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Nonlocal and Fractional Operators

 Springer

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Preface

The study of nonlocal operators is an active field of research in pure and applied mathematics and has been gaining an increasing attention over the last few years. Operators of nonlocal type are used to describe complex systems in which interactions among components are not local, but extend to a neighborhood of each component (space nonlocality). Analogously, they are applied in order to model systems in which the reaction to an external excitation is not instantaneous but depends on the history of the system (time nonlocality).

Due to the large extent of their applications, nonlocal operators are employed with great success and interest in a variety of fields ranging from biology to engineering, image processing, probability theory, physics and so on.

Fractional-order operators (i.e., integrals and derivatives of non-integer order) are maybe the most famous and studied in the literature. Their origin goes back to the end of seventeenth century, but their analysis and applications have flourished only around the middle of the twentieth century.

In this book we have collected a number of invited and refereed contributions illustrating recent developments in theory and applications of Nonlocal and Fractional Operators. The chapters of this book cover different research areas, thus offering an overview of the most updated results and applications of Nonlocal and Fractional Operators.

Most of the contributions are related to talks presented during the Workshop “Nonlocal and Fractional Operators” held at La Sapienza University in Roma on April 12–13, 2019. This meeting was an occasion to bring together researchers working in different areas of mathematics and physics, and to discuss the most recent advancements and applications of Nonlocal and Fractional Operators.

The workshop “Nonlocal and Fractional Operators” was dedicated to Professor Renato Spigler (Department of Mathematics and Physics, Roma Tre University), on the occasion of his retirement, and was an opportunity to celebrate his scientific contributions in the field of applied mathematics and, in particular, of fractional calculus. A transcription of the speech delivered by Professor Michele Caputo and dedicated to the academic and research achievements of Professor Spigler is included, as an introduction to this book.

We wish to forward our special thanks to all authors and coauthors who have contributed, with their articles, to the realization of this volume and to all the anonymous referees, who allowed to select only valuable contributions, as well as to improve their quality with useful and constructive criticisms. A final special thank to the scientific and the organizing committees of the workshop “Nonlocal and Fractional Operators”, which has prompted the realization of this book.

Finally, we are grateful to SIMAI, the Italian Society for Industrial and Applied Mathematics, for hosting this volume in the SIMAI-SEMA series published by Springer.

Rome, Italy
Bari, Italy
Bologna, Italy

Luisa Beghin
Roberto Garrappa
Francesco Mainardi

Presentation of the Workshop “Nonlocal and Fractional Operators” Dedicated to Prof. Renato Spigler (Rome, April 12–13, 2019)

Nonlocal and Fractional Operators
In honour of Prof. Renato Spigler

April 12-13, 2019
Department of Statistical Sciences
Sapienza University of Rome

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Introductory speech by Prof. Michele Caputo (Accademia Nazionale dei Lincei)

I first want to congratulate the organizers of the meeting for celebrating Professor Renato Spigler and also for the selection of the title “Nonlocal and Fractional Operators” which has attracted many excellent mathematicians.

From the program of the workshop, we expect the presentation of many very interesting papers covering different branches of fractional calculus which also show the vitality of nonlocal operators in many fields of mathematics. Indirectly and consequently, they indicate the expansion of the applications of this branch of mathematics in an ever increasing number of different fields of science. I have seen the list of posters which, as sometime happens, seem not less interesting than the papers. Finally I like to thank the organizing committee to have given me the pleasure to open the works of the meeting and to celebrate Professor Renato Spigler.



Professor Spigler was born in Venice in 1947, so say the papers, but certainly his look does not qualify for retiring. The same seems true also when looking at the increasing rate of his current scientific production. He had his first laurea in electronic engineering at the University of Padua in 1972. He specialized in Theory and Applications of Computing Machines at the University of Bologna then he and returned to Padua where he was offered teaching positions but soon he began stages in the US first with the University of Wisconsin in 1980, later at the Courant Institute in New York as Fulbright scholar in 1983 and, after few stages at the same institute, he was there as Associate Research Scientist in 1986.

After these important experiences abroad, at a young age, he became stable in Italy where he had the chair of *Analisi Matematica* at the University of Roma 3 and at the *Università Telematica Internazionale Uninettuno*. That is not only for the registrar's office. Because his stages in different important institutions, particularly those abroad, denote a dynamic style of life, which is reflected in his varied scientific production and collaborations with national and international Agencies. In fact professor Spigler's scientific production, besides the excellent quality, is impressive for the variety of problems treated and for his capability to give essential contributions in problems on the frontier of science.

He proved to be able also to produce first class mathematics in association with eminent colleagues, for instance, in the case of solution of hybrid problems. From existence and uniqueness of classical solutions of certain nonlinear integro-differential Fokker-Planck-type equations, professor Spigler goes to the probabilistically induced domain decomposition methods for elliptic boundary-value problems.

Concerning his recent scientific production and the variety of problems which have been attacked by him, it is worth mentioning that, in 2001, he showed the existence and uniqueness of solutions to the Kuramoto-Sakaguchi parabolic integro-differential equation. The synchronization phenomena in large populations of interacting elements are subject to intense research efforts in biological, chemical also for the study of the evolutions of different competing economies in clubs of economies, in particular of banks and also of social systems. Spigler gave, in 2005, a fundamental

contribution with a successful approach, consisting in modeling each member of the population as coupled phase oscillators.

Renato's papers generally received a large number of citations but this paper had the peak number of 2250 citations. Then in 2007 he finds L1-estimates for the higher order derivatives of solutions to parabolic equations subject to initial values of bounded total variation. In 2012 he lands on fractional calculus with a paper where he applies fractional operators for a numerical solution of two-dimensional fractional diffusion equations, by a high-order ADI method (Alternating Direction Implicit). In 2014, he studies existence, uniqueness and regularity for the Kuramoto-Sakaguchi equation with unboundedly supported frequency distribution (which later led to the Kuramoto-Sivashinsky equation) by introducing also nonlocal operators. More recently, in 2016, again in a different field, he introduces an approximation method by means of neural network operators. Finally, I like to mention that Renato ventured to land also on Earth with the most important problem of our environment, an excellent paper on mathematical models for fighting environmental pollution.

Professor Spigler had also important collaborations with NATO, CNR, EURATOM, UNESCO. He is a Member of the Editorial Board of many international scientific Journal, as well as many important scientific Societies. What at first sight appears remarkable in his splendid scientific carrier and production is the variety of different fields where he operated, not only in different topics, but also conceptually and using the modern fundamental tools of mathematics. He understood the importance of interdisciplinarity and acted at high level, contributing constructively with several eminent scientists in vanguard problems such as those concerning the synchronization in clubs of entities of different kinds, basic in the structure of our society.

As a person, I like his successful sailor behavior, in different seas at times stormy, from an important successful harbor to the next, not the nearest.

Dear Renato, thanks for being with us and congratulations for what you are and have done for all of us and, in all ways, please stay the course: *continua così*.

Rome, Italy
April 2019

Michele Caputo

Contents

On the Transient Behaviour of Fractional $M/M/\infty$ Queues	1
Giacomo Ascione, Nikolai Leonenko, and Enrica Pirozzi	
Sinc Methods for Lévy–Schrödinger Equations	23
Gerd Baumann	
Stochastic Properties of Colliding Hard Spheres in a Non-equilibrium Thermal Bath	57
Armando Bazzani, Silvia Vitali, Carlo E. Montanari, Matteo Monti, Sandro Rambaldi, and Gastone Castellani	
Electromagnetic Waves in Non-local Dielectric Media: Derivation of a Fractional Differential Equation Describing the Wave Dynamics	71
Alessandro Cardinali	
Some New Exact Results for Non-linear Space-Fractional Diffusivity Equations	83
Arrigo Caserta, Roberto Garra, and Ettore Salusti	
A Note on Hermite-Bernoulli Polynomials	101
Clemente Cesarano and Alexandra Parmentier	
A Fractional Hawkes Process	121
J. Chen, A. G. Hawkes, and E. Scalas	
Fractional Diffusive Waves in the Cauchy and Signalling Problems	133
Armando Consiglio and Francesco Mainardi	
Some Extension Results for Nonlocal Operators and Applications	155
Fausto Ferrari	
The Pearcey Equation: From the Salpeter Relativistic Equation to Quasiparticles	189
A. Lattanzi	

Recent Developments on Fractional Point Processes 205
Aditya Maheshwari and Reetendra Singh

**Some Results on Generalized Accelerated Motions Driven
by the Telegraph Process** 223
Alessandra Meoli

**The PDD Method for Solving Linear, Nonlinear, and Fractional
PDEs Problems** 239
Ángel Rodríguez-Rozas, Juan A. Acebrón, and Renato Spigler

**Fractional Diffusion and Medium Heterogeneity: The Case
of the Continuous Time Random Walk** 275
Vittoria Sposini, Silvia Vitali, Paolo Paradisi, and Gianni Pagnini

**On Time Fractional Derivatives in Fractional Sobolev Spaces
and Applications to Fractional Ordinary Differential Equations** 287
Masahiro Yamamoto