

**Guest Editorial for
Special Issue
“Digital Experiences in University Mathematics Education:
Advances and Expectations”**

Guest Editors

Eirini Geraniou (e.geraniou@ucl.ac.uk)

Eleonora Faggiano (eleonora.faggiano@uniba.it)

Janka Medová (jmedova@ukf.sk)

Melih Turgut (melih.turgut@ntnu.no)

The use of digital technologies (DT) in mathematics education has certainly evolved over the years (e.g., Drijvers & Sinclair, 2024; Monaghan et al., 2016), of course due to the advances in digital technology and user expertise and confidence in using them, but also due to other factors, which we may not have been prepared for or even predicted. One such factor was the COVID-19 pandemic. It brought various challenges to mathematics education and teacher practitioners (e.g., Cusi et al., 2023; Chan et al., 2021), but also encouraged (or even forced) an evolution in our mathematics teaching and learning practices (e.g., Bakker et al., 2021; Borba, 2021; Atweh et al., 2023). Mathematics educators had to adapt their practices and gradually became experts in the use of various DT to support the doing, learning and teaching of mathematics remotely (e.g., Donevska-Todorova et al., 2022). Given these developments, researchers have begun to focus on specific aspects of this digital evolution. For instance, Kinnear et al. (2024) have described a collaborative effort to develop an agenda for research on e-assessment in undergraduate mathematics. This agenda provides a framework for a research program, addressing practical concerns and themes (errors, feedback, student interaction with e-assessment, affordances and mathematical skills, etc.). Moreover, Sümmermann et al. (2021) argue that the digital transformation of resources enabled the implementation of computational environments like GeoGebra and Jupyter Notebook(s) could open a door to new possibilities for constructing and experiencing proofs. Regarding the role of undergraduate mathematics courses in engineering education, Pepin et al. (2021) have highlighted the need for more research to explore how engineering students and instructors utilise relevant (digital) resources. Recently, Winslow et al. (2024) argued that there are numerous studies that have examined small-scale experiments involving the integration of pre-existing technology and programming into university mathematics teaching, often drawing on

researchers' personal experiences or discoveries with these tools in their research. Most of these studies focused on ready-made tools and lately on online platforms for teaching and learning mathematics. Regarding the impact of the pandemic in particular, the authors claimed that research on technology, digital resources and media has enabled or even enforced changes (and not just offered an enhancement) in the ways we teach and learn mathematics, as well as how we do mathematics. The authors concluded that the research field regarding digital resources in UME is certainly growing and encouraged for more such contributions, also involving innovative practices with digital resources in UME.

Reflecting upon the impact of the pandemic and recent developments in mathematics education and in fact the inevitable change in everyone's beliefs in, but also usage of DT in mathematics education, this Special Issue called for papers focusing on university mathematics education (UME) in particular. In fact, we believe that this Special Issue contributes to the progress in the so-far small field of research on digital resources on UME (a claim supported by Winslow et al.'s (2024) work, as mentioned earlier).

This Special Issue (SI) aims at investigating questions concerning the design, teaching, learning and assessment with the use of DT within UME, and whether, and how, these aspects have met the expectations of the mathematics education research community before, during and after the pandemic. Originally, the call for papers for this Special Issue welcomed contributions that focused on any of the three following themes: **(1) Instructional design, curriculum and institutional priorities regarding digital technologies in UME.** In more detail, (i) Designs of digital (curriculum) resources supporting the development of individual learning trajectories and collaboration; (ii) Functionalities and roles of Virtual Learning Environments, such as Moodle, in the instructional design and communication, (iii) Machine Learning, Learning Analytics, Artificial Intelligence for instructional design and assessment; and (iv) Computational Thinking and Programming instructional design and policies; **(2) Teaching and learning practices with digital technologies in UME.** In more detail, (i) University teachers and students as (co-)designers of digital technologies for teaching, learning and doing mathematics; (ii) Students' experiences, ways of thinking (algebraic, geometric, computational, etc.) and development of competencies (reasoning, proving, modelling, problem solving, etc.) as affected by the use of digital technologies; and (iii) Students', teachers' and multipliers' experiences with emerging digital technologies such as mobile apps, theorem provers, digital concept maps; **(3) Teacher and student practices with regard to algorithmics, computational thinking and programming / rethinking (digital) assessment in UME.** In more detail, (i) Adaptive feedback in learning and formative and summative assessment; and (ii) Affordances and drawbacks of automated assessment instruments and issues of transfer from pen and paper to automation of test items for digital exams.

We received a high number of proposals and accepted the seven papers collected in this Special Issue. All three themes are evident in these seven papers. The papers present studies carried out in various countries (Germany, Israel, Italy, Sweden, UK, USA), involving the use of a variety of DT, such as digital whiteboards, Miro, googledocs, Computer-Aided-Assessment tools -STACK and Möbius Assessment, Scratch, CalcPlot3D and 3D-printed surfaces and

solids, online videos and Zoom, video games and GeoGebra. In what follows, we summarise the contributions from each and showcase the topics from each theme covered in each paper. Most of the papers address in fact several themes. We have chosen to classify each paper in a single theme, but we also mention the other theme(s) this paper addresses. We conclude by referring to the few topics that are not covered in this Special Issue and therefore call for further work and research to be carried out.

Dilling et al.'s work comes under theme 1 as they argue that investigating teachers' beliefs can offer great insight and explanations for teachers' certain behaviours in the mathematics classroom, which concerns an institutional priority. There has been research on teachers' beliefs about the use of DT in general, but the authors claim that none of the existing research focuses on specific DT. So, they are contributing to filling this gap by focusing on beliefs about visual programming and in particular the use of Scratch in university seminars in Germany, and how such beliefs compare to general beliefs about the use of DT in mathematics education. This foci leads us to classify this paper as one that touches upon theme 2 as well. Qualitative content analysis of seven students' reflection journals identified ten belief categories about visual coding in mathematics. Most beliefs were positive towards visual coding in mathematics education. Some beliefs noted the limits and challenges of visual coding, showing negative associations. Few belief categories matched those identified in previous research on DT in mathematics education. How these beliefs differ from beliefs of experienced in-service teachers remains an open question.

Fahlgren et al. focus on the development of Computer-Aided Assessment (CAA) tasks that target higher-order mathematical skills, particularly through an example-generation task on polynomial function understanding. This study, which concerns both themes 1 and 3, was conducted among first-year engineering students in Sweden and first-year biotechnology students in Italy. This paper's contribution lies in the proposition of a novel approach for CAA and the adoption of an example generation space lens. It explores the effectiveness of the task in extending students' example space in diverse educational contexts. The findings indicate a difference in students' example spaces when performing the task between the two educational settings. Overall, the paper provides insights into how DT can be used to enhance learning and assessment, especially in the context of higher-order mathematical skills. Reflecting on their findings, the authors suggest future work may involve a number of improvements to the task, such as asking for more different examples in a chain of prompts with additional constraints, utilising the algorithmic capabilities of the CAA system. Further research could also focus on students' progressive mathematization in generating example spaces within a qualitative paradigm.

The paper by **Mauntel and Zandieh**, which comes under theme 2, explores how undergraduate students in the Southeast US reason about linear combinations in university-level linear algebra using a combination of GeoGebra and a video game, Vector Unknown, that presents students with symbolic and geometric representations of linear combinations. The authors investigated how these digital resources influence students' thinking and development of mathematical concepts and identified four distinct forms of reasoning, which they called "structuring space" and are influenced by the specific representations used in the video game and GeoGebra. Overall, the paper highlights the significant impact of digital resources on student learning and

the connections between digital resources, student reasoning, and concept development in linear algebra. It contributes to the field of teaching and learning practices with digital resources in UME by demonstrating how video games and dynamic geometry software can influence student understanding of linear algebra concepts as well as provide insights for educators and instructional designers to create learning environments that leverage the affordances of different digital representations. Future research could delve deeper into the connection between the identified reasoning forms and the student success in solving linear algebra problems.

Due to COVID-19 and the shift towards online education, **Wallach and Kontorovich** focused on learner-centred practices at a large university in Israel, in particular online synchronous mathematical discussions, and investigated student-instructor and student-student interactions, a focus which placed their work under theme 2. They looked into the dynamics of online synchronous teaching and learning using a well-known communication platform, focusing on Linear Algebra tutorials during the first year of the pandemic. Employing the commognitive framework, they explored instructional exchanges based on a learning-teaching agreement. Their analysis identified three distinct interactional patterns in the tutorials, referred to as “lecture-ish, instructor-oriented, and cross-student”. The distinction between them was achieved based on the discourse leadership, the duties students were offered and the nature of the expected change involved. The authors claimed that technological affordances inclined the instructor towards taking upon specific roles, while steering the students away from those roles. For example, the tutor was the sole group member with the appropriate technological tools to write “visibly” and “conveniently” in mathematical terms, leading the students to unanimously choose her as the group scribe. Such factors led to a shift towards online tutorials being mainly instructor-led. Future research could look into tutorial teaching and learning beyond the online synchronous setting explored in this paper. Such research could take into account various factors, such as the technological infrastructure and communication affordances, but also the support offered to instructors and students who may not be familiar with the non-traditional teaching, that is online synchronous (or even asynchronous) teaching and learning of mathematics.

Moore-Russo et al. focus on how multivariable calculus instructors in the USA select and use digital and physical resources, their motivations for doing so, and how these resources interact with instructor and students, aspects which place this paper under themes 1 and 2. The paper also examines the role of visualization and other aspects of spatial literacy in multivariable calculus instruction. This paper’s contribution lies in the offering of insights into instructors’ adoption and use of resources for multivariable calculus, an area that has not been extensively delved into. It documents the shifts in instructors’ content coverage and their practices as they have more experience in using resources. In addition, it offers a deeper understanding of how resource selection/adoption could lead to better teaching and learning practice, particularly in terms of visualization and spatial literacy. The authors suggest several directions for future research; delving deeper into how resources are used for particular multivariable calculus topics, challenges for classroom practice in resources use when instructors adopt activities produced by others, and the challenges less technology adopters face in the classroom. A valuable addition to future research directions could be examining - e.g. through a

documentational approach (Trouche, Gueudet, & Pepin, 2020) lens - how instructors interact with various resources and how they select, modify, use and reuse these resources in their teaching practices, in order to provide insights into their professional development and offer clues for enhancing teaching practices and resource utilization.

There is one more paper that focuses on themes 1 and 2 and this is by **Albano et al.** They explore the design principles for mathematics undergraduate students' activities at a university in Italy that involve the "construction of basic concepts in general topology", the "promotion of problem-solving processes", the "development of metacognitive aspects", and the "development of the students' mathematical identity". The authors argued for the use of a technology-enhanced environment being pivotal to the design of activities that encourage students' participation, allowing them to engage in problem solving, while reflecting on their interactions through two lenses. On one hand, students utilize, share, and apply their knowledge to solve the given problem and manage their process. On the other hand, each participant views the process through a unique perspective shaped by their specific cognitive role. The authors extended the 'inside-out' model for problem solving (Albano et al., 2021) which focuses on promoting students' metacognitive reflections and the development of their mathematical identity by integrating epistemological and cognitive elements to the design of activities for undergraduate students. This paper's further contribution is the presentation of preliminary results of piloting an activity designed based on their derived design principles, with second year undergraduate students. Future research can focus on empirical research studies to further evaluate these design principles, but also extend it to different fields in university mathematics involving a distance technological setting, as the authors claimed.

Davies et al. focus on the integration of STACK, a computer-aided assessment (CAA) technology, in a mathematics department of a large university in the UK. Their study, which comes under theme 3, revolves around a department-wide project where instructors were expected to implement STACK into assessment tasks for nearly all core modules across the first two years of undergraduate study. The authors present their findings from interviews with six novice STACK assessment designers and module leaders. The research identifies four themes related to the design of STACK-based assessments by novice tutors: the process of STACKification, technical challenges, users' perspectives on the role of CAA, and variations in assessment designers' approaches to the role of feedback. The authors offered valuable insights into the implementation of CAA by discussing the challenges faced by novice users and reflecting on Sangwin's (2013) design principles for (undergraduate) mathematics. Despite the challenges, CAA's support for formative assessment is highlighted. The authors also emphasize the need for maintenance and ongoing improvements of CAA resources as well as the importance of professional development and the demanding role of the integration. In addition, the authors note the need for further large-scale and empirical studies regarding the role and consequences of CAAs in other contexts.

CONCLUDING REMARKS

Reflecting upon the above summarised contributions of each one of the seven papers, most of the aspects listed in the Call for Papers were discussed. There are, though, some topics that need to be further investigated. These involve: machine learning, learning analytics, artificial intelligence for instructional design and assessment in UME; computational thinking and programming instructional design and policies in UME; but also affordances and drawbacks of automated assessment instruments and issues of transfer from pen and paper to automation of test items for digital exams.

This Special Issue contributed valuable findings to the research field of Digital Experiences in UME, presenting studies carried out in a number of countries as stated earlier, involving a great variety of Digital Technologies. As argued throughout this editorial, further research on digital experiences at university level is definitely needed and we hope our readers appreciate our attempt to put forward some gaps identified in existing work.

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