An introduction to TWG16: Learning mathematics with technology and other resources

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Overview of themes and contributions

The scope of the thematic working group TWG16 was to address opportunities and constraints of digital technology and other resources for students' mathematics learning. As a targeted outcome, we wanted to establish an overview of the current state of the art. We also aimed to suggest important future trends for technology-rich mathematics education, including a research agenda.

Table 1: TWG16 themes and contributions

Theme	# accepted papers	# accepted posters
1. Simulations and animations	3	
2. Embodiment and visuospatial abilities	3	1
3. Computational thinking and programming	4	3
4. Video and interactive textbooks	3	3
5. Students' meaning-making with digital tools	3	2
6. Tablets, apps, and out-of-class learning settings	3	2
7. Online platforms and Padlets	4	1
8. Digital tools design issues	3	3

In the analysis of the TWG16 contributions, we identified eight main themes to be addressed. Table 1 provides an overview of these themes and the number of contributions per theme. Altogether, we had 26 accepted papers and 15 accepted posters. About 50 participants from many countries within and outside Europe took part in the sessions, which took place in a positive and productive atmosphere.

Main results per theme

Simulations and animations: Using simulations and animations as traditional digital tools in learning mathematics is still present in TWG 16. Two studies discussed the role of digital simulations in fostering covariational reasoning (Thompson & Carlson, 2017). The two studies provided evidence that using a digital simulation fosters the emergence of covariational reasoning among young students. In addition, one of the studies that used digital tools to simulate real-world phenomena provided evidence that students who used such digital tools engaged in mathematical discussion and could interpret and mathematically evaluate several real-world phenomena. The paper that discussed the use of an animation presented a new use of such digital tools in which students not only used ready-made animations but were engaged in programming them. This research showed that designing tasks with an animation model in a programming environment encourages students to participate in learning geometrical concepts such as rotation, ratio, and Pythagoras theorem.

Embodiment and visuospatial abilities: Three papers and one poster contributed to this theme, all evoking the role of the body in mathematics meaning-making. Two studies, drawing respectively on embodied instrumentation and the concept of body-artefact system, highlight different "kinds" of embodiment in relation to digital technology: while nomograms (or dynagraphs) offer the students concrete experiences enabling them to "feel" the concept of function, an app recognizing child's manipulations provides opportunities for making sense of numbers and operations in perception-action loops. Two other contributions focus on the development of visuo-spatial abilities: (a) a simulation of moving in a virtual city aims at enhancing students' spatial orientation; (b) a specifically designed learning environment, where students manipulate complex 3D virtual objects, fosters mental rotation abilities. Although these studies do not explicitly draw on theories of embodiment, the body is strongly engaged in the proposed tasks, and it is worth considering its impact on the students' learning. Overall, we remarked a growing interest in considering the body in students' interactions with technology, which leads to the emergence of new theories (e.g., embodied instrumentation) and calls for innovative methodologies for assessing students' knowledge built with these technologies.

Computational thinking and programming: Four papers and three posters were related to this theme. The contributions addressed three main aspects: (1) computational thinking (CT) in mathematics education from the perspectives of CT conceptualization, students' difficulties, and relevant tasks; (2) programming as an object-to-think-with (Papert, 1980); and (3) interactions between mathematics, computer science, and algorithms. The discussions within the working group highlighted that CT is sometimes considered a learning goal and sometimes a tool for solving mathematical tasks. A growing interest can be observed in studying interactions between mathematics and computer science, as a background domain for CT, fueled by challenges brought by the introduction of CT to mathematics curricula. New mathematical problems are arising that can be efficiently solved with computational methods, which requires new types of skills. CT involves designing algorithms, and applying, analyzing, and proving them. Further research directions outlined include identifying tasks fostering CT for mathematical proficiency, and how CT can enhance understanding of mathematical concepts. Therefore, research on CT in relation to mathematics education is expected to keep growing and might give rise to a new thematic working group for the upcoming CERME titled "Teaching and learning of discrete and computational mathematics."

Video and interactive textbooks: Several research topics surfaced through the paper/poster presentation and discussion on videos and digital textbooks in mathematics education. Discussions focused on how digital tasks within digital textbooks and videos effectively boost conceptual understanding and motivate students to develop different argumentation approaches. Since students are not only consumers of existing videos but can produce them, it emerges that creating videos can significantly deepen their grasp of the subject matter. However, it was also highlighted that using digital textbooks and videos causes some concerns with attention, eyesight, etc. Finally, the group discussed the educational impact of dynamic visualizations in animated mathematics videos. Future directions in the research and application of videos and digital textbooks in mathematics education are promising, underpinned by the need for effective digital interaction and collaboration. Participants recognized that Artificial Intelligence (AI) offers immense potential in this context; however, addressing these challenges requires robust research studies.

Students' meaning-making with digital tools: This theme discussed students' meaning-making with digital tools theoretically and practically. One of the papers focused on transitioning from paper-pencil to a dynamic geometry environment (DGE), and another focused on a digital educational game and GeoGebra graphic calculator. The first demonstrated that DGE, like GeoGebra, could support the meaning-making of calculus concepts like parametric functions by revealing features of such objects. The latter presented that a competitive digital educational game could prompt the evolution of mathematical meanings related to trigonometric functions. In the third paper, the digital environment came to the fore with computer-based modelling that integrates mathematical modelling and computer science. While this was an ongoing study, it was suggested that a digital modelling environment (e.g., NetLogo) could support students' ability to use mathematics for modelling socially relevant topics (e.g., sustainable mobility).

Tablets, apps, and out-of-class learning settings: Several contributions of our working group considered educational apps for phones or tablets. Researchers focused on the impact of such apps on students' learning processes in various contexts. In particular, one paper studied how the MathCityMap App, which supports outdoor math trail activities, impacts students' interest, self-efficacy, and performance during the trail. Surprisingly, it only impacts the latter. Another paper studied how the Book Creator App supports students' process of answering Fermi questions. it details how students use various functionalities for different steps in the process. The third paper studied the Blocks app, which is well-known in Germany. The paper applied an interaction analysis with all aspects of interest involved. The last paper on this theme investigated an app that supports the place-value system. Their interest is to uncover how such apps could foster an understanding of the place-value system. Two other contributions about apps were of a more general nature. One presented a literature review on how apps on tablets contribute, in general, to the learning process in primary education. The second paper presented a framework for analyzing out-of-classroom use of apps. We conclude that researchers progress locally, studying apps for specific didactical purposes and trying to make sense of findings on a more global level through literature reviews and developing and extending theoretical approaches to apps.

Online platforms and Padlets: Participants presented research and development on the use of online platforms and padlets. This theme included papers that addressed the design of online platforms, for

example, in the context of self-assessment or concerning professional development for mathematics teachers' noticing of students' mathematical thinking. Therefore, the reconstruction of design elements to support teachers' interpretations of students' mathematical thinking was carried out based on teachers' analysis of a collaborative activity. Further research shows that working with online platforms and padlets can support teaching actions but cannot completely replace the social action of face-to-face teaching. On the other hand, some papers explicitly address the role of collaborative learning and meaningful mathematical discourse as well as the question of whether such complex social and cognitive processes can be theoretically described and how they are supported.

Digital tools design issues: This theme covers designing digital tools that enable the teachers to author the mathematical content. For example, one of the papers discussed the research-based development of a tool to digitally author combinatorial problem-solving tasks and reports on its micro-cyclical and multi-methodical development and evaluation. In addition, designing digital platforms that automatically difficulty-ranked algebraic tasks gained special attention in this theme. Still, the design of digital games for the learning and teaching of mathematics is discussed in this theme. In particular, the design of digital games aims to reduce fear and mathematical anxiety among the students. The participants of this theme concluded that despite the progress in this trend of research, there are still several issues to be discussed and solved. For example, what methodologies can search for math anxiety in a game-based environment or digital tool? Or how could a generic tool for authoring tasks be designed to address various mathematical content?

Conclusion

The first goal of this TWG was to provide an overview of the current state of the art in the domain of technology-rich mathematics learning. Based on the contributions and discussions, we conclude that the field is still in progress and that much attention is paid to the design of digital tools, mobile platforms, digital textbooks and their resources, and embodied environments. The need for further foundations, in terms of new theoretical and methodological frameworks or adaptations of existing ones, and design heuristics is widely recognized. Another issue observed throughout the papers and the discussion is the emphasis on digitalization and less on mathematical content.

A second goal was to identify a research agenda. The above theme, though implicitly, provides an interesting picture: The need for theoretical and methodological approaches is widely acknowledged. We need to reflect on the role of the existing theories and methodologies and how to modify them when using digital tools. Therefore, one of the challenges for the next edition is to invite and receive contributions that discuss these revisions of theoretical and methodological perspectives. In addition, the rapid development of artificial intelligence technology should invite TWG 15 and 16 to explore the potential and the limitations of such technologies in learning and teaching mathematics.

References

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