Prospective mathematics teachers' professional development through meta discussion on a pedagogical model

Michele Giuliano Fiorentino, Maria Alessandra Mariotti, Antonella Montone

Prospective mathematics teachers' professional development through meta discussion on a pedagogical model

Michele Giuliano Fiorentino[1], Maria Alessandra Mariotti[2], Antonella Montone[1]

[1]University of Bari Aldo Moro, Italy antonella.montone@uniba.it

[2]University of Siena, Italy

This paper presents some results emerging during a larger study, still in progress, about prospective teachers' training and concerning their introduction to the particular pedagogical model of Mathematical Discussion. According to our hypothesis, prospective teachers can be introduced to a new pedagogical model through the combination of a living experience with the role of students and a critical reflection on their own experience. Consistently, we designed a teaching experiment involving a class of prospective primary school teachers with the aim of exploring and deepening such hypotheses. The first results focus on a new theoretical construct that we named Meta Discussion on a Pedagogical model, and on its effectiveness in leading prospective teachers make acquaintance, in and from practice, with both theoretical knowledge and practical experience of the pedagogical mode of Mathematical Discussion.

Keywords: Prospective teachers, mathematical discussion, meta discussion on a pedagogical model, mathematics teachers' professional development, training mode.

Introduction

The mathematics teachers' training is a fundamental research topic in the field of Mathematics Education. This contribution aims to provide new insight on how to train prospective teachers with respect to possible pedagogical models and their implementation in a class. In particular, we asked ourselves how to train the prospective teacher on a specific pedagogical model, and in this contribution, we propose a new *training mode*, designed in order to introduce prospective teachers to the pedagogical model of Mathematical Discussion, both from a theoretical and practical point of view. Usually, teachers' practical training is based either on their personal teaching performances (active training) or on their observation of what an expert teacher does (passive training). In this study, we propose to let the prospective teachers experience the pedagogical model at stake in the role of student, then to involve them in a reflective situation guided by the trainer, consisting in a collective discussion focusing on the teacher's role previously played by the trainer. We call such a reflective discussion, Meta Discussion on a Pedagogical model (M-DPm).

Our research has involved 180 pre-service mathematics teachers attending the undergraduate 'mathematics teaching' course in primary education. The analysis of the data collected during the course showed that the training activity carried out by experiencing in first person the pedagogical theories fostered the theory's conceptualisation itself and the future teaching profession's awareness.

In the following, after outlining the conceptual background, in particular the pedagogical model in focus, we present the experimental design, the research methodology, the analysis of some of the data collected and the results obtained.

Conceptual background

During the last years, several research projects have been conducted in this area and several theories emerged, starting from the seminal work of Shulman (1986) who claimed that it is necessary to tackle both the specific knowledge to teach a discipline, not simply seen as a disconnected set of disciplinary knowledge and pedagogical knowledge. Indeed, according to Shulman, teachers need an integrated view of disciplinary content and issues related to its teaching, in order to foster effective teaching-learning processes, e.g. a Pedagogical Content Knowledge, (PCK); moreover, for mathematics, a number of research studies have proposed different conceptualisations and models of specialised mathematical knowledge for teaching, such as the Mathematical Knowledge for Teaching (MKT) model (Ball et al., 2008) or the more recent Mathematics Teacher's Specialised Knowledge (MKTS) model (Carrillo-Yañez et al., 2018). Ball & Even (2009) provide us a prospective for the implementation of educational interventions that takes into account the complex articulation of different contents and their relationships, suggesting ways of intervening in practice. From this follows the idea of training future teachers through practice and providing them an approach to a given task that can serve as a starting point for forming themselves as teachers through understanding and interpreting the actions of their future students (Ball & Even, 2009).-Consistently with these general suggestions, our main hypothesis concerns the effectiveness of a specific training mode to introduce prospective teachers to a definite Pedagogical Model (PM), characterised by combining the living experience of such a PM and a reflective activity on it. In particular, we introduce a new construct that we named Meta Discussion on a Pedagogical model (M-DPm) and that is the focus of our study.-It consists in a collective discussion, involving the prospective teachers and their instructor, on the previous didactical experience of a specific pedagogical model that they lived with the role of students. In other words, such a reflective experience involves the same prospective teachers in a discussion, still conducted by their instructor, but with the motive of reflecting on their previous lived experience and identifying the key aspects characterising the pedagogical model that is at stake. In order to explore the proposed training mode a specific pedagogical model has been selected, which is described in the following section.

The pedagogical model

The pedagogical model we have selected is that of the Mathematical Discussion (MD), firstly elaborated by Bartolini Bussi (1998). The MD was originally defined as a 'polyphony of voices articulated about a mathematical object (concept, problem, procedure, etc.), which constitutes a motive of the teaching-learning activity'. While the teacher has the responsibility of "orchestrating the polyphony", the voices (represented by the signs produced by the students) have to be coordinated with the voice of the mathematical culture (witnessed by the teacher herself)" (Bartolini Bussi & Mariotti, 2008, p. 763). During a MD, the two main teacher' roles are those of mediator and moderator, which are then characterised in the main following teacher' actions: the "back to the task" action (with the aim of reconstructing the context and fostering the (re)emergence of meanings and processes related to the task); "focalizing" action (with the aim of focus on aspects consistent with the didactic objective); "request of synthesis" action (with the aim of supporting students in the process of de-contextualisation and generalisation with respect to specific tasks); "offer of synthesis" action (with the aim of providing a formulation introducing the desired terms; ratifying the

acceptability and mathematical status of a specific meaning). A mathematical discussion activity is differentiated according to its objective (motive). Different types of discussion can therefore be distinguished: the Balance Discussion (MDb) and to a Conceptualisation Discussion (MDc). MDb is defined as the collective process of informing, analysing and evaluating the individual solutions proposed to a given problem. MDc is understood as the collective process of constructing mathematical concepts, building up suitable connections between already lived experiences and particular mathematical terms (Bartolini Bussi et al., 1995, pp. 11-12). It may happen that a Balance Discussion naturally develops into a Conceptualization Discussion. This is actually what happened in our experiment, thus in the following, for clarity reasons, we will simply use the acronym MD to refer to the combination of these two types of discussions. Once selected the MD as our pedagogical model, we can specify our hypothesis as follows: an effective training mode can be based on the combination of a didactical experience lived by prospective teachers as the students in participating to a MD (orchestrated by their instructor with the motive of conceptualising a specific mathematical content) and the reflective experience involving the same prospective teachers in a new discussion, still orchestrated by their instructor, but with a new motive: reflecting on their previous lived experience, identifying the key theoretical aspects characterising the pedagogical model constituted by the MD. Within such a complex conceptual framework the *specific hypothesis of this study* can be synthesised as follows:

Through the combined experience of MD and M-DPm, prospective teachers can be introduced to the pedagogical model of MD, and develop, in and from practice, both theoretical knowledge and practical experience of this mode. Consistently with this hypothesis we designed a teaching experiment with the aim of exploring and deepening such hypothesis. The following section is devoted to describing the research methodology.

The research methodology and experimental setting

This paper focuses on a specific experimental path of a general project that is design-based research (Swan, 2020). In particular, as said, we focus on the design and the implementation of a training mode in which we involve the prospective teachers in the two kinds of activities that develop at two different levels: the practical level where prospective teachers play the role of students and the instructor applies the pedagogical model at stake; the reflective level where, within the frame of the M-DPm, the prospective teachers reflect on their previous experience under the guidance of the instructor. In this way we intend to foster the development, in and from practice, of both practical experience and theoretical knowledge on a specific pedagogical model. The details of the training mode are described in the following. From the huge amount of data collected during the experimentation, we consider the M-DPm transcript and the personal reflective diaries. The excerpts we present in the following sections, have been chosen because they reveal evidences about effectiveness of the training mode and particularly of the construct of the M-DPm. A qualitative analysis of the transcriptions was carried out, according to the criteria of credibility, dependability, transferability and confirmability (Guba, 1981), to ensure trustworthiness. In the following section, we describe the general setting of the specific experimental path in focus. The research involved 180 pre-service prospective teachers in mathematics, fourth-year students in Primary Education, and attending Mathematics Education Course. The training mode and in particular both MD and M-DPm

take place synchronously on Microsoft Teams platform. Both the discussions take place with the whole class group. These discussions have been recorded and then transcribed for the analysis.

The structure of experimented training mode

The design of the training mode consisted of a succession of phases identified by activities informed by our hypothesis on the educational relationship between experiencing a MD and participating in a M-DPm on such an experience. Such phases are described below:

Phase 0: introduction of the MD theoretical model. In a preparatory phase, a lecture on the pedagogical model gave the theoretical elements characterising the MD, in particular the teachers' actions to be performed to orchestrate the discussion.

Phase 1: In order to activate a MD, it is necessary to start by solving an open-ended mathematics problem sufficiently elementary to be understood by anyone and included in school curricula. As a matter of fact, the use of an open-ended problem (Pehkonen, 1997) is crucial for triggering a rich and effective mathematical discussion: it is not solved mechanically, but requires different solving strategies, it is sufficiently challenging to make students engaged both cognitively and emotionally, as solvers of the problem.

We selected the topic of equi-extension which is a complex one, generally based on the procedural activity of calculating the measure of equi-extended areas, and not on the idea of extension as a quantity related to equi-decomposability. The following problem was proposed to be solved in groups.

Two brothers receive a rectangular piece of land as inheritance. In order to divide it into two parts of the same size, one of them suggests planting a stake anywhere on the land and joining it to the four stakes driven into the four vertices of the rectangular land. One of the brothers will take two non-adjacent triangles, the other the remaining part. Are the two parts really equal? Justify your answer.

In the following lesson, starting from the solutions of the above problem a MD was developed. The aim of the MD was bringing out all the problems' solutions strategies, converging towards a shared solution, and making explicit the equi-extension's concept. In the design and then in the realisation of the MD, we referred to the patterns described by the MD pedagogical model. In particular, the instructor who orchestrated the discussion performed the actions expected from the Pedagogical model. In so doing we intended to make the prospective teacher experience the effect of these actions as students, and at the same time, observe these actions as performed by the 'teacher'.

Phase 2: M-DPm on the previous MD. In this phase, engaged in a M-DPm, prospective teachers are asked to reflect on the activity carried out in phase 1: identify the actions performed by the teacher and characterising the pedagogical model, that is the case of the MD. In this M-DPm Mathematics remains in the background, although it is not ignored by the participants. MD, as a pedagogical model and in particular the teacher' actions, become the subject of discussion. A key aspect for the prospective teachers' professional development lies in the transition from the role of student to that of teacher, projecting himself/herself in the actions performed by the instructor.

Analysis of data with respect to the M-DPm

The analysis we present concerns the M-DPm carried out in phase 2. According to the main hypothesis inspiring the research study, the following analysis attempts to highlight the effect of performing such a meta- discussion: how the M-DPm makes the characteristics of the pedagogical model MD emerge and are made explicit by the prospective teachers. The transcript of the M-DPm was analysed with the twofold aim: highlighting the prospective teachers' recognition of MD's characteristics through the recognition of the teacher's actions; and at same time highlighting the importance of experience in practice of prospective teachers during MD. In the following we report the analysis of some excerpts consistent with this twofold aim.

(0:00) Instructor: What did we do in the last lesson [refers to the lesson in which the MD on solving the mathematical open-ended problem on equi-extension was carried out]. Why did I do this lesson with you? What was the teacher's aim?

(1:42) Stefania: we retrace everything that we did together, [...] so both an analysis from the point of view of Mathematical Discussion, as a pedagogical model and on the solutions of the problem [...] so the discussion's steps, the teacher's role who has to act as ... especially as a mediator within the discussion.

The instructor starts the M-DPm, proposing to discuss the previous lesson in which the prospective teachers participated. The aim of the instructor is to bring out the MD's characteristics. For this reason, we could define this action carried out by the instructor as: "Back to the experience". This action aimed at going back over the discussion experience in order to recognise the MD as a pedagogical model. The question is formulated by the instructor in a particular way: "Why did I do this lesson with you?"; there is no longer a reference to the problem to be solved, but to problematize the actions of the teacher in the attempt to make the characteristics of MD emerge. Stefania's answer shows the effectiveness of the instructor's intervention: she recognises and shares with the others her recognition of the MD pedagogical model in action. Following the question that induces and promotes shifting the discussion's focus from the mathematical content to the pedagogical model, other interventions occur.

(3:22) Instructor: OK, so let's see, Eleonora wanted to intervene.

(3:30) Eleonora: Yes, the first thing that came to my mind was actually that...ehmmm that mathematics can be discussed, precisely that through reasoning guided by you and… the various ideas emerged, the various hypotheses that we articulated to solve the problem.

Eleonora acknowledges that the activity in the previous lesson was a discussion and in particular a MD because the motive was that of making the "various ideas" emerge and be shared, and this is a characteristic of MD. A clearer articulation of MD' characteristics is shown in the following excerpt.

(4:16) Instructor: Eleonora said we discussed Mathematics, Stefania said, the role of the teacher is seen to be one of mediator, etc., but can you tell me what were the very elements that allowed you to recognise this activity as a discussion?

(05:42) Stefania: I had written down a few questions that you asked [...] sometimes to investigate more deeply what we were saying and sometimes offering synthesis.... so this aspect is the one that most brought me back to the steps of the MD and above all to the role of the teacher ... so the one who acts as mediator and moderator [...] It happened to me during the MD, when at the beginning I told how we carried out our work with my group, I hadn't been able to make myself understood exactly... you picked up the threads of my speech to... try to understand better what I said...

and you used some words that I later found written in the notes. These words made me understand that ... this is a MD. [...] So your role and therefore the words you used, the questioning and also affirmative tone and our problem's solutions.

(07:43) Eleonora: Yes, following what Stefania said, I think that the most evident element has been the language, that is, an appropriate way that we all noticed to talk about mathematics [...] because you have to master the appropriate language for each discipline that you do.

(09:12) Debora: so, I agree both with Eleonora and Stefania... but another aspect that in my opinion made us understand that this... that it was a MD is the fact that the teacher is not the repository of the truth. Indeed, we never came to a conclusion and we also came to an uncertainty... Let's say you never said "this is the right solution", "this is another right solution" ... you replicated our words...

Here the instructor performs a "meta-focusing" action, which can be considered a key action for di M-DPm, aiming to make explicit the characterising elements that made it possible to recognise a MD in the experienced activity. In the underlined part of the transcript, Stefania explicitly refers to the experience related to the concept of mediator (and moderator) as characteristic of the teacher's roles in a MD. From Stefania's words it emerges that, at the moment when the student speaks about his/her solution, the teacher intervenes to take up and reformulate what he/she said. Stefania therefore recognises this action of linguistic mediation, which we can classify as the action of "offering synthesis", as a teacher's action characteristics of a MD. Stefania highlights the teacher's interventions about the use of language in which it is clarified that it is possible to discuss not only problem's solutions, but also their verbalisation, proposing a linguistic reworking of the same solutions. Recalling what Stefania said, Eleonora highlights the language's function in the evolution of the mathematical concept at stake. In other words, Eleonora emphasises the function of language in the evolution of the mathematical concept as characteristic of the MD pedagogical model. In the same stream, Deborah remarks that the teacher used mirroring student interventions rather than evaluating them, which is another semiotic action of the teacher, characteristic of the MD model.

(10:21) Instructor: So... synthesising... Stefania says... I recognise the discussion, that is, I recognise the elements characterising the MD because there is the role of the teacher who becomes mediator and moderator. The teacher intervened to investigate, sometimes intervened to offer synthesis. Debora says that the teacher never gave us theories, but always waited for us to define the theoretical aspects and then, as Eleonora said, it seemed to us that in this talking of mathematics, the new thing was the language used by the teacher.

(11:57) Rossana: Well, while Daniele was speaking and outlining what was his problem's solution [she refers to the MD], you made an explicit request as to why... you tried to rework that concept several times, so that it wasn't just clear to Daniele but to the whole class. You wanted to make sure that you led everyone on that path.

Here the instructor synthesises what emerged from the three previous interventions. In particular, referring to Stefania's words, she recalls the teacher's role of mediator, which is another characteristic of the MD model. In the first part of the intervention, Rossana makes explicit reference to her experience during MD, highlighting specific teacher's actions as mediator in always asking why. In the second part, Rossana recognises another mediator's action related to the teacher's concern about making all the students involved. Referring to the summary intervention and Rossana's answer, we can observe the evolution from direct reference to the lived experience - what was specifically done and said - to reference to the pedagogical model. As in the following intervention (31:55) by Stefania,

the prospective teachers become increasingly aware of the teacher's role through distancing themselves from the lived situation.

> (31:55) Stefania: So, in the MD the cognitive conflict emerged... there was a constant constructing and deconstructing of what we were saying, going to investigate why, through your questions "But I didn't understand... but are you sure? But can you make me understand? No, but I don't see it…". At one point when I was describing my solution, I distanced myself from what I was saying; that is, I realised that I was able to treat the subject, the solution... with more detachment; I realised that I was treating it with distance and therefore could be more aware and impartial in the discussion. Distancing oneself from oneself and cognitive conflict are the two fundamental aspects of mathematical discussion realised thanks to the teacher's mediation and moderation. Even now in this discussion, I realise that I distanced myself from the fact that I was involved in the discussion in the previous lesson, thanks to your questions and the interventions of my peers.

Stefania states that she experienced a situation in which it was possible for her, thanks to the teacher's interventions, to become aware of the MD pedagogical model, through a distancing process. Moreover, as she describes, she becomes aware of the teacher's role, that is how the teacher's guidance enabled a process of decontextualisation of the solution with respect to the problem.

Preliminary results and concluding remarks

As a preliminary result, it seems that our main hypothesis, implemented in the experimented training mode, has been corroborated. The new theoretical construct M-DPm, applied to the MD pedagogical model, made it possible for prospective teachers to reflect on previous lived experience, identifying the key theoretical aspects characterising the MD's pedagogical model. Analysis of the transcripts revealed evidence how the instructor's interventions trigger the recognition of theoretical aspects from the experienced situation, particularly the identification of the teacher's actions recognised through a detachment process that allows the prospective teachers to leave the student's role and assume the role of future teacher. Indeed, Eleonora in her intervention (03:30) identifies MD as an appropriate pedagogical model that is constructed and/or recognised. In particular, the student recognises and appropriates the patterns of the model, recognises the effectiveness of the model with respect to its function and finally, distinguishes two specific functions: a. solving the problem and constructing the mathematical concept; recognising the nature of mathematics as something that can be discussed. Data analysis seems to show that involving prospective teachers in training activities, in which they can experience and reflect in first person, could foster the theoretical conceptualisation and the practical implementation of a pedagogical model, and in so doing contributing to their effective professional development. In this research study, the implementation of the direct experience reflects some suggestions coming from the literature, but also provides a specification of what kind of experience has to be lived (activities designed according to the pedagogical model) and how reflection can be promoted (M-DPm). The specific training mode presented here, proposes a specific interpretation of the suggestive expression "in and from practice": through the M-DPm, prospective teachers recognise the teacher's actions performed according to the specific pedagogical model, after having personally experienced them as students.

Acknowledgment

The authors were funding by the project HORIZON EUROPE SEEDS – ID. S60 – Qualità e accessibilità della formazione universitaria. Learning Analytics per la prototipazione di un modello di Blended Learning.

References

Ball, D. L., Thames, M. H., Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), pp. 389–407. https://doi.org/10.1177/0022487108324554

Ball, D. L., & Even, R. (2009). Strengthening practice in and research on the professional education and development of teachers of mathematics: Next steps. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics* (pp. 255–259). Springer. https://doi.org/10.1007/978-0-387-09601-8_27

Bartolini Bussi, M.G. (1998). Verbal interaction in mathematics classroom: A Vygotskian analysis, in H. Steinbring, M.G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in mathematics classroom* (pp. 65–84). NCTM. https://doi.org/10.1017/CBO9780511720406.002

Bartolini Bussi, M.G., Boni, M., & Ferri, F. (1995). Social interaction and knowledge at school: The mathematical discussion. Technical report no. 21 NRD of Modena, Municipality of Modena. Available at the link: https://www.comune.modena.it/memo/prodotti-editoriali/saperi-e-discipline/allegati/interazione_sociale_e_conoscenza_a_scuola.pdf

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: artefacts and signs after a Vygotskian prospective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education, second revised edition* (pp. 746–783). Lawrence Erlbaum.

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila,... Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*. https://doi.org/10.1080/14794802.2018.1479981

Guba, E. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Technology Research and Development*, *29*(2), 75–91. https://doi.org/10.1007/BF02766777

Pehkonen, E. (1997). Introduction to the concept "open-ended problem." In *Use of open-ended problems in mathematics classroom (Issue 7).* http://coreylee.me/en/publications/2001_self-efficacy_change.pdf%5Cnhttp://files.eric.ed.gov/fulltext/ED419714.pdf

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(4), 4–14. https://doi.org/10.3102/0013189X015002004

Swan M. (2020). Design research in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 148–152). Springer. https://doi.org/10.1007/978-3-030-15789-0_180