelevant with respect to efficiency. Hence, it is irrelevant whether the project is developed under a PPP, public procurement or full privatization. This entails that, as long as neither moral hazard nor adverse selection arises and both parties fully commit to their contractual obligations, G can attain efficiency by adopting any of those arrangements (see Proposition 1 in Danau and Vinella [5]).[7]

3 Moral hazard and adverse selection

As the first step of the analysis, we identify conditions under which an efficient allocation is implemented contractually under incomplete information. In particular, this will enable us to show that, whereas the contractual length is irrelevant to obtaining efficiency under complete

[7]This conclusion is in line with the Irrelevance Theorem of Sappington and Stiglitz [29]. It is also in line with the view expressed by Engel *et al.* [10] that reliance on PPPs is not justified by the need to avoid disbursing public funds that are costly to society.

information, the presence of moral hazard and adverse selection on the firm's side may impose restrictions on its choice.

Denoting $\Delta\theta = \theta_h - \theta_l$ and $\Delta\nu = \nu_1 - \nu_0$, the moral hazard constraint is given by:

$$\Pi_{l,0} - \Pi_{h,0} \geq \frac{\psi}{\Delta\nu} \tag{3}$$

and ensures that effort is induced by paying the firm more when the cost is low. Because the effort decision is made during the construction of the infrastructure, it depends on the difference between the *total* compensation (for simplicity, the "profit") received in the good and in the bad state, rather than on the number of periods through which that compensation would be cumulated. The difference between profits must be greater the higher the cost of effort, *i.e.* the disutility ψ, and the lower the benefit of effort, *i.e.* the increase $\Delta\nu$ induced in the likelihood of a low cost being realized.

Whereas the effort of F depends only on the profit wedge, information release by F at the outset of the operation phase depends on both the profit wedge and the termination date of the contract. This is evident from the adverse selection constraints, which are formulated as follows

$$\Pi_{l,0} - \Pi_{h,0} \leq \sum_{\tau=0}^{T_l - 1} \frac{\Delta\theta q_l}{(1+r)^{\tau+1}} \tag{4}$$

$$\Pi_{l,0} - \Pi_{h,0} \geq \sum_{\tau=0}^{T_h - 1} \frac{\Delta\theta q_h}{(1+r)^{\tau+1}}, \tag{5}$$

in state h and l, respectively. When the cost is high but a low cost is announced, F incurs a penalty equal to the difference between the true and announced cost, namely $\Delta\theta q_l$, in each period through the termination date T_l. (4) shows that F tells the truth in that state if the discounted cumulated penalty through period T_l is at least as great as the additional profit of $\Pi_{l,0} - \Pi_{h,0}$ it would obtain from that lie. When the cost is low but a high cost is announced, the cost difference of $\Delta\theta q_h$ is a bonus that F obtains in each period through the termination date T_h, and the difference $\Pi_{l,0} - \Pi_{h,0}$ represents the amount of profit F renounces if it lies. (5)

shows that F tells the truth in that state if the discounted bonus through period T_h from a lie does not exceed the foregone profit.[8]

We are interested in identifying conditions under which G decentralizes an efficient allocation, taking into account that the constraints (3) - (5) must be satisfied. Recall that the contractual variables are given by $\{q_i, \Pi_{i,0}, T_i\}$, $\forall i$, and that efficiency is attained if output is set equal to q_i^*, $\forall i$, and $\Pi_{l,0}^*$ and $\Pi_{h,0}^*$ are chosen such that the participation constraint (2) is saturated. Given that there are multiple combinations of profits such that (2) is saturated, once G sets $q_i = q_i^*$, $\forall i$, it remains to choose $\left\{\Pi_{i,0}^*, T_i\right\}$, $\forall i$, to achieve the goal. However, because the constraints (3) - (5) all depend on the profit wedge at date 0, rather than on the exact values of the profits $\Pi_{l,0}^*$ and $\Pi_{h,0}^*$, we can use (2) as an equality to express the profits as follows:

$$\Pi_{l,0}^* = I + \psi + (1 - \nu_1)\,\Delta\Pi \tag{6}$$

$$\Pi_{h,0}^* = I + \psi - \nu_1 \Delta\Pi, \tag{7}$$

where $\Delta\Pi \equiv \Pi_{l,0}^* - \Pi_{h,0}^*$. Based on these expressions, in the presentation of results hereafter we will refer to the triplet $\{\Delta\Pi, T_l, T_h\}$ in place of the pairs $\left\{\Pi_{i,0}^*, T_i\right\}$, $\forall i$. According to (6) and (7), once $\Delta\Pi$ is set, the profits are such that, apart from recovering the initial investment of $I + \psi$, F receives a "reward" of $(1 - \nu_1)\,\Delta\Pi$ in state l and incurs a "punishment" of $\nu_1 \Delta\Pi$ in state h. Overall, F obtains a positive payoff of $\Pi_{l,0}^* - (I + \psi)$ in the good state and a negative payoff of $\Pi_{h,0}^* - (I + \psi)$ in the bad state, being exposed to an amount of risk equal to $\Delta\Pi$, which is just enough for G to retain all the expected surplus.

Proposition 1 *The triplet* $\{\Delta\Pi^*, T_l^*, T_h^*\}$ *that implements an efficient allocation under* $(3)-(5)$

[8]The formulation of (4) and (5) is reminiscent of that found in repeated adverse selection problems *à la* Baron and Besanko [2]. In those problems, private information is not persistent, and the agent makes a new report to the principal in each subsequent period. In our model, the unit cost of operation is drawn once for all, and the firm reports to the government only at date 0. However, a lie at that date can be assimilated to a repetition of the same lie, yielding the same output obligation and the same compensation right, in each subsequent period through the termination date. Essentially, what differs in our setting is that the number of periods through which the firm could benefit from that lie is endogenous to the contract.

is such that:

$$\Delta\Pi^* \in \left[\frac{\psi}{\Delta\nu}, \frac{\Delta\theta q_l^*}{r}\right] \tag{8}$$

$$T_l^* \geq \underline{T}(\Delta\Pi^*) \equiv \frac{\ln \frac{\Delta\theta q_l^*}{\Delta\theta q_l^* - r\Delta\Pi^*}}{\ln(1+r)} \tag{9}$$

$$T_h^* \leq \overline{T}(\Delta\Pi^*) \equiv \frac{\ln \frac{\Delta\theta q_h^*}{\Delta\theta q_h^* - r\Delta\Pi^*}}{\ln(1+r)} \quad \textit{if} \;\; \Delta\Pi^* < \frac{\Delta\theta q_h^*}{r}. \tag{10}$$

The fact that efficiency may be implemented contractually with an appropriate choice of the contractual variables does not come as a surprise. Indeed, as is well known from the literature, neither moral hazard nor adverse selection is an issue when the agent is risk neutral and not yet informed about the state of nature at the time when the contract is signed.[9] The peculiarity of our model with respect to most of the literature is that the agency relationship is long term, rather than being one shot. For this reason, in addition to restrictions on the choice of the profit wedge, there are also restrictions on the choice of the contractual length. We first interpret these restrictions, as listed in the proposition, and then explain what they imply in terms of the desirability of a state-dependent contractual term.

Let us begin with the profit wedge**.** According to (8), not only is it necessary that the profit wedge not fall below $\psi/\Delta\nu$ to prevent shirking in the construction of the infrastructure, but it must also be the case that the profit wedge does not exceed $\Delta\theta q_l^*/r$. Otherwise, the cumulated penalty would be too low to prevent understatement of a high cost, even if F were delegated the activity forever when the cost is low.[10] Consider now the termination dates. First, (9) imposes a lower bound on T_l^*. For F to be unwilling to understate a high cost, it must be the case that F incurs the penalty associated with that lie, namely $\Delta\theta q_l^*$, for a sufficiently large number of periods. This restriction is weak when the per-period penalty is high, and hence it is enough if it is incurred over a limited number of periods. The restriction is instead tight

[9] See Ch. 2 and Ch. 4 of Laffont and Martimort [24], for instance.

[10] In addition, it must be the case that effort is not too costly, *i.e.*, $\psi \leq \Delta\nu\Delta\theta q_l^*/r$. Otherwise, it would be impossible for G to set the profit wedge in such a way that (3) is satisfied jointly with (4), and efficiency would be beyond reach. Because we focus on implementation of an efficient allocation, we neglect this possibility and assume the above condition to hold.

when the additional profit of $\Delta\Pi^*$, which F obtains by pretending l, is high, thus making the lie particularly attractive. Considering that $\Delta\Pi^*$ must take a higher value the more severe moral hazard is, we can conclude that the restriction on T_l^* is stronger the tighter (3) is. This emphasizes that, albeit the effort of F does not depend on the choice of the contractual length, moral hazard has bite in that choice by restricting the possibility for G to induce information release through the choice of the profit wedge. Second, (10) imposes an upper bound on T_h^*. For F to be unwilling to overstate a low cost, it must be the case that F obtains the bonus associated with that lie, namely $\Delta\theta q_h^*$, for a sufficiently low number of periods. The restriction is tight when the per-period bonus is low; it is weak when the foregone profit $\Delta\Pi^*$ is high. The restriction completely disappears when $\Delta\Pi^* \geq \Delta\theta q_h^*/r$, *i.e.*, when the foregone profit is so high that it is not compensated for, even if F obtains the bonus for an infinite number of periods.

The foremost implication of Proposition 1, for the purpose of this study, is that, in the implementation of an efficient allocation under incomplete information, G gains flexibility if it switches from a fixed to a state-dependent contractual term. Suppose for a moment that the term is fixed, such that $T_l = T_h \equiv T$. Then, (9) and (10) taken together imply that T should be chosen within the range $\left[\underline{T}(\Delta\Pi), \overline{T}(\Delta\Pi)\right]$ for some given $\Delta\Pi$. If the term of the contract is set too short, then the firm is prompted to claim l in the bad state because, by doing so, it will incur the cost penalty only through a limited number of periods. If instead the term of the contract is set too long, then the firm is prompted to claim h in the good state to accumulate a bonus (in cost reimbursement) over the long contractual period. Conditioning the term of the contract on the state of nature enables G to lessen these restrictions. Extending the contract by $T_l - \overline{T}(\Delta\Pi)$ periods in the good state does not make the firm more eager to exaggerate cost because the benefit associated with that lie depends on T_h rather than on T_l. Shortening the contract by $\underline{T}(\Delta\Pi) - T_h$ periods in the bad state does not make F more eager to understate cost because the penalty associated with that lie depends on T_l rather than on T_h. Overall, G can set $T_h < \underline{T}(\Delta\Pi) \leq T_l$ and $T_h \leq \overline{T}(\Delta\Pi) < T_l$, for some $\Delta\Pi$ sufficiently high to satisfy the moral hazard constraint, without violating the adverse selection constraints.

3.1 Flexibility gain and per-period profits

The flexibility gain to G can also be understood by examining how the per-period profit of F is determined in each state of nature when the optimal decisions are made. To that end, it is first useful to recall the essential features of the flexible-term contracts proposed by the literature, which also have a state-dependent duration. In those contracts, the per-period profit of the firm is determined exogenously and the duration of the contract is adjusted to ensure that, by cumulating that profit in all periods through the termination date, the firm attains its reservation utility, regardless of the realized state. In our model, that would involve allowing F to obtain an amount equal to the cost of investment $I + \psi$ in all periods of operation through the termination date. In each state i, given a per-period profit of π_i, T_i would be chosen to satisfy the following:

$$\pi_i = \frac{I + \psi}{\gamma_{T_i}}. \tag{11}$$

By contrast, the per-period profit is determined endogenously in our setting. When G chooses a pair $\left\{\Pi^*_{i,0}, T^*_i\right\}$, G is indirectly choosing the per-period profit that F will receive in each period through the termination date T^*_i, according to the formula $\pi_i = \Pi^*_{i,0}/\gamma_{T^*_i}$.[11] Because the pair $\left\{\Pi^*_{i,0}, T^*_i\right\}$ serves to address incentive problems under incomplete information, the per-period profit will include a component related to the incentives of F, which is missing in (11). To see the exact formulation of the per-period profit and identify this incentive component, we rewrite (4) and (5) to express $\Delta\Pi^*$ as follows:

$$\Delta\Pi^* = \Delta\theta z_j \gamma_{T_j}, \tag{12}$$

where j can be taken to be either l or h, not necessarily coinciding with the true state i, and with the additional requirement that $z_l\gamma_{T_l} = z_h\gamma_{T_h}$ ensuing from the two constraints. Taking $j = l$ in (12), one can interpret $\Delta\theta z_l$ as the wedge between the profits that F will obtain in

[11] With the output being set to the efficient level of q^*_i, the revenues that F collects from the market do not depend on the choice of $\Pi^*_{i,0}$ and T^*_i, which only affects the transfer to F, determined residually as $t^*_i = K + \Pi^*_{i,0}/\gamma_{T^*_i}$.

each period through the termination date, "normalized" as if that date were T_l in both states. As in a one-period agency relationship, a profit wedge of $\Delta\theta z_l$ is associated with a reward of $(1-\nu_1)\,\Delta\theta z_l$. However, as we are concerned with a multi-period interaction, $(1-\nu_1)\,\Delta\theta z_l$ is here the per-period reward through date T_l, where $z_l \in \left(\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^*\right]$. Analogously, taking $j = h$ in (12), one can interpret $\Delta\theta z_h$ as the per-period wedge between profits, normalized as if the termination date were T_h in both states, and yielding a per-period punishment of $\nu_1\Delta\theta z_h$, where $z_h \in [q_h^*, \infty)$. When deciding $\Delta\Pi$ and T_l, G is essentially choosing z_l and, hence, the per-period reward $(1-\nu_1)\,\Delta\theta z_l$ in a contract with termination date T_l. Symmetrically, when deciding $\Delta\Pi$ and T_h, G is choosing z_h and, hence, the per-period punishment $\nu_1\Delta\theta z_h$ in a contract with termination date T_h. Replacing (12) into (6) and (7), we derive the expressions of the per-period profits in the two states:

$$\pi_l^* = \frac{I+\psi}{\gamma_{T_l^*}} + (1-\nu_1)\,\Delta\theta z_l \tag{13}$$

$$\pi_h^* = \frac{I+\psi}{\gamma_{T_h^*}} - \nu_1\Delta\theta z_h. \tag{14}$$

In either expression, the first term is the counterpart of the per-period profit in the flexible-term contract (as displayed in (11)). It represents the "cost of capital" that G promises to repay to F in each period of operation to induce private investment up front and is inversely related to the contractual length in the given state. The second term, which serves to incentivize the firm, is the per-period reward to F if the state is good and the per-period punishment if the state is bad. Considering that an increase in T_l is paired with a reduction in z_l and a reduction in T_h with an increase in z_h, and noticing that $z_l = z_h \equiv z \in [q_h^*, q_l^*]$ in a fixed-term contract, the following result obtains.

Corollary 1 *If* $T_l^* > \overline{T}\,(\Delta\Pi)$, *then* $z_l < q_h^*$ *and the per-period reward to the firm is lower than in a fixed-term contract satisfying the conditions of Proposition* 1. *If* $T_h^* < \underline{T}\,(\Delta\Pi)$, *then* $z_h > q_l^*$ *and the per-period punishment to the firm is higher than in a fixed-term contract satisfying the conditions of Proposition* 1.

Therefore, by decomposing the per-period profits as in (13) and (14), one can identify the exact root of the flexibility that G enjoys when switching from a fixed-term contract to a contract with a state-dependent duration in the presence of moral hazard and adverse selection. That is, given the amount of risk to which G must expose F to avoid shirking in construction, adverse selection can be addressed by either granting a lower reward for a longer time or imposing a higher punishment for a shorter time.

One may wonder why this flexibility is useful to G. According to Proposition 1, it is possible to attain efficiency both when the flexibility is exploited and when it is not, or, in other words, both when the contract has a state-dependent duration and when it has a fixed term. As we will see in the next section, it is under limited commitment that G may want to take advantage of the flexibility. To give a hint, whereas the incentives of F to shirk and cheat depend only on the second component of the per-period profit (reward or punishment), the incentives of the partners to honor the contract depend on the overall per-period profit of F. When the contractual length is chosen, not only the per-period reward/punishment but also the per-period repayment of the cost of capital is determined accordingly. Thus, by adjusting T_i, G may be able to attain some desirable value of π_i^*, such that both partners are motivated to remain in the relationship, without violating the information constraints through the changes thereby induced in the per-period reward/punishment.

4 Limited commitment

We identified the optimal contractual triplet $\left\{q_i^*, \Pi_{i,0}^*, T_i^*\right\}$, $\forall i \in \{l, h\}$, where T_i^* is the termination date such that an efficient allocation $\left\{q_i^*, \Pi_{i,0}^*\right\}$ is implemented under incomplete information. In this section, we consider the situation in which the partners are unable to commit and the contract is exposed to their opportunism during the operation phase. We raise the question of what conditions the termination date T_i must satisfy to ensure that, in addition to addressing moral hazard and adverse selection, the triplet $\left\{q_i^*, \Pi_{i,0}^*, T_i\right\}$ is robust to the partners' incentives to renege. To answer this question, we first need to explore the renegotiation process in which G and F engage hypothetically. We will then formalize the additional (self-enforcing) constraints entailed by the renegotiation-proofness of the contract and draw conclusions on the contractual implementation of an efficient allocation under incomplete information and limited commitment.

4.1 Renegotiation process and renegotiation payoffs

Suppose that one party (whether F or G) reneges on the contract in state $i \in \{l, h\}$ at date $\tau \in \{0, ..., T_i - 1\}$. Following contractual reneging, the partners engage in a renegotiation game, and one needs to assess whether the partnership is terminated or, rather, a new agreement is reached. In the latter case, one would also need to establish what variables the partners renegotiate on and what payoffs they will obtain as a result of that negotiation. The payoffs will depend on the period τ in which the contract is first reneged upon. Moreover, the payoffs also depend on whether reneging is expected to occur again in any of the subsequent periods. Whereas we develop this part of the analysis assuming that the renegotiation process takes place under complete information about the realized cost θ_i, we show in the proof of Proposition 2, to be stated in Section 4.3 below, that information is indeed released even if F anticipates that one partner will renege during the execution of the contract.

4.1.1 No break-up following contractual reneging

When one party reneges, a court of law cannot oblige it to execute the contract. However, the court can impose sanctions following a breach, in the same vein as in the recent literature on non-enforceable contracts (see Levin [25]). Specifically, when G breaches the contract, the court can impose a penalty of $P > 0$ in favor of F because G appropriates (a part of) the private investment. Instead, there is no sanction for F if it terminates the partnership. This is because, in that case, break-up is assimilated to private default, entailing that G relieves the SPV and the private partner cannot be called upon to contribute further resources in addition to those already allocated to construct the infrastructure. Because F renounces the ability to recoup a part of its initial investment if it stops abiding by the contract, this loss will naturally serve as a penalty for the firm in the event of a breach. Taking this into account, when the PPP contract is signed, it is complemented by a termination clause, according to which G will pay P to F if the PPP is terminated on the former's initiative. In addition to the sanction, the break-up entails a cost of $R > 0$ to G. The following result is then obtained.

Lemma 1 *Conditional on one party reneging on the contract, the outcome of the renegotiation game is a new agreement rather than the break-up of the partnership.*

Intuitively, G prefers to negotiate a new deal with F and save on the cost of break-up; F is eager to return to the negotiating table and share that saving with G, rather than terminating the partnership.

4.1.2 What partners renegotiate on

Because the renegotiation process takes place under complete information, renegotiation concerns the contractual triplet $\left\{q_i^*, \Pi_{i,\tau}^*, T_i\right\}$ in effect in the realized state i at the beginning of period τ, where:

$$\Pi_{i,\tau}^* = \pi_i \gamma_{T_i-\tau} = \Pi_{i,0}^* \frac{\gamma_{T_i-\tau}}{\gamma_{T_i}} \tag{15}$$

is the residual contractual profit of F, yielding G a residual contractual return of:

$$V_{i,\tau}^* = w_i\left(q_i^*\right)\gamma_{T_i-\tau} - \Pi_{i,\tau}^*. \tag{16}$$

Notice that $\Pi_{i,\tau}^*$ appears in the contractual triplet in place of the profit $\Pi_{i,0}^*$ stipulated at date 0. This is because, during the first $\tau - 1$ contractual periods, F has already accumulated a part of that profit, which is obviously already disbursed at the beginning of period τ. In fact, the partners do not renegotiate on all three values. To see this, consider first $\Pi_{i,\tau}^*$ and T_i. Because $\Pi_{i,\tau}^*$ is the present value of the stream of profits to be accumulated in future periods through date T_i, renegotiating on $\Pi_{i,\tau}^*$ is tantamount to renegotiating on a combination of the per-period transfer and the termination date. Given the substitutability between the two, it is without loss of generality to assume that any new agreement on $\Pi_{i,\tau}^*$ is a new agreement on the per-period transfer (and, thereby, on the per-period profit), whereas the termination date is unchanged. Accordingly, we will hereafter neglect the possibility of a new deal being made on T_i. We are left with clarifying whether the partners have an interest in modifying the level of output (and, hence, the price). The following result makes this point.

Lemma 2 *Suppose that one party reneges in state i in period τ Under the new agreement reached by F and G, F will continue to produce q_i^* in each period thereafter.*

Intuitively, any output (or price) distortion away from the efficient level would cause a reduction in the value of $w(\cdot)$ and, hence, in the size of the pie to be shared in the renegotiation process.

Having clarified what the partners would renegotiate on, we can now explore their incentives to renege on the contract.

4.1.3 Repeated reneging and renegotiation payoff of F

Let us first examine the behavior of the firm. F may have incentives to renege at the beginning of the period, before producing and incurring the associated cost. Once F reneges in some state i at some date τ, G reasonably expects F to renege again at any $\tau' > \tau$.

Lemma 3 $\forall \tau, \tau' \in \{0, ..., T_i - 1\}$ *such that $\tau < \tau'$, if F reneges at τ and G believes that F will not renege at τ', then F does renege at τ'.*

This result is essential to understand how much surplus will be shared in the renegotiation game. Provided that, following reneging at τ, renegotiation will occur in all periods through date $T_i - 1$, the available surplus is such that

$$X_\tau + \frac{X_{\tau+1}}{1+r} + ... + \frac{X_{T_i-1}}{(1+r)^{T_i-\tau}} = R, \ \forall \tau \in \{0, ..., T_i - 1\}, \tag{17}$$

where X_τ is the surplus to be shared in period τ, $X_{\tau+1}$ is the surplus to be shared in period $\tau + 1$, and so forth. In essence, (17) means that the maximum amount that F can extract from

G, if they renegotiate repeatedly from date τ through date $T_i - 1$, sums, in discounted terms, to the saving R that the continuation of the partnership grants to G. This is explained as follows. In each period τ, the firm knows that, if it abjures the contract, then the government will agree to renegotiate to avoid the break-up, which is costly. If there were no possibility of reneging again beyond date τ, then F could afford to force G to share its entire continuation benefit R. However, G is aware that, in the subsequent period, F will be motivated to also breach the new contract to once again cash in the cost of break-up. In light of this, rather than sharing R with F repeatedly, G would prefer to terminate the partnership and incur that cost once and for all. Being unable to convince G that it will not renege again beyond date τ, F agrees to renegotiate on a lower amount X_τ to avoid that outcome. Accordingly, a new agreement can be reached only if the parties negotiate on a surplus of $X_\tau < R$ at the end of period τ, $X_{\tau+1} < R$ at the end of period $\tau + 1$, and so forth, such that, overall, they share no more than the cost of break-up, in discounted terms.

Lemma 4 *Suppose that F reneges at τ. Under the reasonable belief that F will renege again in each subsequent period, the surplus to be shared in renegotiation amounts to $Rr/(1+r)$, if $\tau < T_i - 1$, and to R, if $\tau = T_i - 1$.*

Essentially, when F reneges before the last period of operation, the partners negotiate only on a part of the cost of break-up equal to $[R - R/(1+r)] = Rr/(1+r)$. The greater the discount rate r is, the less the firm can profit from future renegotiations, in discounted terms, and hence the more surplus will be shared at date τ. There is one interesting implication of this result. Because F cannot take advantage of repeated renegotiation, the exact duration of the contract is irrelevant in this respect. This explains why the expected profit of F from current and future renegotiations, in discounted terms, is independent of the contractual length T_i and equal to:

$$\Pi_{i,\tau}^{rn} = \alpha R, \ \forall \tau \in \{0, ..., T_i - 1\}. \tag{18}$$

where $\alpha \in (0,1)$ is the probability of F making a take-it-or-leave-it offer to G in the renegotiation game.[12]

4.1.4 One-shot reneging and renegotiation payoff of G

Let us next examine the behavior of the government. Unlike F, G may have incentives to renege at the end of the period, when it is supposed to compensate the firm. If G reneges on the contract and the partnership is terminated, then G incurs not only the cost of break-up but also the breach penalty. Thus, if the partners renegotiate after G reneges, then F cashes in a quota α of the cost of break-up R, as in the case in which F itself reneges, together with the entire penalty P, which is the reservation utility of F when G reneges. This makes it onerous for G to renege repeatedly, which explains the following result.

Lemma 5 *Suppose that G reneges at τ. Then, it does not renege at any $\tau' > \tau$.*

Based on this result and recalling that output is maintained at the efficient level in the new agreement, the earnings of G, if it reneges on the contract in state i and period τ, are determined as follows:

$$V_{i,\tau}^{rn} = w_i(q_i^*)\gamma_{T_i-\tau} - \frac{P+\alpha R}{1+r}, \ \forall \tau. \tag{19}$$

As in the initial contract, G appropriates the entire net surplus from the activity, $w_i(q_i^*)\gamma_{T_i-\tau}$. This amount is then diminished by the social value of the expected compensation that G owes to F, namely $(P+\alpha R)/(1+r)$. This expression is understood as follows: should the partnership be terminated at the end of period τ, G would pay P to F and incur the cost R. In $V_{i,\tau}^{rn}$, these values are discounted at the beginning of the period.

We have now established how each of the partners makes its reneging decisions, neglecting the possibility of the other partner having previously reneged. The following result completes

[12] Notice that the expression of $\Pi_{i,\tau}^{rn}$ in (18) is derived under the assumption that F avoids incurring not only the variable cost but also the fixed cost, if the partnership is interrupted. One can imagine that the fixed cost is strictly related to the management of the activity and that it remains with the latter when the partnership is terminated. Then, it passes to the firm that replaces F, with no effect on the overall net benefit of G from the activity. However, it is easy to verify that, should F actually bear the fixed cost in the break-up period, then the renegotiation payoff would be equal to $\alpha R - (1-\alpha)K$ (or to zero, when this expression is negative), rather than to αR, with no qualitative impact on the results.

this part of the analysis.

Lemma 6 *Neither F nor G reneges following reneging by the partner.*

This result is intuitive in that no partner can achieve a higher payoff than already obtained through the previous renegotiation.

4.2 Renegotiation-proofness of the contract

Based on the analysis developed hitherto, we can formulate the *self-enforcing constraints*, which must be satisfied for the contract that stipulates an efficient allocation to be robust to contractual reneging. There are two sets of constraints to be considered, namely those whereby both partners prefer to abide by the initial contract rather than terminating the partnership, and those whereby both partners refer to abide by the initial contract rather than reneging on it to reach a new agreement.

4.2.1 Initial contract *vs* break-up

Under the assumption that each partner has a zero outside option, neither of them prefers to terminate the contract and take that option as long as the following conditions hold:

$$\frac{w_h\left(q_h^*\right)}{r} \geq I+\psi; \quad \frac{w_l\left(q_l^*\right)}{r} \geq \frac{P+\alpha R}{1+r}; \quad I+\psi \geq \frac{\psi}{\Delta\nu}. \tag{20}$$

Under the first two conditions, the benefit that G obtains from the project is sufficiently high relative to the cost it incurs for the realization of the project and to the cost that it expects to bear if it reneges on the contract and the relationship is terminated. Under the last condition, the cost of investment is, in turn, sufficiently high to warrant that the profit $\Pi_{h,0}^*$ (as defined in (7)) is not too low to motivate F to terminate the relationship during operation. Henceforth, we take (20) to be satisfied such that the contract is not reneged upon, unless some partner expects to reach a better deal within the relationship.

4.2.2 Initial contract *vs* renegotiation

Under the conditions in (20), the only self-enforcing constraints to be considered are those whereby both F and G prefer the initial contract to the new agreement they could reach through renegotiation. The constraints are given by

$$\Pi_{i,\tau}^{*} \geq \Pi_{i,\tau}^{rn}, \ \forall i, \tau, T_i \tag{21}$$

$$V_{i,\tau}^{*} \geq V_{i,\tau}^{rn}, \ \forall i, \tau, T_i. \tag{22}$$

Using (15), (16), (18) and (19) and noticing that (21) is tightest for $\tau = T_i - 1$ and (22) for $\tau = 0$ and for $i = l$, we can rewrite (21) and (22), respectively, as follows:

$$T_i \leq \frac{\ln \frac{\alpha(1+r)R}{\alpha(1+r)R - r\Pi_{i,0}^{*}}}{\ln(1+r)}, \ \forall i, \quad \text{if } \alpha R \geq \frac{r\Pi_{i,0}^{*}}{1+r} \tag{23}$$

$$\Pi_{l,0}^{*} \leq \frac{P + \alpha R}{1+r}. \tag{24}$$

From (23), we see that the temptation for F to renege is *related* to the duration of the contract such that (23) potentially conflicts with (9) in state l. From (24), we further see that, by contrast, the temptation for G to renege is *independent* of the duration of the contract. The two conditions are explained as follows. (23) reflects the circumstance in which, as time passes, the residual contractual profit $\Pi_{i,\tau}^{*}$ becomes smaller relative to the expected renegotiation profit $\alpha Rr/(1+r)$. This makes it more difficult to motivate F to honor the contract through the choice of the termination date, regardless of the state of nature. This issue is exacerbated when the duration is long because, in that case, the contractual profit is diluted over a greater period. (24) reflects the circumstance in which G is most tempted to renege at the outset of the operation stage regardless of the contractual term. This temptation is stronger in the good state, in which G owes a higher profit to F ($\Pi_{l,0}^{*} > \Pi_{h,0}^{*}$), which explains why (24) is stated for $i = l$.

Based on the irrelevance of the contractual length in (24), we take this condition to hold when drawing our subsequent results concerning the optimal choice of T_l and T_h. However, before turning to that, we further inspect (24) to learn more about the incentives of G. Unsurprisingly, P and R act as substitutes in the trade-off that G faces between complying with the initial contract and renegotiating. Thus, for G to be motivated to honor the contract, at least one of P or R must be sufficiently high. This is more clearly viewed by substituting (6) into (24) and remarking that (24) is weakest when $\Delta\Pi = \psi/\Delta\nu$, in which case it is specified as follows:

$$P \geq (1+r)\left[I + (1-\nu_0)\frac{\psi}{\Delta\nu}\right] - \alpha R. \tag{25}$$

This condition shows that the minimum penalty that a court must be able to impose is lower the higher the cost of break-up is to G. Notice, however, that the minimum necessary penalty also depends on the initial investment. Given R, the more the firm contributes to the project up front, the more attractive the break-up is to the government and the higher the sanction that the government must face for not terminating the partnership. Whereas we draw our subsequent results assuming that P and I are such that (25) holds for any given R, we will reconsider the restrictions that limited penalties impose on the size of the private investment when stating Corollary 3 below.

4.3 Enforcement of the contract under limited commitment

We now return to the choice of the contractual length and, particularly, to the potential conflict between (9) and (23) with regard to the choice of T_l. This conflict reflects the circumstance in which it might be difficult to motivate F to abide by the contract while also extracting information from F in state h. Proceeding as in the previous section, in the results below, we refer to the triplet $\{\Delta\Pi, T_l, T_h\}$ as to the decision variables through which an efficient allocation is decentralized.

Lemma 7 *Assume that* (20) *and* (25) *are satisfied. There exists a triplet* $\{\Delta\Pi, T_l, T_h\}$ *such* *that the contract stipulating an efficient allocation is renegotiation-proof if and only if:*

$$\alpha R \leq \left[I + (1 - \nu_0)\frac{\psi}{\Delta\nu}\right]\frac{\Delta\theta q_l^* / (1 + r)}{\psi / \Delta\nu}. \tag{26}$$

To see why (26) is necessary, suppose it is violated. Then, it is impossible to identify a value of T_l such that adverse selection is addressed in state h, together with the firm's opportunism in state l. Whereas T_l must be sufficiently long to comply with (10) for F to reveal the high cost, T_l must also be sufficiently short to comply with (23) in state l. Otherwise, F could extract more from G than the residual contractual profit, cashing in the high cost of break-up. Interestingly, (26) is also sufficient for implementation of an efficient allocation, provided that different termination dates can be chosen for different states, if it is convenient to do so. Indeed, by separating the choice of T_h from that of T_l, one can satisfy the self-enforcing constraint (23) in state h, without being concerned with the lower bound that (9) imposes on the duration of the contract, provided that bound only applies in state l. The core result of our study, to be presented in a moment, is that enforcement of the contract that stipulates an efficient allocation does require relying on a state-dependent duration for some intermediate values of R. Before formalizing this result, it is useful to define the following:

$$T_h^{lc}(\Delta\Pi) \equiv \frac{\ln \frac{\alpha(1+r)R}{\alpha(1+r)R - r(I + \psi - \nu_1 \Delta\Pi)}}{\ln(1 + r)}.$$

This is the contractual length such that (23) is saturated in state h whenever the cost of break-up is sufficiently high for that to occur for a finite value of T_h (*i.e.*, whenever $\alpha R > (I + \psi - \nu_1 \Delta\Pi) r / (1 + r)$).

Proposition 2 *Assume that* (20) *and* (25) *are satisfied. Under limited commitment:* (*i*) *If*

$$\alpha R \leq \left(I - \nu_0 \frac{\psi}{\Delta\nu}\right)\frac{\Delta\theta q_l^* / (1 + r)}{\psi / \Delta\nu}, \tag{27}$$

then an efficient allocation is implemented contractually by setting $\Delta\Pi = \Delta\Pi^*$ *and* $T_l^{**} = T_h^{**} \in$

$\left[\underline{T}(\Delta\Pi^*), T_h^{lc}(\Delta\Pi^*)\right]$. *(ii) If*

$$\left(I - \nu_0 \frac{\psi}{\Delta\nu}\right) \frac{\Delta\theta q_l^* / (1+r)}{\psi/\Delta\nu} < \alpha R \leq \left[I + (1-\nu_0) \frac{\psi}{\Delta\nu}\right] \frac{\Delta\theta q_l^* / (1+r)}{\psi/\Delta\nu}, \tag{28}$$

then implementation of an efficient allocation implies that $T_h^{**} \leq T_h^{lc}(\Delta\Pi^*) < \underline{T}(\Delta\Pi^*) \leq T_l^{**}$, $\forall \Delta\Pi^* \in \left[\frac{\psi}{\Delta\nu}, \frac{\Delta\theta q_l^*}{r}\right]$.

The main result of the proposition, stated in part *(ii)*, is that if G wants to attain efficiency when the cost of break-up is sufficiently - though not too - high (such that (28) holds), then G should set $T_h^{**} < T_l^{**}$. That is, the contract should stipulate that F will run the activity for a greater number of periods if the cost is low, whereas the partnership will have a shorter duration if the cost is high. To appraise the relevance that moral hazard and adverse selection, on one side, and limited commitment, on the other, have for this result, it is useful to restate (27) as follows:

$$\alpha R \leq \frac{r}{1+r} \frac{\Delta\theta q_l^* / r}{\Delta\Pi / \Pi_{h,0}^*}. \tag{29}$$

First, (29) is tighter the higher the rate $\Delta\Pi/\Pi_{h,0}^*$ at which the profit grows from the bad state to the good state. As this rate is lowest when (3) is binding ($\Delta\Pi = \psi/\Delta\nu$), the extent to which (29) can be relaxed depends on the severity of the moral hazard problem. Second, the higher the risk to which F is exposed, the more severe the punishment that F faces in state h, and the stronger the temptation of F to understate the cost to escape punishment. To eliminate this temptation, F should face a sufficiently high penalty if it claims l in state h. The highest value the penalty can attain is $\Delta\theta q_l^*/r$ in a contract with infinite duration. The lower this value, the more difficult it is to induce truth-telling in the bad state, the tighter (29) is. Third, a harsher punishment makes F more eager to renege on the contract during its execution. Then, (23) is satisfied in state h only if the expected benefit to F from renegotiation, namely αR, is sufficiently low. The more beneficial renegotiation is, the more difficult it is to motivate F to abide by the contract and, again, the tighter (29) is. This all explains why the severity of moral hazard and adverse selection and the limits to commitment matter in the result of

Proposition 2. If exerting effort were costless, then G would not need to punish F in the bad state. The profits could be set equal in the two states and (29) would hold trivially, entailing that the adverse selection problem would be addressed together with the commitment problem in a fixed-term contract. When effort is, instead, sufficiently costly, if the penalty $\Delta\theta q_l^*/r$ is not high enough or the renegotiation payoff αR is not low enough, then those two problems cannot be addressed simultaneously unless $T_h^{**} < T_l^{**}$.

Recalling from Proposition 1 that switching from a fixed to a state-dependent duration enhances the possibility of changing the duration to compensate for changes in the per-period reward/punishment to F, we can state the following corollary.

Corollary 2 *If $T_h^{lc}(\psi/\Delta\nu) < \underline{T}(\psi/\Delta\nu)$, then the contract that stipulates an efficient allocation is made renegotiation-proof by setting z_h above q_l^* :*

$$z_h \geq \frac{\alpha R}{I + \psi - \nu_1 \Delta\Pi^*} \frac{\psi}{\Delta\nu} \frac{1+r}{\Delta\theta} > q_l^*.$$

By reducing the contractual length below $\underline{T}(\psi/\Delta\nu)$ in the bad state, the per-period punishment to F can be raised above the maximum value of $\nu_1\Delta\theta q_l^*$ that would be consistent with the adverse selection constraints in a fixed-term contract.

An important observation is in order to complete this part of the analysis. We argued that under limited commitment it might be necessary to decrease T_h because, by doing so, the compensation to the firm is not diluted over too many periods, and hence it is not too low in each period. Thus, at each τ, the residual profit is sufficiently high to induce F to continue executing the contract. Based on Corollary 2, one might wonder how it is possible that F obtains a higher profit in each period if the contract is shortened, provided that F faces a punishment in state h. This is because the per-period cost of capital is increased as T_h is reduced. Hence, even if the firm is subject to a greater per-period punishment, sufficient to keep the adverse selection constraints satisfied, it will receive a higher per-period profit overall. This effect cannot occur in Engel *et al.* [8] - [9] because the cash flow is exogenous in their framework, as we said, and hence the per-period profit is not an instrument to shorten the duration of the contract. Obviously, the component of the per-period profit related to the cost of capital is also higher the greater the investment is. Therefore, the nexus between I and P can be used to assess whether a state-dependent duration is necessary, depending on how substantially the court is able to fine G. Indeed, using (25), one can see that, for any given value of P, there is a maximum level of investment that the firm can be required to make up front, defined as follows:

$$\overline{I}\left(P\right) \equiv \frac{P + \alpha R}{1+r} - \left(1-\nu_0\right)\frac{\psi}{\Delta\nu}. \tag{30}$$

Evidently, $\overline{I}\left(P\right)$ is lower the lower P is. Because $T_h^{lc}\left(\psi/\Delta\nu\right)$ increases with I, whereas $\underline{T}\left(\psi/\Delta\nu\right)$ is independent of the cost of investment, it is clear that conditioning the contractual length on the state of nature is more important the weaker the ability of the court to fine the government is. Intuitively, if the investment is low, then the cost of capital to be repaid to the firm in each period will also be low, unless the relationship has a short duration.

Corollary 3 *If*

$$P < \left[\frac{\psi/\Delta\nu}{\Delta\theta q_l^*}\left(1+r\right)^2 - 1\right]\alpha R + \left(1+r\right)\frac{\psi}{\Delta\nu}, \tag{31}$$

then an efficient allocation is contractually implemented only if $T_h^{**} \leq T_h^{lc}\left(\Delta\Pi^*\right) < \underline{T}\left(\Delta\Pi^*\right) \leq T_l^{**}$.

4.4 An example

To illustrate the analysis developed thus far, we provide a numerical example. Take $S\left(q\right) = 20q^{(\varepsilon-1)/\varepsilon}\varepsilon/\left(\varepsilon-1\right)$ such that $p\left(q\right) = S'\left(q\right) = 20q^{-1/\varepsilon}$ is an inverse demand function with constant elasticity equal to ε. Further take $\varepsilon = 1.2$, $\psi = 40$, $\nu_1 = 0.7$, $\nu_0 = 0.3$, $\theta_l = 5$, $\theta_h = 8$, $I = 130$, $r = 0.03$. The profit wedge $\Delta\Pi$ such that (3) is saturated is equal to 100 and the profits in the two states are $\Pi_{l,0} = 200$ and $\Pi_{h,0} = 100$. Accordingly, $\underline{T}\left(100\right) = 7.1$. Further, taking $R = 50$ and $\alpha = 0.5$ yields $T_h^{lc}\left(100\right) = 4.2$. This means that, for the contract that implements an efficient allocation to be enforced, the partnership should last at least 7 years in the good state and at most 4 years in the bad state. Starting from a fixed term of 7 years, a reduction to 4 years in state h is matched with an increase in the per-period profit π_h from 16 to 26.9. The increase in π_h reflects the fact that the firm is allowed to obtain a profit of $\Pi_{h,0} = 100$ in a shorter time. However, neither shirking in construction nor cost exaggeration in operation is encouraged because the per-period punishment $\nu_1 \Delta\theta z_h$ is increased, in turn, from 11.23 to 18.83. One can verify that the necessary conditions in (20) are all satisfied and that (25) is satisfied if $P \geq 181$.

Now, let R, ε, I and P vary one by one, while all other parameters remain unchanged. Proceeding in this way, a state-dependent duration is found to be necessary if (i) the cost of break-up is not too small, $R > 30$, in which case $T_h^{lc}(100) < 7.1$; (ii) demand is not very elastic, $\varepsilon < 1.55$, in which case $\underline{T}(100) > 4.2$; (iii) the cost of investment made by F is not too high, $I < 194$, in which case $T_h^{lc}(100) < 7.1$; and (iv) the penalty that the court can impose on G is not very high, $P < 246$, and hence the investment must be such that $I < 194$ (*i.e.,* (31) holds), entailing $T_h^{lc}(100) < 7.1$.

5 Declining cost of break-up

In PPPs, reputation concerns represent an essential component of the cost of a break-up. The government may face a loss of reputation and/or credibility for not being authoritative enough to have the contract executed by the project partner and/or for breaking promises *vis-à-vis* current and prospective investors, customers and voters (see Guasch *et al.* [14] and Irwin [22]). One expects this loss to decline during the execution of the contract because expropriation of the private investor after two years is likely to have a stronger impact on reputation than expropriation after twenty years. To consider this possibility, in this section, we assume that the cost of break-up to G, now denoted $R_{T_i-\tau}$ in state i, is higher the greater the residual contractual period $T_i - \tau$ is (see Danau and Vinella [5] for a similar approach). That is, for any given τ, the cost of break-up is higher the greater T_i is; for any given T_i, it is lower the greater τ is. For convenience, we also assume that the function $R_{T_i-\tau}$ is invertible. Based on the same arguments as above, the maximum that F can expect to obtain, in equilibrium, from all perspective renegotiations starting from period τ amounts to $\alpha R_{T_i-\tau}$.

The fact that not only the residual profit of F but also the cost of break-up to G declines as the residual contractual length declines is essential to determining whether, in each period, the contract is executed, a new agreement is reached, or the partnership is interrupted at all. Depending on the speed at which the cost of break-up declines relative to the residual profit, either F or G becomes more eager to renege on the contract. To see this, it is convenient to rewrite (21) and (22), respectively, as follows:

$$\Pi_{i,0}^{*}\frac{\gamma_{T_i-\tau}}{\gamma_{T_i}} \geq \alpha R_{T_i-\tau},\ \forall i,\tau,T_i \tag{32}$$

$$\Pi_{i,0}^{*}\frac{\gamma_{T_i-\tau}}{\gamma_{T_i}} \leq \frac{P+\alpha R_{T_i-\tau}}{1+r},\ \forall i,\tau,T_i. \tag{33}$$

For the analysis to be meaningful, we assume that the court can impose a sufficiently large penalty $P \geq r\alpha R_{T_i-\tau}$, $\forall \tau, T_i$, which is necessary for (32) and (33) to hold simultaneously, for all values of T_i that are relevant for the subsequent findings.

Unlike under a constant cost of break-up, it is no longer clear that the self-enforcing constraint of F becomes tighter as one moves from period τ to the subsequent periods. Indeed, in either state, not only the residual profit but also the potential renegotiation benefit to F decreases as time passes. A similar issue arises with regard to G. It is no longer clear that G is more eager to renege on the contract at the outset of the operation phase, provided that breaking up the partnership becomes less costly as time passes. Thus, everything depends on how sharply the cost of break-up decreases from one period to the next.

First, suppose that the rate at which the cost of break-up decreases is small enough to satisfy

$$\alpha\left(R_{T_i-\tau}-R_{T_i-(\tau+1)}\right) \leq \Pi_{i,0}^{*}\frac{\gamma_{T_i-\tau}-\gamma_{T_i-(\tau+1)}}{\gamma_{T_i}},\ \forall i,\tau,T_i. \tag{34}$$

One can interpret this condition as follows. Starting from $\tau=0$ and from some cost R_{T_i} of terminating the partnership at the outset of the operation phase, the cost of break-up remains relatively high in all periods as τ increases. Under this condition, (32) is tightest for $\tau = T_i - 1$

and (33) is tightest for $\tau = 0$, and they are, respectively, rewritten as follows:

$$\begin{aligned} T_i &\leq T_i^1 (\Delta\Pi) \equiv \frac{\ln \frac{\alpha R_1 (1+r)}{\alpha R_1 (1+r) - r \Pi_{i,0}^*}}{\ln (1+r)}, \text{ if } \alpha R_1 > \frac{r \Pi_{i,0}^*}{1+r} \\ \Pi_{i,0}^* &\leq \frac{P + \alpha R_{T_i}}{1+r}, \end{aligned}$$

where $T_i^1 (\Delta\Pi)$ is the counterpart of $T_i^{lc} (\Delta\Pi)$ with R_1 replacing R. Denote $\Upsilon (\cdot)$ as the inverse function of R_{T_i} to further reformulate the latter constraint as follows:

$$T_i \geq \Upsilon \left(\frac{(1+r) \Pi_{i,0}^* - P}{\alpha} \right).$$

This shows that a lower bound may now appear on the contractual length. Intuitively, for G to be unwilling to terminate the partnership prematurely, the interruption must occasion a sufficiently high cost in each period of operation. When the cost of break-up decreases over time, it might be necessary to set the duration long enough to ensure that the cost remains sufficiently high as time passes. Taken together, (32) and (33) may define a range of admissible termination dates for each state i :

$$\Upsilon \left(\frac{(1+r) \Pi_{i,0}^* - P}{\alpha} \right) \leq T_i \leq T_i^1 (\Delta\Pi).$$

Obviously, the range is bounded from both below and above as long as both $\Upsilon (\cdot)$ and $T_i^1 (\cdot)$ are finite values. Note that the existence of a range of termination dates T_i is guaranteed by the existence of profits $\Pi_{i,0}^*$ jointly satisfying (32) and (33). Because $\Pi_{h,0}^* < \Pi_{l,0}^*$, the range of admissible values of T_h is shifted downward relative to the range of admissible values of T_l. In particular, when the upper bound on T_h lies below the lower bound on T_l, namely:

$$T_h^1 (\Delta\Pi) < \Upsilon \left(\frac{(1+r) \Pi_{l,0}^* - P}{\alpha} \right), \tag{35}$$

the admissible values of T_h are all smaller than those of T_l. This shows that, unlike in situations in which the cost of break-up is constant, here, the self-enforcing constraints *alone* may require shortening the duration in the bad state relative to the good state.[13]

Of course, the restrictions resulting from the self-enforcing constraints must be considered jointly with the upper bound and the lower bound imposed on T_l and T_h by (9) and (10), respectively. When (35) is violated and the self-enforcing constraints can be satisfied with $T_l = T_h$, it remains necessary to set $T_h < T_l$ if either $T_h^1(\Delta\Pi) < \underline{T}(\Delta\Pi)$ or $\Upsilon\left(\left[(1+r)\Pi_{l,0}^* - P\right]/\alpha\right) > \overline{T}(\Delta\Pi)$. Starting from a fixed term of $T_h = T_l \equiv T$ set in such a way that moral hazard and adverse selection are addressed, either T_h should be decreased below T_l to prevent F from reneging in state h or T_l should be raised above T_h to make reneging costly enough to G in state l. Statement (i) in Proposition 3 below mirrors this reasoning.

A similar reasoning applies to the subsequent statements (ii) and (iii), which refer to cases in which $R_{T_i-\tau}$ decreases more sharply with τ. Based on this similarity, we only provide a brief description of such cases before formalizing the proposition. First, when the cost of break-up decreases sharply enough to satisfy:

$$\alpha\frac{R_{T_i-\tau} - R_{T_i-(\tau+1)}}{1+r} \geq \Pi_{i,0}^* \frac{\gamma_{T_i-\tau} - \gamma_{T_i-(\tau+1)}}{\gamma_{T_i}}, \ \forall i, \tau, T_i, \tag{36}$$

(32) is tightest for $\tau = 0$ and (33) is tightest for $\tau = T_i - 1$, and they respectively become:

$$\begin{aligned} T_i &\leq \Upsilon\left(\frac{\Pi_{i,0}^*}{\alpha}\right) \\ T_i &\geq T_i^2(\Delta\Pi) \equiv \frac{\ln\frac{P+\alpha R_1}{P+\alpha R_1 - r\Pi_{i,0}^*}}{\ln(1+r)}. \end{aligned}$$

The range of admissible termination dates for each state i is given by:

$$T_i^2(\Delta\Pi) \leq T_i \leq \Upsilon\left(\frac{\Pi_{i,0}^*}{\alpha}\right).$$

As the inequality $\Pi_{l,0}^* > \Pi_{h,0}^*$ entails that both $T_i^2(\Delta\Pi)$ and $\Upsilon(\Pi_{i,0}^*/\alpha)$ are lower in state h than in state l, the range of values of T_h is shifted downward relative to that of values of T_l, with a

[13] Notice however that it is to satisfy (3) and (5) that the profits are such that $\Pi_{h,0}^* < \Pi_{l,0}^*$.

similar conclusion to the previous case. Furthermore, a similar conclusion is also reached when:

$$\alpha\frac{R_{T_i-\tau}-R_{T_i-(\tau+1)}}{1+r}\leq\Pi_{i,0}^{*}\frac{\gamma_{T_i-\tau}-\gamma_{T_i-(\tau+1)}}{\gamma_{T_i}}\leq\alpha\left(R_{T_i-\tau}-R_{T_i-(\tau+1)}\right),\ \forall i,\tau,T_i. \tag{37}$$

Then, both (32) and (33) are tightest for $\tau=0$, and the following range of values of T_i is identified:

$$\Upsilon\left(\frac{(1+r)\,\Pi_{i,0}^{*}-P}{\alpha}\right)\leq T_i\leq\Upsilon\left(\frac{\Pi_{i,0}^{*}}{\alpha}\right).$$

We can now summarize the results in a proposition.

Proposition 3 *For an efficient allocation to be implemented contractually, it is necessary that the duration of the contract be state-dependent and such that* $T_h<T_l$ *if,* $\forall i,\tau,T_i$:

(*i*) (34) *holds together with*

$$\begin{aligned}\textit{either}\ \ T_h^1\left(\frac{\psi}{\Delta\nu}\right) &< \min\left\{\Upsilon\left(\frac{(1+r)\,\Pi_{l,0}^{*}-P}{\alpha}\right);\ \underline{T}\left(\frac{\psi}{\Delta\nu}\right)\right\}\\ \textit{or}\ \ \overline{T}\left(\frac{\psi}{\Delta\nu}\right) &< \Upsilon\left(\frac{(1+r)\,\Pi_{l,0}^{*}-P}{\alpha}\right)\end{aligned}$$

(*ii*) (36) *holds together with*

$$\begin{aligned}\textit{either}\ \ \Upsilon\left(\frac{\Pi_{h,0}^{*}}{\alpha}\right) &< \min\left\{T_l^2\left(\frac{\psi}{\Delta\nu}\right);\ \underline{T}\left(\frac{\psi}{\Delta\nu}\right)\right\}\\ \textit{or}\ \overline{T}\left(\frac{\psi}{\Delta\nu}\right) &< T_l^2\left(\frac{\psi}{\Delta\nu}\right)\end{aligned}$$

(*iii*) (37) *holds together with*

$$\begin{aligned}\textit{either}\ \ \Upsilon\left(\frac{\Pi_{h,0}^{*}}{\alpha}\right) &< \min\left\{\Upsilon\left(\frac{(1+r)\,\Pi_{l,0}^{*}-P}{\alpha}\right);\ \underline{T}\left(\frac{\psi}{\Delta\nu}\right)\right\}\\ \textit{or}\ \ \overline{T}\left(\frac{\psi}{\Delta\nu}\right) &< \Upsilon\left(\frac{(1+r)\,\Pi_{l,0}^{*}-P}{\alpha}\right).\end{aligned}$$

Proposition 3 is the counterpart of Proposition 2 in the case in which the cost of break-up declines as the contract approaches its termination. It shows that, just as in the case of a constant cost of break-up, it might be necessary to adopt a contract with a state-dependent duration and that, when this is the case, the termination dates must be set such that $T_h < T_l$. However, unlike in the case of a constant cost of break-up, there are three alternative reasons for $T_h < T_l$, two of them being new. First, a fixed-term contract might fail to simultaneously eliminate the incentives for the two partners to renege. A low value of T_h is more likely to motivate F to abide by the contract because recovery of the initial investment occurs rapidly, and this makes the contract more beneficial to F relative to renegotiation. A high value of T_l is more likely to motivate G to comply with the contract because reputation concerns are especially important when the contract is long-term. Second, and in line with Proposition 2, it might be necessary to couple a low value of T_h with a high value of T_l to be able to address the information problems together with the commitment problem on the firm's side in the bad state. Third, it might be necessary to couple a high value of T_l with a low value of T_h to be able to address the information problems together with the commitment problem on the government's side, provided that T_h cannot be set too high under Proposition 1.

6 Further considerations

We characterized an efficient allocation and identified conditions under which it can be implemented contractually. Provided that the efficient allocation does not depend on the specific length of the contract, we could investigate how the contractual term should be set to address moral hazard, adverse selection and limited commitment altogether. In practice, there might be cases in which efficiency itself depends on the contractual term. Thus, when moral hazard and adverse selection have bite together with commitment problems, the optimal contractual length would reflect the trade-off between efficiency and the costs of incentive provision. We hereafter provide a synthetic overview of possible reasons for this trade-off to arise and leave further investigation for future research.

Different discount rates In situations in which the discount rate of the government differs from that of the firm, the desirability of PPP arrangements in the realization of infrastructure projects may be affected.[14] Let r_F and r_G be the discount rates of F and G, respectively, which are potentially different. Accordingly, use the notation γ_x^F and γ_x^G with the same meaning as before. Under complete information, the profit of F at date τ amounts to $\Pi_\tau = \pi\gamma_{T-\tau}^F$, whereas the return of G is given by $V_\tau = w(q)\gamma_{T-\tau}^G - \Gamma(\tau, T)\,\Pi_\tau$, where $\Gamma(\tau, T) \equiv \gamma_{T-\tau}^G / \gamma_{T-\tau}^F$. Focusing for simplicity on a fixed contractual term T, the expected payoff of G from the entire life of the project is given by

$$\mathbb{E}\left[W_i^*\right] = \frac{\mathbb{E}\left[w\left(q_i^*\right)\right]}{r_G} - \Gamma\left(0, T\right)\left(I + \psi\right).$$

When $r_G \neq r_F$, this is not independent of the choice of T. In particular, a government that is more impatient than the firm ($r_G > r_F$) will find it optimal to set $T \to \infty$, which is tantamount to privatizing the activity, to minimize $\Gamma(0, T)$. Intuitively, an impatient government valuing future consumption less than current consumption delays the social cost of the payments to the less impatient firm by diluting those payments over as long a time interval as possible. Whereas efficiency calls for privatization, incentive issues are addressed only if the contract has a limited duration, at least in the bad state of nature. Therefore, the government would face a trade-off between efficiency and incentive provision.

No transfers Public transfers to firms are sometimes forbidden. For instance, the EU requires that build-operate-transfer concession holders rely exclusively on revenues from market sales.[15] Having thus excluded transfers, the quantity, and hence the price, and the contractual length are the only available incentive tools, entailing that, as before, the choice of the contractual length is not neutral with respect to efficiency. For each state i, there is an optimal price-duration pair $\{p(q_i), T_i\}$ trading off intertemporal budget balancing, under which the firm recovers both the initial cost of investment and the costs of production during the execution of the contract,

[14] For instance, the discount rate may be *lower* for governments that are perceived to be reliable and thus have cheaper access to financial resources than their private partners. The discount rate may be *higher*, instead, for governments with an urgent need for revenue.

[15] For instance, Auriol and Picard [1] mention the Channel Tunnel project.

and average cost pricing, under which the provider operating the activity after the end of the contract breaks even. To clarify this trade-off, let us write the payoff of G from the entire life of the project:

$$\mathbb{E}\left[\left(S(q_i) - p\left(q_i\right) q_i\right)\gamma_{T_i} + \frac{S(q_i^{ac}) - p\left(q_i^{ac}\right) q_i^{ac}}{r\left(1+r\right)^{T_i}}\right],$$

where $p\left(q_i^{ac}\right) = \theta_i + K/q_i^{ac}$ and the superscript ac is appended to denote *a*verage *c*ost. The intertemporal budget constraint of F, when it exerts effort in construction, is given by

$$\mathbb{E}\left[\left(p\left(q_i\right) q_i - \theta_i q_i - K\right)\gamma_{T_i}\right] \geq I + \psi.$$

Letting μ be the Lagrange multiplier associated with this constraint, the efficient pricing scheme is characterized by the standard Ramsey-Boiteux rule, with the Ramsey number depending on μ, whereas a corner solution emerges from optimization of the Lagrangian with respect to the contractual length. That is, T_i should be set either very high or close to zero, according to the sign of the following expression:

$$-\left[\left(S(q_i^{ac}) - p\left(q_i^{ac}\right) q_i^{ac}\right) - \left(S(q_i) - p\left(q_i\right) q_i\right)\right] + \mu\left(p\left(q_i\right) q_i - \theta_i q_i - K\right),$$

which in turn depends on the magnitude of μ. Intuitively, when the initial investment is important (μ is high), it is preferable to let the private partner recover the associated cost through an infinite number of periods to minimize the price distortion in each period. This is tantamount to a full privatization of the activity. By contrast, when the initial investment is small (μ is low), it is better to let the firm recover the associated cost as soon as operation begins and then revert to average cost pricing in each subsequent period under new management (whether public or private). This is very similar to traditional procurement. Our analysis indicates that incentive considerations impose lower and/or upper limits on the choice of the contractual length. Hence, once incentive issues are accounted for, a corner solution is unlikely to be suitable. Particularly, when the duration of the contract cannot be too long, the desirability of a PPP arrangement is restored.

Fixed price In some instances, public authorities prefer to fix the consumer fee prior to the activity being operated. That fee is then adopted by the firm during the management phase regardless of the specific operating conditions. If the price were required to be fixed in our model, then the firm's profit could still be conditioned on the state of nature through a proper choice of the transfers and the termination dates, but the resulting mix of price distortions and costly transfers would no longer be optimal even under complete information. If public transfers were also excluded (one may consider concession contracts, for instance), then the same issues as above would arise in the determination of the optimal contractual length.

In addition to situations in which efficiency depends on the duration of the contract, a few observations are in order regarding situations in which the infrastructure becomes obsolete after a certain number of years. Obsolescence of the infrastructure might impose difficulties in setting a long duration when the operating conditions are good, as our analysis recommends for incentive purposes. One can think of obsolescence of the infrastructure in two different ways. First, the operation and maintenance cost increases over time and becomes very high after a certain period of time. Second, the demand for the good decreases over time and, at a certain point, becomes too low to justify the continuation of the activity. Allowing for any such possibility would shift our analysis toward a different issue, namely whether a PPP is at all an appropriate contractual arrangement for the development of the project, which seems to be highly questionable as far as fast-evolving sectors are concerned.[16]

7 Concluding remarks

There is one essential lesson regarding PPP contracts to be drawn from our analysis. Under asymmetric information and limited commitment, a contract with a state-dependent duration may perform better than a fixed-term contract. In situations in which this is the case, the duration of the contract should be shorter if the operating conditions are unfavorable and

[16] See HM Treasury [19], which specifies that due to fast-changing service requirements and given the poor evidence of past projects, PPPs are not used by the UK Government for information technology projects (p.32).

longer if they are favorable.

There are two effects that jointly explain why the contractual length should be set in this way. First, if the choice of the termination date is separated in the two states, then the firm can be punished more in the bad state than would be possible if the contract had a fixed term while remaining in compliance with the information constraints. Similarly, the firm can be rewarded more in the good state. Second, by increasing the per-period punishment/reward to the firm through the adoption of a state-dependent duration, the self-enforcing constraints are relaxed. This is because the profit assigned to the firm in each period of operation includes two components, one intended to dissuade the firm from exploiting its private information and the other to repay the firm for its initial contribution to the project. For instance, if the duration of the contract is shortened in the bad state relative to the good state, then the power of incentives is reinforced, as the firm is punished more, and the contract is more easily enforced, as the firm is repaid a higher quota of its investment in each period. Therefore, commitment is enhanced without exacerbating moral hazard and adverse selection.

The discrepancy between this contractual approach, which emerges in an environment with endogenous and non-contractible risk, and the flexible-term proposal, which is advanced in the context of exogenous risk, demonstrates the need for a better understanding of the nexus between contractual non-enforceability and contractual incompleteness as distinct sources of limited commitment. For the optimal design of PPP contracts, it would thus be useful to investigate further in that direction.

In terms of policy implications, our findings suggest that the duration of a PPP should be stipulated in the contract rather than being determined during the operation phase. A recommendation of this kind is found in the document on "Standardisation of PF2 contracts" published by HM Treasury in the UK, which even provides a standard for bounds on the contractual length.[17] Our results shed light on the factors that should be considered in contractual standardization. More generally, they offer a theoretical foundation for what seems to be com-

[17] At the beginning of Chapter 3 of the draft document, one reads the following: "The Contract must specify its duration." The draft is available at https://www.gov.uk/government/publications/private-finance-2-pf2

mon sense in PPPs, namely that governments and firms both fair better when partnerships take place under favorable contingencies. It is then natural that, if the term of a partnership is to be specified in a contract, it must be set longer in the former case, unless environmental conditions are such that a fixed-term contract can be expected to perform as well as a contract with a state-dependent duration.

References

[1] Auriol, E., and P.M. Picard (2013), "A Theory of BOT Concession Contracts," *Journal of Economic Behavior and Organization*, 89(C), 187-209

[2] Baron, D.P., and D. Besanko (1984), "Regulation and information in a continuing relationship," *Information Economics and Policy*, 1, 267-302

[3] Bennett, J., and E. Iossa (2006), "Building and Managing Facilities for Public Services," *Journal of Public Economics*, 90, 2143-2160

[4] Dahlby, B. (2008), *The Marginal Cost of Public Funds: Theory and Applications*, Cambridge, MA: MIT Press

[5] Danau, D., and A. Vinella (2015), "Public-private contracting under limited commitment," *Journal of Public Economic Theory*, 17(1), 78-110

[6] Dewatripont, M. (1989), "Renegotiation and information revelation over time: the case of optimal labor contracts", *Quarterly Journal of Economics*, 104, 589-619

[7] Ehrhardt, D., and T. Irwin (2004), "Avoiding Customer and Taxpayer Bailouts in Private Infrastructure Projects. Policy toward Leverage, Risk Allocation, and Bankruptcy," *World Bank Policy Research* Working Paper No. 3274

[8] Engel, E., R. Fischer and A. Galetovic (1997), "Highway Franchising: Pitfalls and Opportunities," *American Economic Review*, vol. 87(2), 68-72

[9] Engel, E., R. Fischer and A. Galetovic (2001), "Least-Present-Value-of-Revenue Auctions and Highway Franchising," *Journal of Political Economy*, 109(5), 993-1020

[10] Engel, E., R. Fischer and A. Galetovic (2013), "The Basic Public Finance of Public-Private Partnerships" *Journal of the European Economic Association*, 11(1), 83-111

[11] Engel, E., R. Fischer and A. Galetovic (2014), "Finance and Public-Private Partnerships: A Roadmap," background paper for the *Global Financial Development Report 2015-2016*, Washington, D.C.: World Bank Group

[12] Estache, A., and L. Wren-Lewis (2009), "Towards a Theory of Regulation for Developing Countries: Following Laffont's Lead," *Journal of Economic Literature*, 47(3), 729–770

[13] Guasch, J. L. (2004), *Granting and Renegotiating Infrastructure Concessions: Doing it Right*, WBI Development Studies, Washington, DC. World Bank

[14] Guasch, J. L., J.J. Laffont and S. Straub (2006), "Renegotiation of concession contracts: A theoretical approach," *Review of Industrial Organization*, 29, 55-73

[15] Guasch, J. L., J.J. Laffont and S. Straub (2008), "Renegotiation of Concession Contracts in Latin America, Evidence from the Water and Transport Sectors," *International Journal of Industrial Organization*, 26(2), 421-442

[16] Harris, M., and A. Raviv (1979), "Optimal incentive contracts with imperfect information," *Journal of Economic Theory*, 20, 231-259

[17] Hart, O. (2003), "Incomplete Contracts and Public Ownership: Remarks, and an Application to Public-Private Partnerships," *Economic Journal*, 113(486), C69-C76

[18] Hart, O., and J. Tirole (1988), "Contract renegotiation and Coasian dynamics," *Review of Economic Studies*, 55, 509-540

[19] HM Treasury (2006), *PFI: strengthening Long-Term Partnerships*, London, March 2006

[20] Iossa, E., and D. Martimort (2012), "Risk allocation and the costs and benefits of public–private partnerships," *RAND Journal of Economics*, 43(3), 442-474

[21] Iossa, E., and D. Martimort (2015), "The Simple Micro-Economics of Public-Private Partnerships," *Journal of Public Economic Theory*, 17(1), 4-48

[22] Irwin, T.C. (2007), *Government Guarantees: Allocating and Valuing Risk in Privately Financed Infrastructure Projects*, Washington, DC: The World Bank

[23] Laffont, J.J. (2003), "Enforcement, Regulation and Development," *Journal of African Economies*, 12, 193-211

[24] Laffont, J.J., and D. Martimort (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton and Oxford

[25] Levin, J. (2003), "Relational incentive contracts", *American Economic Review*, 93, 835-857

[26] Martimort, D. (2006), "An agency perspective on the costs and benefits of privatization," *Journal of Regulatory Economics*, 30, 5-44

[27] Martimort, D., and J. Pouyet (2008), "To build or not to build: normative and positive theories of Public-Private Partnerships," *International Journal of Industrial Organization*, 26(2), 393-411

[28] Rey, P., and B. Salanié (1996), "On the value of commitment with asymmetric information," *Econometrica*, 64, 1395-1414

[29] Sappington, D.E.M., and J.E. Stiglitz (1987), "Privatization, Information and Incentives," *Journal of Policy Analysis and Management*, 6(4), 567-582

[30] Trebilcock, M., and M. Rosenstock (2015), "Infrastructure public-private partnerships in the developing world: Lessons from recent experience," *Journal of Development Studies*, 51(4), 335-354

A Renegotiation process and payoffs

Suppose that a party reneges at date τ in state i. The partners return to the negotiating table. With probability $\alpha \in (0,1)$, F makes a take-it-or leave-it offer to G; with probability $1-\alpha$, G makes a take-it-or leave-it offer to F. Renegotiation either fails, in which case the partnership is broken up; or renegotiation succeeds and the partnership is continued according to the new agreement reached by the partners. The party that takes the initiative optimally makes the offer that leaves the partner just indifferent between renegotiation and break-up.

Hereafter, payoffs are determined at the beginning of period τ, taking into account that reneging by F takes place at the beginning of the period, reneging by G takes place at the end of the period.

Break-up payoffs Suppose that renegotiation fails and break-up follows. The payoff of F is $\Pi_{i,\tau}^{b,G} = P/(1+r)$, if G reneges; it is $\Pi_{i,\tau}^{b,F} = 0$, if F itself reneges. The payoff of G is:

$$V_{i,\tau}^{b,G} = w_i(q_i^*)\gamma_{T_i-\tau} - \frac{P+R}{1+r} \tag{38}$$

if G itself reneges; it is:

$$V_{i,\tau}^{b,F} = w_i(q_i^*)\gamma_{T_i-\tau} - R, \tag{39}$$

if F reneges.

Renegotiation payoffs Suppose that renegotiation succeeds. The pair of variables $\{q_i^{ar}, \Pi_{i,\tau}^{ar}\}$, on which the partners renegotiate, replaces the pair $\{q_i^*, \Pi_{i,\tau}^*\}$ stipulated in the contract (according to Lemma 2, T_i is not renegotiated). Whereas F obtains $\Pi_{i,\tau}^{ar}$, the payoff of G is:

$$V_{i,\tau}^{ar} = w_i(q_i^{ar})\gamma_{T_i-\tau} - \Pi_{i,\tau}^{ar}. \tag{40}$$

The values of q_i^{ar}, $V_{i,\tau}^{ar}$ and $\Pi_{i,\tau}^{ar}$ are determined in equilibrium when renegotiation succeeds.

A.1 Proof of Lemma 1

(1) Suppose that F reneges.

(1a) With probability α, F makes a take-it-or leave-it offer to G, according to which G is left with $V_{i,\tau}^{b,F}$; hence, $V_{i,\tau}^{ar} = V_{i,\tau}^{b,F}$. Replacing (40) in this equality, the payoff of F from renegotiation is computed as $\Pi_{i,\tau}^{ar} = w_i(q_i^{ar})\gamma_{T_i-\tau} - V_{i,\tau}^{b,F}$, which is highest for $q_i^{ar} = q_i^*$. Replacing $q_i^{ar} = q_i^*$ together with (39) in this expression yields $\Pi_{i,\tau}^{ar} = R$.

(1b) With probability $1-\alpha$, G makes a take-it-or-leave-it offer to F, according to which F is left with $\Pi_{i,\tau}^{b,F} = 0$. G obtains $V_{i,\tau}^{ar} = w_i(q_i^{ar})\gamma_{T_i-\tau}$, which is highest for $q_i^{ar} = q_i^*$. Replacing $q_i^{ar} = q_i^*$ yields $V_{i,\tau}^{ar} = w_i(q_i^*)\gamma_{T_i-\tau}$.

Based on (1a) and (1b), the expected payoff of F from renegotiation is:

$$\begin{aligned}\Pi_{i,\tau}^{rn} &= \alpha\Pi_{i,\tau}^{ar} + (1-\alpha)\Pi_{i,\tau}^{b,F} \\ &= \alpha R > 0 = \Pi_{i,\tau}^{b,F}.\end{aligned}$$

Hence, F strictly prefers renegotiation to break-up. The expected payoff of G from renegotiation is:

$$\begin{aligned} V_{i,\tau}^{rn} &= \alpha V_{i,\tau}^{b,F} + (1-\alpha) V_{i,\tau}^{ar} \\ &= w_i(q_i^*)\gamma_{T_i-\tau} - \alpha R \\ &> w_i(q_i^*)\gamma_{T_i-\tau} - R = V_{i,\tau}^{b,F}. \end{aligned}$$

Hence, G strictly prefers renegotiation to break-up.

(2) Suppose that G reneges.

(2a) With probability $1-\alpha$, G makes a take-it-or-leave-if offer to F, according to which F is left with $\Pi_{i,\tau}^{b,G} = P/(1+r)$. The payoff of G at the beginning of period τ is given by $V_{i,\tau}^{ar} = w_i(q_i^{ar})\gamma_{T_i-\tau} - P/(1+r)$, which is highest for $q_i^{ar} = q_i^*$. Replacing $q_i^{ar} = q_i^*$ yields $V_{i,\tau}^{ar} = w_i(q_i^*)\gamma_{T_i-\tau} - P/(1+r)$.

(2b) With probability α, F makes a take-it-or-leave-it offer to G, according to which G is left with $V_{i,\tau}^{b,G}$. F obtains $\Pi_{i,\tau}^{ar} = w_i(q_i^{ar})\gamma_{T_i-\tau} - V_{i,\tau}^{b,G}$, which is highest for $q_i^{ar} = q_i^*$. Replacing $q_i^{ar} = q_i^*$ together with (38) yields $\Pi_{i,\tau}^{ar} = (R+P)/(1+r)$.

Based on (2a) and (2b), the expected payoff of G is:

$$\begin{aligned} V_{i,\tau}^{rn} &= (1-\alpha) V_{i,\tau}^{ar} + \alpha V_{i,\tau}^{b,G} \\ &= w_i(q_i^*)\gamma_{T_i-\tau} - \frac{P+\alpha R}{1+r} \\ &> w_i(q_i^*)\gamma_{T_i-\tau} - \frac{P+R}{1+r} = V_{i,\tau}^{b,G}. \end{aligned}$$

Hence, G strictly prefers renegotiation to break-up. The expected payoff of F is:

$$\begin{aligned} \Pi_{i,\tau}^{rn} &= (1-\alpha)\Pi_{i,\tau}^{b,G} + \alpha \Pi_{i,\tau}^{ar} \\ &= \frac{P+\alpha R}{1+r} > \frac{P}{1+r} = \Pi_{i,\tau}^{b,G}. \end{aligned}$$

Hence, F strictly prefers renegotiation to break-up.

A.2 Proof of Lemma 2

In the proof of Lemma 1, $q_i^{ar} = q_i^*$, $\forall i$, regardless of whether F or G reneges on the contract.

A.3 Proof of Lemma 3

The per-period profit $\pi_{i,\tau}^{rn}$ that F would obtain under the new agreement following reneging in period τ is such that:

$$\Pi_{i,\tau}^{rn} = \pi_{i,\tau}^{rn}\gamma_{T_i-\tau}. \tag{41}$$

Recall from the proof of Lemma 1 that, when renegotiation follows reneging by F, the payoff of F is $\Pi_{i,\tau}^{rn} = \alpha R$. Using this in (41), the residual profit of F at any $\tau' > \tau$ is:

$$\widetilde{\Pi}_{i,\tau',\tau} = \pi_{i,\tau}^{rn} \gamma_{T_i - \tau'} = \Pi_{i,\tau}^{rn} \frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}} = \alpha R \frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}}$$

If F reneges again at τ', then it obtains:

$$\Pi_{i,\tau'}^{rn} = \Pi_{i,\tau}^{rn} = \alpha R > \alpha R \frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}} = \widetilde{\Pi}_{i,\tau',\tau}.$$

Therefore, the belief that F will not renege at τ' leads to F reneging again at τ'.

A.4 Proof of Lemma 4

Using (17), the following results are obtained proceeding by backward induction:

$$X_{i,T_i-1} = R$$

$$X_{i,T_i-2} = R - \frac{X_{i,T_i-1}}{1+r} = R - \frac{R}{1+r}$$

$$X_{i,T_i-3} = R - \frac{X_{i,T_i-2}}{1+r} - \frac{X_{i,T_i-1}}{(1+r)^2} = R - \frac{R}{1+r}$$

$$...$$

$$X_{i,T_i-\tau} = R - \frac{X_{i,T_i-\tau+1}}{1+r} - \frac{X_{i,T_i-\tau+2}}{(1+r)^2} - ... - \frac{X_{i,T_i-1}}{(1+r)^{\tau+1}} = R - \frac{R}{1+r}.$$

A.5 Proof of Lemma 5

The per-period benefit of G if the contract is renegotiated in period τ, is the value of $\upsilon_{i,\tau}$ satisfying the following:

$$V_{i,\tau}^{rn} = \upsilon_{i,\tau} \gamma_{T_i-\tau}.$$

The residual return of G at $\tau' > \tau$ is given by

$$\widetilde{V}_{i,\tau',\tau}^{rn} = \upsilon_{i,\tau} \gamma_{T_i-\tau'} = V_{i,\tau}^{rn} \frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}}.$$

Recall from the proof of Lemma 1 that, if G reneges and renegotiation follows, then the payoff of G at the beginning of period τ is $V_{i,\tau}^{rn} = w_i(q_i^*) \gamma_{T_i-\tau} - (P + \alpha R)/(1+r)$. Hence, $\widetilde{V}_{\tau',\tau}^{rn}$ is rewritten as follows:

$$\widetilde{V}_{\tau',\tau}^{rn} = w_i(q_i^*) \gamma_{T_i-\tau'} - \frac{P + \alpha R}{1+r} \frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}}.$$

If G reneges at τ', then it will have to again forego $(P+\alpha R)/(1+r)$, thus obtaining $V_{i,\tau'}^{rn}$ instead of $\widetilde{V}_{i,\tau',\tau}^{rn}$. Because

$$\begin{aligned}\widetilde{V}_{i,\tau',\tau}^{rn} &= w_i(q_i^*)\gamma_{T_i-\tau'}-\frac{P+\alpha R}{1+r}\frac{\gamma_{T_i-\tau'}}{\gamma_{T_i-\tau}} \\ &> w_i(q_i^*)\gamma_{T_i-\tau'}-\frac{P+\alpha R}{1+r}\gamma_{T_i-\tau'}=V_{i,\tau'}^{rn},\end{aligned}$$

G prefers $\widetilde{V}_{i,\tau',\tau}^{rn}$ to $V_{i,\tau'}^{rn}$.

A.6 Proof of Lemma 6

As $P+\alpha R\geq\alpha R$, F gains less if it reneges at the beginning of period T_i-1 following reneging by G at the end of period T_i-2.

As $(P+\alpha R)(1+r)>\alpha[R-R/(1+r)]$, F gains less if it reneges at the beginning of any $\tau\in\{1,...,T_i-2\}$ following reneging by G at the end of period $\tau-1$.

The two above inequalities further imply that, if G reneges at the end of any period $\tau\in\{0,...,T_i-1\}$, then it must concede a higher profit to F than if F reneges at the beginning of period τ.

B Enforcement under limited commitment

B.1 Proof of Lemma 7

(23) in state l is specified as follows:

$$T_l\leq T_l^{lc}(\Delta\Pi)\equiv\frac{\ln\frac{\alpha(1+r)R}{\alpha(1+r)R-r[I+\psi+(1-\nu_1)\Delta\Pi]}}{\ln(1+r)}.$$

$\exists T_l$ satisfying both $\underline{T}(\Delta\Pi)\leq T_l$ and $T_l\leq T_l^{lc}(\Delta\Pi)$ if and only if $\underline{T}(\Delta\Pi)\leq T_l^{lc}(\Delta\Pi)$ or, equivalently:

$$\alpha R\leq\frac{I+\psi+(1-\nu_1)\Delta\Pi}{\Delta\Pi}\frac{\Delta\theta q_l^*}{1+r}.$$

This condition is weakest when $\Delta\Pi=\psi/\Delta\nu$. Accordingly, it is rewritten as (26).

B.2 Proof of Proposition 1

We first derive conditions (27) and (28). We then prove that cheating off the equilibrium path (*i.e.*, misrepresentation of θ_i anticipating reneging in period τ) is not an issue.

B.2.1 Derivation of (27) and (28)

Take $T_l = T_h = T$. According to Proposition 1, $\Pi^*_{l,0} > \Pi^*_{h,0}$. Using (7), (23) is specified as $T \leq T^{lc}_h(\Delta\Pi)$. Using the definitions of $T^{lc}_h(\Delta\Pi)$ and $\underline{T}(\Delta\Pi)$, we verify that $T^{lc}_h(\Delta\Pi) \geq \underline{T}(\Delta\Pi)$ is equivalent to:

$$\alpha R \leq \frac{I + \psi - \nu_1 \Delta\Pi}{\Delta\Pi} \frac{\Delta\theta q^*_l}{1+r}. \tag{42}$$

This condition is weakest when $\Delta\Pi$ takes the lowest feasible value according to (8): $\Delta\Pi = \psi/\Delta\nu$. Using this value, (42) is rewritten as (27).

Suppose now that (27) is violated, which combined with the necessary condition (26) leads to (28). From (9) and $T \leq T^{lc}_h(\Delta\Pi)$, it follows that $T_h \leq T^{lc}_h(\Delta\Pi) < \underline{T}(\Delta\Pi) \leq T_l$.

B.2.2 Information release when reneging is anticipated

Let $\Pi^{RN}_{i,\tau}$ denote the payoff that F would obtain in state i, discounted at time τ, if it were to cheat at the outset of the operation phase and reneging were to occur at τ. We only consider the case in which F might renege at τ, provided the case of G reneging is analogous. Moreover, let $\pi^*_{i,x}$ be the per-period profit in state i in period $x \in \{0, ..., \tau\}$. F has no incentive to lie if and only if

$$\Pi^*_{l,0} \geq \sum_{x=0}^{\tau} \frac{\pi^*_{h,x} + \Delta\theta q^*_h}{(1+r)^{x+1}} + \Pi^{RN}_{l,\tau} \tag{43a}$$

$$\Pi^*_{h,0} \geq \sum_{x=0}^{\tau} \frac{\pi^*_{l,x} - \Delta\theta q^*_l}{(1+r)^{x+1}} + \Pi^{RN}_{h,\tau}. \tag{43b}$$

We hereafter show that (43a) is satisfied. If F reports h at date 0, in state l, and the contract is renegotiated at some $\tau \in \{0, ..., T_h - 1\}$, the profit of F through the residual periods $\{\tau, ..., T_h - 1\}$ is given by

$$\Pi^{RN}_{l,\tau} = \Pi^{rn}_{h,\tau} + \sum_{x=\tau}^{T_h-1} \frac{\Delta\theta q^*_h}{(1+r)^{T_h-x}}$$

where $\Pi^{rn}_{h,\tau}$ is the profit that a high-cost firm can extract in case of renegotiation. Hence:

$$\Pi^{RN}_{l,\tau} = \sum_{x=\tau+1}^{T_h} \frac{\pi^{RN}_{l,\tau}}{(1+r)^{T_h-x}} = \Pi^{rn}_{h,\tau} + \sum_{x=\tau+1}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^{T_h-x}}.$$

(43a) becomes:

$$\begin{aligned} \Pi^*_{l,0} \geq\ & \Pi^*_{h,0} + \sum_{x=1}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^x} \\ & + \frac{1}{(1+r)^\tau} \left[\Pi^{rn}_{h,\tau} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^{T_h-x}} - \left(\Pi^*_{h,\tau} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^{T_h-x}} \right) \right]. \end{aligned} \tag{44}$$

Recalling that (23) in state h is equivalent to $\Pi^*_{h,\tau} \geq \Pi^{rn}_{h,\tau}$, we see that

$$\Pi^*_{h,\tau} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^{T_h - x}} \geq \Pi^{rn}_{h,\tau} + \sum_{x=\tau}^{T_h} \frac{\Delta\theta q^*_h}{(1+r)^{T_h - x}}.$$

Hence, (43a) is implied by (5) and (23).

Similarly, one can prove that (43b) is implied by (4) and (23).

B.3 Proof of Corollary 2

For $j = h$ and $\Delta\Pi = \psi/\Delta\nu$, (12) is written as follows:

$$\Delta\theta z_h \gamma_{T_h} = \frac{\psi}{\Delta\nu},$$

which is reformulated as follows:

$$T_h = \frac{\ln \frac{\Delta\theta z_h}{\Delta\theta z_h - r\psi/\Delta\nu}}{\ln(1+r)}.$$

Hence, $T_h \leq T^{lc}_h(\psi/\Delta\nu)$ if and only if

$$z_h \geq \frac{\alpha R}{I - \nu_0 \psi/\Delta\nu} \frac{1+r}{\Delta\theta} \frac{\psi}{\Delta\nu}.$$

Moreover, $T^{lc}_h(\psi/\Delta\nu) < \underline{T}(\psi/\Delta\nu)$ if and only if

$$\frac{\alpha R}{I - \nu_0 \psi/\Delta\nu} \frac{\psi}{\Delta\nu} \frac{1+r}{\Delta\theta} > q^*_l.$$

The result follows.

B.4 Proof of Corollary 3

Recall the condition to the left in (28):

$$\alpha R > \left(I - \nu_0 \frac{\psi}{\Delta\nu}\right) \frac{\Delta\theta q^*_l/(1+r)}{\psi/\Delta\nu}.$$

This is tightest for $I = \overline{I}(P)$. We replace the expression of $\overline{I}(P)$ from (30) to write

$$\alpha R > \frac{P + \alpha R}{1 + r} \frac{\Delta \theta q_l^* / (1 + r)}{\psi / \Delta \nu} - \frac{\Delta \theta q_l^*}{1 + r}.$$

Rearranging yields (31).