

CORRIGENDUM TO
“LEAST ENERGY RADIAL SIGN-CHANGING SOLUTION
FOR THE SCHRÖDINGER-POISSON SYSTEM IN \mathbb{R}^3
UNDER AN ASYMPTOTICALLY CUBIC NONLINEARITY”
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ABSTRACT. The aim of this paper is to show as in [1] an assumption made on the nonlinearity can be removed. Moreover we give a better explanation in a step of the proof which seems to have a gap.

In this note we would give some light on two facts concerning the paper [1], that however do not affect the main result, neither the statements of the auxiliary lemmas, which remains true.

(I) The first one is on the assumption $\lim_{t \rightarrow \infty} [f(t)t - 4F(t)] = +\infty$. This is never required in the computations, so it can be removed. In particular we can allow now a nonlinearity of type

$$f(t) = \frac{t^3}{1 + e^{-t}} \text{ when } t \geq 0; \text{ } f \text{ odd function.}$$

(II) The second fact concerns a proof which contains an omission we would like to remedy. In the Appendix A, Section A.1 on page 567, we choose $T_0 > 1$ large enough such that $T_0 > \max\{T_1, T_2\}$ and $\left[\frac{r_1}{T_0}, \frac{r_4}{T_0}\right] \cap [r_1, r_4] = \emptyset$. However the fact that it is possible to satisfy also the inequality $\lambda\phi_u(T_0r_1) < 1$ needs an explanation since u also depends on T_0 .

At page 564 from $\|\mathbf{u}\|^2 = \int_{\mathbb{R}^3} (f(\mathbf{u})\mathbf{u} - \lambda\phi_u\mathbf{u}^2)dx$ we deduce

$$0 < \frac{1}{2}\|\mathbf{u}\|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |\nabla\mathbf{u}|^2 = \int_{\mathbb{R}^3} \left(f(\mathbf{u})\mathbf{u} - \lambda\phi_u\mathbf{u}^2 - \frac{1}{2}\mathbf{u}^2 \right) < \int_{\mathbb{R}^3} \left(\mathbf{u}^4 - \lambda\phi_u\mathbf{u}^2 - \frac{1}{2}\mathbf{u}^2 \right).$$

Consequently we choose now $0 < r_1 < r_2 < r_3 < r_4$ and $\delta > 0$ such that $\mathbf{u}^2 - \lambda\phi_u - \frac{1}{2} > 0$ on $[r_1, r_4]$, $\frac{\lambda}{r_1} \int_{\mathbb{R}^3} \mathbf{u}^2 = \frac{\lambda 4\pi}{r_1} \int_{r_1}^{r_4} \mathbf{u}^2(s)s^2 ds < \frac{1}{2}$ and

$$\int_{A_{r_1, r_4}} (\mathbf{u}^2 - \lambda\phi_u)\mathbf{u}^2 > \int_{A_{r_1, r_4}} \left(\mathbf{u}^2 - \lambda\phi_u - \frac{1}{2} \right) \mathbf{u}^2 > \frac{3}{2}\delta.$$

$$\int_{A_{r_2, r_3}} \left(\mathbf{u}^2 - \lambda\phi_u - \frac{1}{2} \right) \mathbf{u}^2 > \delta \text{ and } \int_{A_{r_i, r_{i+1}}} \left(\mathbf{u}^2 + \lambda\phi_u + \frac{1}{2} \right) \mathbf{u}^2 < \frac{\delta}{4}, \text{ for } i = 1, 3.$$

Then we define $\mathbf{v} := \nu\mathbf{u}\eta$ and $e_t := \sqrt{t}\mathbf{v}$ for $t > 1$. The functional G remains unaltered as well as the statement of Lemma 12. Its proof is easily adapted.

The definition of the functional H_t changes into

$$H_t(u) := t \int_{\mathbb{R}^3} |\nabla u|^2 + \left(\frac{1}{t} + \frac{t^2}{2} \right) \int_{\mathbb{R}^3} u^2 + \frac{\lambda}{t} \int_{\mathbb{R}^3} \phi_u u^2 - \frac{1}{t^2} \int_{\mathbb{R}^3} f(tu)u$$

and again the proof of Lema 13 is easily adapted. Finally the computations are adapted by considering $u := \sqrt{T_0} \nu u \eta = \sqrt{T_0} \mathbf{v}$ with $T_0 > \max\{T_1, T_2, r_4/r_1\}$ and $[r_1/T_0, r_4/T_0] \cap [r_1, r_4] = \emptyset$, since in this case is $\lambda \phi_u(T_0 r_1) < 1/2$.

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REFERENCES

- [1] E. G. Murcia, G. Siciliano, *Least energy radial sign-changing solution for the Schrödinger-Poisson system in \mathbb{R}^3 under an asymptotically cubic nonlinearity*, J. Math. Anal. Appl. (474) 2019, 544–571.