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NORMALIZED SOLUTIONS FOR AN HORIZONTAL TRANSMISSION PROBLEM

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ABSTRACT. Let Ω be a bounded and smooth domain in $\mathbb{R}^N, N \geq 3$. We study in this paper the following nonlinear transmission problem

$$\begin{cases} -\Delta u + \lambda u = |u|^{p-2}u & \text{in } \Omega, \\ -\Delta v + \mu v = a(x)|v|^{q-2}v & \text{in } \mathbb{R}^N \setminus \Omega, \\ u = v \ \text{ and } \ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega, \end{cases}$$

where the unknowns are $u : \Omega \to \mathbb{R}, v : \mathbb{R}^N \setminus \Omega \to \mathbb{R}$ and the real numbers λ, μ. Actually we are looking for solutions u, v with prescribed L^2-norm. The problem has a variational formulation and indeed, under minimal assumptions, we are able to find infinitely many solutions by using the Krasnoselkii genus, whatever the L^2-norm of u and v *a priori* is.

1. Introduction

This paper is concerned with the following transmission problem

$$\begin{cases} -\Delta u + \lambda u = |u|^{p-2}u & \text{in } \Omega, \\ -\Delta v + \mu v = a(x)|v|^{q-2}v & \text{in } \mathbb{R}^N \setminus \Omega, \\ u = v \ \text{ and } \ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega, \\ \displaystyle\int_\Omega u^2 = \rho_1 > 0, \quad \int_{\mathbb{R}^N \setminus \Omega} v^2 = \rho_2 > 0, \end{cases} \tag{1.1}$$

where Ω is a smooth and bounded domain in $\mathbb{R}^N, N \geq 3$ and where we are denoting with $\mathbf{n}$ the outward unit normal vector to the domain Ω. In particular since we are interested in solutions $u \in H^1(\Omega), v \in H^1(\mathbb{R}^N \setminus \Omega)$ with prescribed L^2-norm, the unknowns of the problem are, beside u and v, the real numbers λ, μ. The function $a : \mathbb{R}^N \setminus \Omega \to \mathbb{R}$ has to be considered assigned and will satisfy very mild assumptions listed below. The numbers ρ_1, ρ_2 are arbitrary, but fixed.

These kind of problems appear in physical phenomena such as inverse scattering theory in inhomogeneous media (for example with discontinuous coefficients) or in electromagnetic waves propagation in fibres. In such a cases, two quantities live in different regions but on the common boundary they have to have a sort of continuity. For example, mathematically a fiber is a domain of type $\Omega \times \mathbb{R} \subset \mathbb{R}^3$, Ω bounded in $\mathbb{R}^2$, with a "core" $\Omega_1 \times \mathbb{R}$, $\Omega_1 \subset \Omega$. In many physical situations the dielectric constants of the fiber are different in and outside the core, and moreover do not depend on the $z-$variable (that is, they depend only on the

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cross section of the fiber). In this sense a discontinuity is created along the boundary $\partial\Omega \times \mathbb{R}$, which is translated into the mathematical requirement that the normal derivatives of the wave functions coincide on $\partial\Omega$. For more information on such physical models, see e.g. [11] and the more recent papers [2–5, 7–10]. We have to say that mathematically this kind of problems were already addressed in the fifties by Picone, Lions, Stampacchia, Campanato, see e.g. [14] and the references therein.

However, to the best of our knowledge, this is the first paper where normalised solutions are found for such a nonlinear transmission problem, which are important also from a physical point of view due to the interpretation of the unknowns u, v as the modulus of the wave function of the electromagnetic propagation and λ, μ as the related frequencies.

Then our aim is to investigate for the transmission problem the existence of normalised solutions and indeed we are able to give a positive answer.

We point out that recently we have studied other kinds of problems and we looked for solutions have prescribed norm, let us say in L^2 which usually has a physical meaning. See e.g. [1, 12, 13].

Beside the fact that we are looking for normalized solutions, the function v is defined in the unbounded domain $\mathbb{R}^N \setminus \Omega$ and this adds a further difficulty to our problem. To this respect, we emphasise that we are not in the radial setting, so that the compactness is a quite hard issue to manage. With this in mind, the assumptions on the function a have to be seen as a way to overcome the lack of compactness.

More specifically our few assumptions on the problem are:

(a0) $p \in (2, 2 + 4/N), q \in (2, 2^*)$,

(a1) $a \in C(\mathbb{R}^N \setminus \Omega) \cap L^\infty(\mathbb{R}^N \setminus \Omega)$ and $a < 0$ in $\mathbb{R}^N \setminus \Omega$,

(a2) there exists $R > 0$ such that $\sup_{|x|\geq R} |a(x)||x|^2 < \infty$,

Hereafter, for $N \geq 3$ we set $2^* = 2N/(N-2)$. The hypothesis on the function a appeared for the first time in [15] and are used to recover some compactness concerning integrals on the unbounded domain $\mathbb{R}^N \setminus \Omega$.

Our main result is the following.

Theorem 1. *For any $\rho_1, \rho_2 > 0$ there exist infinitely many solutions*

$$u_n \in H^1(\Omega), \quad v_n \in H^1(\mathbb{R}^N \setminus \Omega), \quad \lambda_n, \mu_n \in \mathbb{R}$$

of problem (1.1).

In order to prove our result we use variational methods. In particular an energy functional I can be defined in such a way that the solutions of (1.1) are its critical points restricted to a suitable manifold M. However to use classical arguments in critical point theory we need to face with the compactness issue. To this aim we recall once for all a classical definition. We say that I satisfies the Palais-Smale condition ((PS) for sort) on M if any sequence $\{w_n\} \subset M$ such that

$$I(w_n) \to c \in \mathbb{R}, \qquad I'_{|M}(w_n) \to 0$$

admits a convergent subsequence in M.

The (PS) condition is proved in the Proposition 4. The major difference with respect to classical papers, is that we need to face with two Lagrange multipliers since, as we will see, the manifold M will have codimension two.

We recall also the notion of Krasnoselki genus: given a closed and symmetric set C of a topological space, the genus of C, denoted by $\gamma(C)$, is defined as the smallest integer $k \in \mathbb{N}$

for which there exists an odd and continuous map $h : C \to \mathbb{R}^k \setminus \{0\}$. If there is no finite such k we set $\gamma(C) = \infty$ and, finally, $\gamma(\emptyset) = 0$.

The paper is organised as follows.

In Section 2 we give the variational framework for the problem and some preliminaries. The main result is proved in Section 3.

1.1. **Notations.** We introduce here few basic notations:

- $c_1, c_2, C, C' \dots$ denote suitable positive constants,
- B_R is the ball in $\mathbb{R}^N$ centered in the origin with radius R,
- $o_n(1), o(R)$ denote generic quantities which tend to zero as $n, R \to +\infty$,
- finally we set,

$$|a|_\infty = \sup_{x\in\mathbb{R}^N\setminus\Omega} |a(x)|.$$

2. Preliminary Results

We fix from now the two constants $\rho_1, \rho_2 > 0$ and in all the paper assumptions (a0)-(a2) are tacitly assumed. Let us define

$$E = \left\{(u, v) \in H^1(\Omega) \times H^1(\mathbb{R}^N \setminus \Omega) : u = v \text{ on } \partial\Omega \text{ in the sense of trace}\right\}$$

which is an Hilbert space with the natural (squared) norm

$$\|(u, v)\|^2 = \int_\Omega |\nabla u|^2 + \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 + \int_{\mathbb{R}^N\setminus\Omega} v^2.$$

Observe that this is indeed a norm (not just a seminorm) due to the transmission condition $u = v$ on $\partial\Omega$. Moreover it is easy to see as in [7, Lemma 1] and using the Trace Theorem (see e.g. [6, Theorem 15.23]) that it is equivalent to the usual norm in $H^1(\Omega) \times H^1(\mathbb{R}^N \setminus \Omega)$.

Let

$$M = \left\{(u, v) \in E : \int_\Omega u^2 = \rho_1 \quad \int_{\mathbb{R}^N\setminus\Omega} v^2 = \rho_2\right\}.$$

It is known that M is a smooth manifold of codimension two in E with tangent space, in a point $(u, v) \in M$, given by

$$\mathrm{T}_{(u,v)}M = \left\{(w, z) \in E : \int_\Omega uw = \int_{\mathbb{R}^N\setminus\Omega} vz = 0\right\}.$$

It is not difficult to show that the weak solutions of problem (1.1) are characterised as critical points of the C^1 functional on E given by

$$I(u, v) = \frac{1}{2}\int_\Omega |\nabla u|^2 + \frac{1}{2}\int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 - \frac{1}{p}\int_\Omega |u|^p - \frac{1}{q}\int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q$$

restricted to the manifold M. In fact λ, μ appear as Lagrange multipliers. This can be seen like in [11].

It will be useful the following result which is a special case of [13, Lemma 3.1].

Lemma 1. *Given a smooth domain $D \subset \mathbb{R}^N, N \geq 3$ and numbers*

$$2 < p < 2^*, \quad 0 < r \leq N\left(1 - \frac{p}{2^*}\right),$$

there exists $C > 0$ such that, for any $u \in H^1(D)$

$$\int_D |u|^p \leq C\left(\int_D |\nabla u|^2 + \int_D u^2\right)^{(p-r)/2}\left(\int_D u^2\right)^{r/2}.$$

Then we have the basic result.

Lemma 2. *The functional I is bounded from below and coercive on M. Moreover if $\{(u_n, v_n)\} \subset M$ is such that $\{I(u_n, v_n)\}$ is bounded, then the sequences*

$$\left\{\int_\Omega |\nabla u_n|^2\right\}, \quad \left\{\int_\Omega |u_n|^p\right\}, \quad \left\{\int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2\right\} \quad \textit{and} \quad \left\{\int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^q\right\}$$

are all bounded.

Proof. Since by assumption $2 < p < 2 + 4/N$, it is easy to see that

$$0 < p - 2 < N\left(1 - \frac{p}{2^*}\right)$$

and then it is possible to choose a real r such that

$$0 < p - 2 < r < N\left(1 - \frac{p}{2^*}\right). \tag{2.1}$$

Hence applying Lemma 1 with $\int_\Omega u^2 = \rho_1$ and an elementary inequality, we have

$$\begin{aligned}\int_\Omega |u|^p &\leq C\left(\int_\Omega |\nabla u|^2 + \rho_1\right)^{(p-r)/2}\rho_1^{r/2}\\ &\leq c_1\left(\left(\int_\Omega |\nabla u|^2\right)^{(p-r)/2} + \rho_1^{(p-r)/2}\right)\rho_1^{r/2}\end{aligned}$$

so that

$$I(u,v) \geq \frac{1}{2}\int_\Omega |\nabla u|^2 - c_2\left(\int_\Omega |\nabla u|^2\right)^{(p-r)/2} - c_3 + \frac{1}{2}\int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 - \int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q$$

from which, using that $1 > (p-r)/2$ and that $a < 0$, the conclusions hold. □

In particular

$$m = \inf_M I > -\infty$$

is well defined and any minimising sequence is bounded in E. The next result will be important in order to obtain the compactness in $H^1(\mathbb{R}^N \setminus \Omega)$. It strongly uses assumption (a2).

Lemma 3. *Assume that $v_n \rightharpoonup v$ in $H^1(\mathbb{R}^N \setminus \Omega)$. Then*

$$\begin{aligned}\int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^q &\to \int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q,\\ \int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^{q-2}v_n v &\to \int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q.\end{aligned}$$

Proof. We prove here just the first convergence since the second one is very similar.

By assumption there is a bound $C > 0$ on the norm of $\{v_n\}$ and v in $H^1(\mathbb{R}^N \setminus \Omega)$. Now given $\varepsilon > 0$ there is an $R > 0$ sufficiently large such that, for $r = 2^*/(2^* - q)$, we have (note that $r > N/2$ and $r' = 2^*/q$)

$$\begin{aligned}\int_{\mathbb{R}^N\setminus B_R} \frac{|v_n|^q}{|x|^2} &\leq \Big(\int_{\mathbb{R}^N\setminus B_R} \frac{1}{|x|^{2r}}\Big)^{1/r} \Big(\int_{\mathbb{R}^N\setminus B_R} |v_n|^{2^*}\Big)^{q/2^*} \\ &\leq \varepsilon \left(\int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 + \int_{\mathbb{R}^N\setminus\Omega} v_n^2\right)^{q/2} \\ &\leq C\varepsilon.\end{aligned}$$

Analogously one shows that

$$\int_{\mathbb{R}^N\setminus B_R} \frac{|v|^q}{|x|^2} \leq C\varepsilon.$$

Then for $R > 0$ sufficiently large we have the estimate

$$\begin{aligned}\int_{\mathbb{R}^N\setminus\Omega} |a(x)|\Big||v_n|^q - |v|^q\Big| &\leq |a|_\infty \int_{(\mathbb{R}^N\setminus\Omega)\cap B_R} \Big||v_n|^q - |v|^q\Big| + \int_{\mathbb{R}^N\setminus B_R} |a(x)|\Big||v_n|^q - |v|^q\Big| \\ &= o_n(1) + \int_{\mathbb{R}^N\setminus B_R} |a(x)|\Big||v_n|^q - |v|^q\Big| \\ &\leq o_n(1) + \sup_{x\in\mathbb{R}^N\setminus B_R} |a(x)||x|^2 \Big(\int_{\mathbb{R}^N\setminus B_R} \frac{|v_n|^q}{|x|^2} + \int_{\mathbb{R}^N\setminus B_R} \frac{|v|^q}{|x|^2}\Big) \\ &= o_n(1) + C'\varepsilon,\end{aligned}$$

where we used the compact embedding of $H^1\big((\mathbb{R}^N \setminus \Omega) \cap B_R\big)$ into $L^q\big((\mathbb{R}^N \setminus \Omega) \cap B_R\big)$. The conclusion then follows since ε is arbitrarily small. □

3. Proof of the Main Result

We prove here the main theorem, by implementing the Ljusternick-Schnirelmann Theory. We already know that the functional I is bounded below on M

Let us show the compactness condition for the functional I on the manifold M.

Proposition 4. *The functional I satisfies the (PS) condition on M.*

Proof. Let $\{(u_n, v_n)\} \subset M$ and $\{(\lambda_n, \mu_n)\} \subset \mathbb{R}^2$ be such that

$$I(u_n, v_n) \to c, \quad I'(u_n, v_n) - \lambda_n u_n - \mu_n v_n = o(n) \quad \textit{in } E^{-1},$$

being E^{-1} the dual of E. Due to the homogeneous Neumann condition, we have

$$I'(u_n, v_n)[u_n, 0] = \int_\Omega |\nabla u_n|^2 - \int_\Omega |u_n|^p - \lambda_n \rho_1 = o(n)$$

and by Lemma 2 the boundedness of $\{\lambda_n\}$ holds.

Similarly from

$$I'(u_n, v_n)[0, v_n] = \int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 - \int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^q - \mu_n \rho_2 = o(n)$$

we get the boundedness of $\{\mu_n\}$. In particular we may assume that

$$\lambda_n \to \lambda, \ \mu_n \to \mu, \quad u_n \rightharpoonup u \text{ in } H^1(\Omega) \ \text{ and } \ v_n \rightharpoonup v \text{ in } H^1(\mathbb{R}^N \setminus \Omega).$$

Let us show the strong convergence of $\{u_n\}$ and $\{v_n\}$.

As in [11] it is easy to see that (u,v) is a weak solution of the problem. Clearly since Ω is bounded we have

$$u_n \to u \quad \text{in } L^\alpha(\Omega), \quad 1 \le \alpha < 2^* \tag{3.1}$$

so that $\int_\Omega u^2 = \rho_1$. But

$$\begin{aligned} o(n) &= I'(u_n,v_n)[u_n,0] - I'(u_n,v_n)[u,0] \\ &= \int_\Omega |\nabla u_n|^2 - \int_\Omega \nabla u_n \nabla u - \int_\Omega |u_n|^p - \int_\Omega |u_n|^{p-2} u_n u - \lambda_n \rho_1 + \lambda_n \int_\Omega u_n u \\ &= \int_\Omega |\nabla u_n|^2 - \int_\Omega |\nabla u|^2 + o(n) \end{aligned}$$

which, joint with (3.1) gives

$$u_n \to u \quad \text{in } H^1(\Omega). \tag{3.2}$$

Being $u \not\equiv 0$ the boundary conditions give $v \not\equiv 0$. Hence by expliciting $I'(u,v)[0,v] = 0$ we have

$$0 \not\equiv \|v\|^2 = \mu\rho_2 + \int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q$$

which implies, being $a(x) < 0$, that $\mu > 0$.

Now

$$\begin{aligned} o(n) &= I'(u_n,v_n)[0,v_n] \\ &= \int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 - \int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^q + \mu_n\rho_2 \end{aligned} \tag{3.3}$$

and, by using Lemma 3

$$\begin{aligned} o(n) &= I'(u_n,v_n)[0,v] \\ &= \int_{\mathbb{R}^N\setminus\Omega} \nabla v \nabla v_n - \int_{\mathbb{R}^N\setminus\Omega} a(x)|v_n|^{q-2} v_n v + \mu_n \int_{\mathbb{R}^N\setminus\Omega} v_n v \\ &= \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 - \int_{\mathbb{R}^N\setminus\Omega} a(x)|v|^q + \mu_n \int_{\mathbb{R}^N\setminus\Omega} v^2. \end{aligned} \tag{3.4}$$

By (3.3), (3.4) and Lemma 3 we deduce

$$\int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 + \mu_n\rho_2 \longrightarrow \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 + \mu \int_{\mathbb{R}^N\setminus\Omega} v^2. \tag{3.5}$$

Since $\mu_n \to \mu > 0$ in virtue of (3.5) and using that $\int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 \le \liminf \int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2$ we have

$$\begin{aligned} \Big|\mu_n\rho_2 - \mu\int_{\mathbb{R}^N\setminus\Omega} v^2\Big| &= \Big|\mu_n\rho_2 - \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 + \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 - \mu\int_{\mathbb{R}^N\setminus\Omega} v^2\Big| \\ &\le \Big|\mu_n\rho_2 - \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2 + \int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 - \mu\int_{\mathbb{R}^N\setminus\Omega} v^2\Big| \\ &\longrightarrow 0 \quad \text{by (3.5)}. \end{aligned}$$

But then,

$$\int_{\mathbb{R}^N\setminus\Omega} v^2 = \rho_2$$

showing that we have the strong convergence of $\{v_n\}$ in $L^2(\mathbb{R}^N \setminus \Omega)$, and coming back to (3.5) the convergence

$$\int_{\mathbb{R}^N\setminus\Omega} |\nabla v_n|^2 \to \int_{\mathbb{R}^N\setminus\Omega} |\nabla v|^2.$$

holds. Summing up we have also

$$v_n \to v \quad \text{in} \quad H^1(\mathbb{R}^N \setminus \Omega), \tag{3.6}$$

so that (3.2) and (3.6) conclude the proof. □

As a consequence of the previous results, the functional I achieves its minimum on M.

The next result is fundamental in order to apply the Ljusternick-Schnirelmann theory and deduce the multiplicity result.

Proposition 5. *It holds $\gamma(M) = +\infty$.*

Proof. Indeed, setting

$$S_{\rho_1} = \left\{ u \in H^1(\Omega) : \int_\Omega u^2 = \rho_1 \right\}$$

and fixed $v \in H^1(\mathbb{R}^N \setminus \Omega)$, it is

$$\gamma(S_{\rho_1} \times \{v, -v\}) = +\infty$$

and we conclude. □

Remark 6. *Actually we have that for every $k \in \mathbb{N}$ there exists a compact set in M with genus equals to k.*

Summing up, we have an even functional bounded below and satisfying the Palais-Smile condition on an even manifold with infinite genus. Classical abstract critical point theorem guarantees that the functional has infinitely many (pairs of) critical points on M with increasing energy. The proof of Theorem 1 is concluded.

As a final remark, since the functional has diverging critical values, the solutions u_n and v_n cannot be both bounded in norm.

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