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Policies and implementations for technology integration in mathematics education: perspectives from around the world

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Abstract

In this chapter, we examine the role of policies and other factors affecting digital technology (DT) integration in mathematics education. In particular, we develop a cross-national analysis of the impact on DT implementation in four countries: two countries in Europe (Italy and Portugal) and two countries in Latin America (Colombia and Mexico). We analyse the role that policies, political changes, reforms, curricula, educational organisation and systems, socio-cultural aspects, and teachers' training, knowledge, and beliefs, play towards possible DT implementations. We observe that there is a discourse in policies to promote digital technologies' use, but in practice the availability and integration of such resources in mathematics classrooms is still scarce. We also note that the efforts done during the pandemic didn't change this, promoting general ICT use, rather than DT resources that might enhance mathematics teaching and learning.

1 Introduction

For decades, governments around the world have promoted initiatives, policies and curricular reforms seeking to prepare students with the knowledge, skills and competences, particularly digital ones, needed for the future. In fact, the Sustainable Development Goal 4 (SDG 4) of the Education 2030 Agenda (UNESCO, 2016) aims to "ensure inclusive and equitable quality education and promote lifelong learning opportunities for all" by 2030. However, the effectiveness of policies in changing and impacting educational practices, as well as achieving their goals, is to be determined.

In this chapter, through a cross-national analysis (Jablonka et al., 2018), we aim to identify historic, socio-cultural and policy-related factors that affect (meaningful) integration of digital technologies (DT) for mathematics education. Specifically, we examine different aspects of the educational systems of two countries in Europe (Italy and Portugal) and two in Latin America (Colombia and Mexico), to find and elicit similarities and differences (Clarke, 2003) that impact DT integration. Such analysis was done for the writing of this chapter and is not part of a separate study.

Our work may not be comprehensive or generalizable to other countries, but it may provide insights into the challenges of DT integration in mathematics education, as well as on the impact and limitations of policies. We will show, on the one hand, how political changes and decisions, and other social aspects (e.g., COVID-19) impact DT use and integration; on the other hand, that there are factors (e.g., lack of professional development –PD) that prevent policies from impacting DT implementations as intended.

1.1 Background: issues of implementation of DT in mathematics education

Despite various national policies and curricular recommendations for technology use, in practice, DT resources have been found to be underused in mathematics education. Also, as has been documented elsewhere (e.g., Faggiano et al., 2021; Julie et al., 2009, Sacristán, 2017, Sacristán et al., 2021), the prevailing use of technology in mathematics classrooms in many countries (exacerbated recently by the COVID-19 pandemic) is for presentation and communication (such as PowerPoint or videos) rather than of mathematics-enhancing digital resources. Many teachers have been found to prefer traditional teaching practices (e.g., teacher-centred instruction, dependence on white/blackboard and textbooks) and shown difficulties in changing those practices to integrate innovative approaches (this is what Sacristán, in press, calls the "inertia of classroom cultures"). Furthermore, we argue that, in general, few teachers (and few textbooks) offer students opportunities to engage in solving open mathematical problems (as reported by Boero & Dapueto, 2007) and, less so, by using technologies to do so.

Thus, the reasons for the lack of DT integration in mathematics classes are complex. Such integration is affected by many factors at different levels, ranging from the world and country-wide levels of society, the government and policies level, the regional and schools' levels, and the teacher level.

For instance, at the teacher level, some of the reasons include barriers for the uptake of DT (Clark-Wilson et al., 2011; Sacristán, 2017) that may relate to teachers' interpretations of policies; the influence of school cultures and traditions (see Sacristán, in press); issues of teachers' abilities, professional knowledge and personal orientations (e.g., confidence and beliefs) –as in Thomas & Palmer's, 2014, Pedagogical Technological Knowledge (PTK) model– (see also Remillard & Heck, 2014, discussed in the next section); as well as time constraints and lack of institutional or continuing professional development (CPD) and support.

At other levels, there are, for instance, issues of a lack of guidelines in official documents and curricula on how to use recommended resources or decide when to use them. Researchers have analysed, in particular, the interaction between policy and implementation of innovations in mathematics education, and the inherent challenges. Krainer (2021) identified that while policymakers drift to technical aspects, there is a social need to have negotiation processes between the policy, practice and research communities. On their part, Trouche et al. (2012) have described how policies related to the development of educational technology, are of two

dimensions: One, the *top-down/bottom-up dimension*, refers to the differences between policies imposed on the mathematics education community (top-down), which result from political choices made by top-level administrations, versus those that emerge (bottom-up) from the needs of mathematics education actors (students, teachers, parents, etc.). The other, the *access/support dimension*, refers to the difference between policies which focus primarily on access to technology (e.g., providing hardware) versus policies that focus on supporting teachers and students in integrating technology (Trouche et al., 2012).

1.2 Framework and method

International comparative studies have several kinds of methodological approaches, as categorised by Jablonka et al. (2018). The work done for this chapter is a type of documentary analysis, comparing categories in a range of aspects that encompass contexts, educational culture and policies (including those concerning PD), research literature results, as well as curricular materials.

We started our work discussing the situations and national educational systems of each of our countries, which helped identify similar or different aspects impacting DT integration that could lead to categories of analysis. We were also inspired by some categories used in various documents and studies: such as those of the European Union (Eurydice, n.d. a, and, specifically, its listed topics given in Eurydice, n.d. b); and the factors identified by Remillard and Heck (2014). The latter authors provided an analysis of the system and relationships between curriculum policies, the design of instructional materials and the curriculum enactment. They identified factors that influence each of these. We consider that their identified factors fall into three categories (i) the needs and views of society, of professional and educational groups and of stakeholders (including those of curriculum developers and publishers); (ii) access, constraints and support; and (iii) teachers' knowledge, beliefs, and practices. Additionally, we believe that policies and implementations have been affected by historical (Popkewitz et al., 2001) as well as cultural factors, and thus consider this historical dimension in one of our categories.

Table 1. Categories of analysis

Category	*Description*
Organisation, Governance and Practices	In this category, we consider: - Educational systems' organisations - General curricular structures - School practices / cultures and teachers' roles and practices (including their autonomy to integrate digital technologies). (Inspired by Eurydice topic "Organisation and Governance" and parts of the "Teaching and learning" subtopic of primary and secondary school levels involving how curriculum is structured, as well as teacher practices such as "whether or not the teacher is free to choose the methodology")
Political, Socio-Cultural and Historical Background and Trends	In this category, we consider, in particular, historical turning points and trends in educational policy and in the use of digital technologies. (Inspired by Eurydice* topic "Political, Social and Economic Background and Trends")
Teacher education and recruitment	In this category, we consider teacher education and professional development (initial and continuing education) and conditions of service. (Inspired by Eurydice "Teachers and Education Staff" topic)
Textbooks and other teaching resources	In this category, we consider: Textbooks and other teaching resources (including DT resources and repositories), policies related to them, their adoption possibilities and practices, and their influence on the integration of digital technologies into teaching practices. (Inspired by parts of Eurydice sub-subtopic "Teaching methods and materials" of the "Teaching and learning" subtopic of primary and secondary school levels, which includes Use of new technologies in class Whether or not the teacher is free to choose the methodology Whether teaching materials are compulsory or recommended and the body responsible for preparing such materials)
Assessment	In this category, we consider: National systems for assessing student learning and their impact on teaching practices using digital technologies. ("Assessment" is one of the Eurydice sub-subtopics in the "Teaching and learning" subtopic of primary and secondary school levels).

* https://eurydice.eacea.ec.europa.eu/chapter-topics

In the next sections, we present results from our cross-national analysis of educational policies and systems analysing factors that affect DT integration within each of the following categories (see Table 1): 1. *Organisation, governance and practices*, where we examined how educational structures and systems can limit (or promote) DT integration, including schools' and teachers' degrees of autonomy. 2. *Political, socio-cultural and historical background and trends*, where we analysed the role that historical and political changes factors have had. 3. *Teacher education and recruitment*, where we address how the way in which teachers are trained and recruited affects their knowledge and beliefs, in turn impacting teaching practices. 4. *Textbooks and other teaching resources*. And 5. *Assessment,* two categories where we focus on how reliance on textbooks and required assessments influence practices. We finish by describing how DT are included in our countries' curricula and repositories, and presenting examples from each country of how one middle-school mathematics topic is addressed using DT resources.

Before we continue, we need to point out that because each country defines educational stages differently (e.g., with different uses for terms such as "secondary"), we have adopted certain naming conventions. In this chapter, we consider that pre-university schooling consists of three main stages (even if in Portugal there are four), which we call primary (starting at age 6), middle and high-school, although the grades in each stage vary from country to country –see Table 2. However, in all of our countries, grades 1-9 (up to 8, in Italy) are called "basic school". We also note that, unless explicitly stated, in this chapter we mostly refer to basic school, and do not consider university level at all, nor what happens in private schools.

Table 2. School levels and compulsory education in each country

		Italy	Portugal	Colombia	Mexico
Basic Level	Primary-school	Grades 1-5	Grades 1-4 (1st cycle)	Grades 1-5	Grades 1-6
	Middle-school	Grades 6-8	Grades 5-9 (2nd cycle 5-6, 3rd cycle 7-9)	Grades 6-9	Grades 7-9
High-school		Grades 9-13	Grades 10-12	Grades 10-11	Grades 10-12
Compulsory education		Grades 1-10 (ages 6-16)	Grades 1-12	Grades K-11	Grades K-12

In the next sections, we delve into factors that affect educational practices (including DT integration), such as curricular guidelines, school and teacher autonomy, historical and social changes and trends, professional development, textbooks and assessments. We begin with details of the organisation of our educational systems.

2 Organisation, governance and practices in our countries' current educational systems

In our four countries, the national or federal authority has exclusive legislative competences on the general organisation of the education system (e.g., it sets the minimum standards of education and the policies concerning digital technologies, including those regarding hardware and connectivity in schools). Thus, the regulatory oversight of the national educational systems, not only sets educational guidelines, but also affects school staff, quality assurance, and financial resources in state-run schools.

In larger countries, regional authorities also have some jurisdiction, even if there is a central governing authority (such as in Mexico). An extreme example of regionalization is the USA, where no single government agency controls education, and guidelines are set by regional (state) authorities; however, for mathematics, the USA's National Council of Teachers of Mathematics (NCTM) has issued recommendations and standards (Dossey et al., 2016) that many other countries in the world (including some of ours) use as reference.

In our countries, national reforms and policies determine certain guidelines (norms of reference) on how content is to be delivered in schools through curricula and official textbooks, providing objectives and structure (goals and learning aims) for what is to be taught in each area (e.g., mathematics), sometimes also including some methodological suggestions. They are meant to be used by schools and teachers in their didactic planning activity, with attention to context, didactic and organisational conditions. All state-run (and many private) schools follow these national policies and curricular guidelines. Learning aims in those national guidelines identify knowledge and skills deemed indispensable in order to reach the goals for the development of skills.

In this way, the national guidelines, curricula and textbooks represent unavoidable references for teachers that indicate didactic paths to be followed and help determine educational actions necessary for students' learning. Nevertheless, schools and/or teachers have the responsibility and some freedom to organise themselves and choose the most appropriate path to allow students to achieve the best results. In Colombia and Italy, in particular, schools and teachers have a high degree of autonomy (see Table 3).

Table 3. Governance and autonomy

		Italy	Portugal	Colombia	Mexico *for basic level only
	Central authority	Ministries of education (at least for basic level*) set general educational structures and policies (including assessments and teacher recruitment), provide official curricular guidelines, approve textbooks, etc. *specially in Mexico			
Level of autonomy	**Regional autonomy**	Administrative only	Only in the autonomous regions (Azores and Madeira)	Regional ministries develop regional programs following national guidelines, sometimes with the support of some universities	Regional ministries (33, one for each state), sometimes develop complementary materials (particularly for middle-school)
	School autonomy	(Math) teachers' collective decide on textbook to adopt (for middle-school in Mexico)			
		High. Each school designs own curriculum following national guidelines	Limited. Programs & structure are defined. School decides on assessments. Teachers collective decides on sequence of content delivery	High. Each school designs their curriculum following national guidelines	Limited. Programs & structure are defined
	Teacher autonomy	High. Teacher's autonomy is stated in the Constitution. Teachers "collectively" decide	Limited. Teachers can decide how and when to use DT	High. Teachers have autonomy to make decisions at school and classroom levels	Medium. Teachers can decide sequence of contents & whether if they use DT or not

In Italy, teachers have the opportunity to experiment with new approaches involving technologies in extracurricular activities (sometimes funded by the EU). Extracurricular activities help teachers appreciate digital technologies, which often result in their continued use in everyday classroom activities (it is a way that helps teachers' PD). Concerning these opportunities, a noteworthy example is that of the ongoing project "*Liceo Matematico*" (Capone et al., 2017), which began in 2015, and is widespread in many high-schools across the country (www.liceomatematico.it), with the collaboration of the Italian Mathematics Union and universities: students are guided to see mathematics as a culture, discovering its main characteristics and also its relationship with other disciplines; the activities are mainly developed in laboratory environments (Bartolini Bussi et al., 2004) and based on open problems and the use of technologies. In Colombia and Italy, the educational system is organised according to the principles of subsidiarity and of autonomy of institutions. Schools and teachers have a high degree of autonomy and determine or even develop their own curriculum in accordance with the national guidelines (to ensure equal education for all). The guidelines are considered as an open text, which the professional community is called upon to assume and contextualise, elaborating specific choices relating to contents, methods, organisation and evaluation, consistent with the educational goals. Thus, individual schools can define curricula, choose the textbooks, widen the educational offer, organise teaching (school time and groups of pupils) as well as determine their assessment systems. However, the Colombian national government is responsible for curricular guidelines, teacher selection and PD programs (see next sections).

As outlined in Table 3, in Portugal and Mexico, the national curricula or guidelines are more prescriptive. They define the content to be addressed, the goals to achieve and may also suggest

methodological approaches. However, there are some decisions usually left to the teachers and the schools. In Portugal, the sequence of contents to be addressed is decided by the teachers and the assessment criteria are decided by the school, based on a proposal of the mathematics teachers.

In Mexico, schools have to follow the national guidelines, but different regions have some autonomy to develop their own programs and exams. School autonomy also depends on the school level, with much more autonomy in high-school. Basic school-teachers have some methodological autonomy: for instance, they can choose the sequence of contents and whether to use a particular digital resource, while still following the guidelines.

In all our countries, often mathematics teachers, in each school, work together to decide on the approach to be taken in a mathematics course, such as the sequence of contents to be covered, or possible resources to be used. Sometimes this also includes the choice of textbooks and some assessments (but may be subject to school approval).

3 General historical educational turning points and technology integration policies

In this section we refer to some factors from another category of analysis, "political, socio-cultural and historical background and trends", that have impacted education in general, and DT policies and implementations. Educational policies in each country have a history related to political changes and initiatives, and are driven, as well, by socio-geo-economic factors, and, to a varying degree, by international resolutions (e.g., from UN, OECD or from the European Union– EU –, in particular) and assessments (the OECD's Programme for International Student Assessment –PISA, https://www.oecd.org/pisa/ – and the Trends in International Mathematics and Science Study – TIMSS, https://timssandpirls.bc.edu/timss-landing.html). We also note, but do not discuss, that in our four countries (partially in Italy) teacher and/or labour unions have also had important socio-political influences (e.g. who can be a teacher, teacher work hours, etc., albeit not mathematical content).

Some of the main historical turning points in each of our countries' educational histories have had lasting consequences –we outline and detail further some of these in a timeline (see Appendix), that highlights how political and educational changes are intertwined, as well as in Table 4, showing the comparison of events in our countries relevant to DT integration.

Table 4: Historical key policies, programmes, trends and ideas for DT integration

	Italy	Portugal	Colombia	Mexico
Trends and revisions for schooling & curriculum in 20th c.: relevance of problem-solving in mathematics & DT use	- 1979 & 1985: National programmes for MS & PS, include problem-solving - 1989: National Plan (PNI) for CS in HS	- Mid 1980s: Focus on problem-solving & DT (trend; not a reform). - 1985-1994: Project MINERVA - 1991: curricular reform - 1994: document for teachers includes GC & DT - 1997: GC in HS exam	- 1994: Schools' curricular autonomy led to Informatics inclusion - 1998: Curricular guidelines (BS & HS) recommend GC & DT use	1993 BS Reform 1997-2006: EMAT (MS) Enciclomedia), as well as for *Telesecundaria* schools (for further details, see Trouche et al., 2012; Sacristán et al., 2021)
Mathematics laboratories (experimenting with math ideas)	- A suggested teaching-learning methodology - From 2001 onwards (still in national guidelines)	- Dedicated physical spaces - Emphasised in 1997 syllabus - Variable implementation but idea still used as *Sala de aula do futuro*	- Proposed in PINTCM (2000-2004) - Popular idea still present today.	- Central to EMAT (1997-2006) - Mostly abandoned afterwards
Initiatives in 21st c. promoting DT access and use	- 2009: MIUR - CI@ssi 2.0 provision of devices, and Internet access in classrooms. - 2015: National Plan for Digital Education (PNSD)	2007: *e-escola* (laptops & Internet) 2020: digital school	2002 - onwards: "*Computadores para Educar*" to provide infrastructure & develop teachers' digital skills	Some initiatives like 2011's "Digital abilities for all" HDT, but with few results
Teacher training & support: **- PD programs with some focus on DT** **- DT resources and repositories**	- 2009-2015: M@t.abel & PQM. - Their resources for teachers are still available. - Other resources currently on the UMI-CIIM or DiFiMa websites	- 1998: PD programme & booklets for DT use according to new syllabus - 2022-2023: PD & repository for new syllabus with focus on computational thinking	2000-2004: PINTCM 2007-onwards: network & DT repository "Colombia Aprende" 2014-2019: "National System of Educational Innovation with the use of ICT" PD programme	- 1997-2006: Linked to EMAT & Enciclomedia - Afterwards: - Little PD - Repositories not math specific, or videos
Coding & computational thinking	Some general projects (not for math)	Included in 2021 & 2023 syllabi for BS & HS	No policies, nor programmes	No policies, nor programmes

Codes in table: PS: primary-school, MS: middle-school; BS: basic-school; HS: high-school; GC graphing calculators

Most of the first major educational changes took place in Europe in the mid-19th century, linked to key political changes there, which included the first initiatives in Italy and Portugal towards compulsory and free education. Then, a hundred years ago, in all of our countries, major educational resolutions were taken, seeking to further promote education for all; at that time, top-down structures were created to oversee educational implementations and policies were decreed governing educational practices. The educational evolution and "education for all" policies led to a gradual increase in compulsory education, which varies slightly in our different countries (see Table 2) but is considered a right and thus can be free in all of them.

Our cross-national analysis shows that at a first level, socio-political changes (with variable views on education) affect educational policies and reforms. At a second level, policies and reforms, in order to be implemented, require efforts, financial support (subject to variable economic conditions) and time; thus, they often face difficulties in being realised and are rarely fulfilled before a new political change upturns the policies. For example, in Portugal, the 25th April revolution of 1974, that resulted in a democracy in that country, was a significant political change that promoted equal education and had great impact on schools. After 1991, curricular

revisions in Portugal have been done often (with almost each change of the government's political force) subject, in the past decade, to public discussion from teachers, university researchers, etc., with comments considered by policymakers. Also, in Mexico, educational policies (particularly those for technology integration in schools) have been subject to political currents, leading them to be "uneven", with related consequences in their corresponding implementations (for a brief account, see Sacristán et al., 2021).
We now describe briefly details of key historical turning points that we consider have had influences on initiatives to integrate digital technologies in education, in general, and for mathematics and/or science.

3.1 Mid 20th century to turn of the century

After the Second World War, teachers and teacher educators in many countries were involved in various reforms and initiatives revolutionising education. In the 1960s and early 1970s, there were important changes, when mathematicians started to take interest in education (Blum et al., 2019). For example, in Portugal, the 1963 "reform of Sebastião e Silva", addressed not only the mathematics to be taught but also how it should be taught. In Italy, the University Didactical Research Groups (Nuclei di Ricerca Didattica –NRD) assumed an important role in elaborating, experimenting and analysing studies and teaching activities for mathematics education at school level. In Colombia, important reforms took place at that time with the creation of high-schools and technical schools. And in Mexico, a group of mathematicians developed the mathematics compulsory textbooks and launched research in mathematics education (Filloy Yagüe, 1981).
From the end of the 1970s to the beginning of the 21st century, initiatives to reform school curricula, as well as large projects to integrate the then "new" digital technologies, were launched in all of our countries. At least for mathematics, these latter initiatives were led by teams that included researchers and members of higher education institutes, mathematics teachers and policy makers. We now focus on some of these.

3.1.1 Major revisions to education programs and integration of DT for mathematics at the end of the 20th century

In Italy and Portugal, significant initiatives took place in the early to mid 1980s. In Italy, national programmes were launched (see timeline in Appendix); the ones for mathematics were developed taking into account the work and results of the NRDs (Mariotti et al., 2019). A particularly important trend in those programmes was the problem-solving approach (Boero & Dapueto, 2007) and, in this context, new technologies also acquired a prominent role. In 1985, with the experimental project PNI, "*Piano Nazionale per l'Informatica"*, Computer Science was introduced in high-schools (Ciarrapico, 2002). More generally, and to this day, in the national curriculum guidelines "Indicazioni Nazionali per il curricolo" (Ministero dell'Istruzione [MIUR], 2007; 2018), the role of technology is embedded in the notion of the mathematics laboratory (see next section). Likewise, in Portugal, the relevance of problem solving in mathematics and the use of technology began to be emphasised in that decade. In particular, in 1985, an important project (MINERVA) was launched to introduce digital technologies in Portuguese schools; it lasted nearly a decade and had a deep impact (Ponte, 1994). In 1991, other curricular changes in Portugal emphasised the use of DT, with mathematics laboratories created and teaching materials developed; later, a PD program to support teachers (including in DT use) was launched in 1998. The latter, together with, the resources and books developed, as well as the inclusion, in 1997, of

graphing calculators in the final high-school exam (Rocha, 2008), all contributed to increase the use of DT in Portugal.

Starting in the 1990s and until the beginning of the 21st century, both Colombia and Mexico in response to social trends (and similar to what had happened in Italy and Portugal), launched major initiatives for DT integration in several disciplines, with some specifically aimed at enhancing mathematical learning in primary and middle schools. In Colombia, schools, teachers and the general public pushed for the government to consider DT in education seriously: Thus, the Colombian government launched in 1993 the *Misión de Sabios* ("Mission of the Wise"), which recommended providing access to all students to digital resources, as well as teacher education, so that schools and teachers could focus on the development of creativity and innovation through the uses of technology to do science (including mathematics), (Aldana et al., 1996). That initiative was the precursor of many educational policies in Colombia on the use of DT, such as the 1994 "Educational General Law" (*Ley 115* - Congreso de la República, 1994), still valid today, which proposed a compulsory "Technology and Informatics" subject for all students (while also giving content autonomy to schools) and fostered DT-equipped rooms in schools. Later, national curricular guidelines (Ministerio de Educación Nacional [MEN], 1998; and MEN, 2006) recommended the use of digital technologies in mathematics lessons with the creation of mathematics laboratories in schools.

In Mexico, as a consequence of the general 1993 educational reforms, the Mexican Ministry of Education (SEP) launched initiatives to integrate computational technologies to the middle-school level in 1997 (with the Teaching Physics and Mathematics with Technology –EFIT-EMAT– programmes, which lasted for 11 years), and a few years later to the primary level (with Enciclomedia), as well as for *Telesecundaria* schools (for further details, see Trouche et al., 2012; Sacristán et al., 2021). For those programmes, thorough and research-based curricula and digital resources were developed for mathematics. The idea of mathematics laboratories, as well as problem-solving, were also central to the EMAT programme.

3.1.2 The notion of Mathematics laboratories: a common trend

It is interesting how the curricular idea of a *mathematics laboratory*, which emerged in all of our countries during the 1990s, and which would be inevitably linked to the use of DT, was, conceived as both a methodology of how learning is to take place, as well as a space for that learning (e.g., a physical space equipped with adequate resources).

In Italy, this notion of mathematics laboratory was described in the following way:

> In mathematics, as in other scientific disciplines, the laboratory is a fundamental element, understood both as a physical place and as a moment in which the pupil is active, formulates his/her own hypotheses and checks their consequences, designs and experiments, discusses and argues their choices, learn to collect data, negotiate and build meanings, lead to temporary conclusions and new openings the construction of personal and collective knowledge. (MIUR, 2018, p.12)

From this point of view, a mathematics laboratory is somewhat similar to a Renaissance workshop, where apprentices learn by doing and seeing things done, communicating with each other and with experts: the construction of meanings is closely linked, on the one hand, to the use of the tools, and on the other, to the interactions between people. In this sense, resources and

technologies are seen as *tools to learn and do mathematics,* to "experiment" with mathematical ideas, as in a chemistry lab.

However, whereas in the Italian conception, mathematics laboratories can take place in any space or classroom, in the other countries they were conceived as dedicated physical spaces, containing a set of resources for mathematics lessons. In Italy and Portugal, these laboratories include DT, but are not limited to them. The Portuguese idea of mathematics laboratories has been variable, changing even from school to school, with financial and other difficulties encountered that has led to limited implementations. In Colombia and Mexico, in contrast to the general Italian and Portuguese conception of mathematics laboratories, their emergence was linked to the initiatives for DT integration (PINTCM –see section 3.2 below–, and EMAT, respectively).

In Italy, Colombia and Portugal, the mathematics laboratories idea continues. In Portugal, some schools are creating classrooms equipped with a diversity of digital resources, each called "Sala de aula do futuro" ("Future Classroom") (Tudella & Brunheira, 2016).

The mathematics laboratory trend and how they were actually implemented (or not) is a reflection of the forces at play, since time and diverse efforts (e.g., economic) are needed to change teaching practices (e.g., through PD), equip the labs, etc. It also illustrates how digital integration is a consequence of both political decisions and the possibilities of their implementation.

3.2 The 21st century: infrastructure initiatives, resources availability, and the computational thinking trend

In this century, particularly in the last fifteen years, there have been initiatives to promote access and use of DT and Internet. Simultaneously, DT resources have been made available online. On a global level, there has been a trend pushing for computational thinking and coding learning.

In Italy, a noteworthy ministerial initiative launched in 2009 and still active today, Cl@ssi 2.0 (MIUR, 2009), aims at supporting daily school teaching through sustained and widespread use of technology-based learning environments: through such initiative, some students and teachers have been provided with technological and multimedia devices, and classrooms progressively equipped with Internet connections. In 2015, a set of policies known as "La Buona Scuola" was launched by the Ministry of Education, to promote learning scenarios, devices, platforms, research, training, and teaching methodology and competencies (Legge 13 luglio 2015 n. 107, 2015). Part of these was the National Plan for Digital Education (MIUR, 2015) –a comprehensive innovation strategy across the school system to bring it into the digital age (Faggiano et al., 2021). Further opportunities to improve the digital infrastructures, renovating school laboratories to adapt them to innovation and creativity, have been given to schools, although with inadequate investments. For example, when projects were called in 2021 for creating laboratories and providing DT for STEM learning, only 8% of the state-run schools (half of those that applied), received a small amount of funding (MIUR, 2021). On the other hand, two seminal national projects, funded by the MIUR and developed in collaboration with the NRD: the m@t.abel project (Progetto m@t.abel, 2015) and PQM project (Progetto Qualità e Merito, 2015) have provided mathematics teachers with free access to online digital resources; in these projects, technology was an added value to support well-defined teaching strategies, focused on the students' learning process (Arzarello & Bartolini Bussi, 1998). More recently, initiatives have sought to foster the teaching of coding, computational thinking and digital literacy across

different disciplines; however, none have been planned specifically for mathematics education (Nardelli & Ventre, 2015).

In Portugal, a 2007 initiative, *e-escola* (e-school), aimed to provide students with free laptops and internet access at home at reduced price (Presidência do Conselho de Ministros, 2015); but this did not change school practices, particularly considering that even electrical outlets were scarce in many schools. The 2013/2014 curricular reform (Ministério da Educação [ME], 2013 for basic school, and ME, 2014 for high-school) gave priority to mathematical formalism that led to a reduced emphasis on technology use. This was reversed in the 2018 curricular adjustment (ME, 2018), where guidelines gave importance again to the role of technology; and the 2021/2022 curricular reform (ME, 2021 for basic school, and ME, 2023 for high-school) highlights DT use and integrates computational thinking (among other aspects). Also, by 2024, all national assessments will be done online (some already began in 2021).

It is also worth noting that, nowadays, for the case of European countries, the EU promotes collaborative initiatives, collaborations, PD, as well as infrastructure development for innovative approaches to education and uses of technology, where experts, policymakers, and practitioners rethink the role of DT in the classrooms (see European Schoolnet, 2017).

In Colombia, the 1995 and 2007 TIMSS results identified problems that led to future policies for providing schools with computers and teachers with PD. From 2000 to 2004, the Colombian ministry of education launched a national PD program (the "*Proyecto de Incorporación de Nuevas Tecnologías al Currículo de Matemáticas*" –PINTCM) focused on integrating DT into mathematics classrooms (Castiblanco et al., 1999). Although a PD programme, it had a strong impact on the schools and universities involved (see Faggiano et al., 2021) and popularised the idea of mathematical laboratories (see similar, also, to Italy's conception of those). Another large program that has helped provide infrastructure, and also promotes teachers' digital skills, has been the "*Computadores para Educar*" (Computers to educate) programme, launched in 2002 and still ongoing (Ministerio de Tecnologías de la Información y las Comunicaciones [MinTIC], 2022). In 2007, an educational network and repository "Colombia Aprende" (Universidad Nacional de Colombia, 2018) was created, where there is currently a variety of content, tools and resources, and where teachers can also share their own digital resources. In 2014, the "National System of Educational Innovation with the use of ICT" (Ministerio de Educación Nacional [MEN], 2014) promoted the creation of regional educational innovation centres in which teacher training was provided with an emphasis on the production of digital content; however, this was largely abandoned in 2019 when the government changed. Since then, other national and regional policies have been proposed to promote "digital education with the use of ICT" (MEN, 2014, but have failed to lead to high-impact programs and initiatives).

In Mexico, despite a May 2006 decree that recommended DT integration and explicitly stated that schools and students should have the necessary access to the EFIT-EMAT programmes (Secretaría de Educación Pública [SEP], 2006, p. 39), most of those projects (and accompanying resources) were abandoned at the end of 2006 when the federal government changed. Nevertheless, important lessons can be learned from those initiatives. Since 2007, with each change of government, similar to what happens in Portugal, there have been changes to educational policies every six years (see Gómez Collado, 2017; Latapí, 2004), with a lack of continuity to previous policies; this has affected DT integration in schools. A 2013 major educational reform extended compulsory education until the end of high-school, addressing teacher recruitment, and pushing for national assessments; but DT were barely considered. The

latest 2019 reform, called the "New Mexican School" (*Nueva Escuela Mexicana,* SEP, 2019a), revamped (yet again) the entire curriculum (meant to replace the current 2017 curriculum, SEP, 2017); although this new model claims to integrate thousands of digital resources (https://nuevaescuelamexicana.sep.gob.mx/), an analysis reveals that these are mostly video lessons ("*Aprende en Casa"*) produced during the COVID-19 pandemic (see section 6). So, since 2007, meaningful technology integration is mostly lacking in all school subjects, including mathematics, with only very general policies for digital integration. On the other hand, for high-schools, a recent reform (SEP, 2022), resulting from a collective discussion between teachers, experts and authorities, defined common curricular guidelines (MCCEMS); these include innovative learning approaches, emphasise technologies and, for mathematics, problem-solving and modelling.

3.3 Remarks on our countries' latest initiatives

It is interesting that after the push to change pedagogy through the use of DT at the end of the 20th century, many recent initiatives focus more on providing infrastructure. However, providing infrastructure does not address how DT can enhance (mathematical) learning. And, as shown above, government changes lead to changes in policies and programmes.

Concerning access to digital resources, in the past decade, with the spread of the Internet, there are more opportunities for teachers to collaborate, as well as more available digital resources; however, this also has created more difficulty for teachers to choose quality resources for mathematics. Therefore, teacher training is very important (see section 4 below).

Finally, it is worth noting that the global trend of coding and computational thinking has only been promoted in Portugal and only tangentially in Italy.

4 Teacher education and recruitment

We cannot talk about policy and implementation regarding digital technologies in mathematics education without considering how teachers are trained and selected, discussing PD both for pre-service and in-service teachers. So, we focus now on issues from the category "Teacher education and recruitment". In Table 5, we summarise the general structure and policies, in our countries, regarding teacher PD (including those concerning DT) and recruitment for state-run schools.

Continuing professional education (CPD), as well as having support mechanisms (e.g., from experts and/or universities and from educational systems) have also been found to be important for the integration of DT recommended in policies and curricula, or beyond. Nowadays, particularly since the pandemic, there are also online opportunities for CPD (such as in the case of Italy –see section 6) and communities of practice that can support teachers in their work. However, we find that PD and teacher support is uneven in our different countries (which illustrates the impact of both policies and socio-economic possibilities for teacher education).

Table 5. Teacher PD and recruitment for public schools

	Italy	Portugal	Colombia	Mexico
Training institutions	Between 1999-2009, there were regional teacher training schools (within larger universities) for obtaining that postgraduate qualification. Stopped for political reasons but will be re-introduced, in a slightly different form.	For K-6, training is done mainly in Polytechnic schools. For grades 9-12, training is done in universities at Faculties of Sciences or Education.	There are specialised pre-service teacher training universities, such as the Normal Schools, or the National Pedagogical Universities. "Normal" schools are responsible for pre-service teacher education (for PS in Colombia; in Mexico, there are such schools for PS and for MS). Faculties of Education in universities or the National Pedagogical Universities provide training for teachers at all levels (K-12) and all teaching areas.	
DT in pre-service PD	Depends on level and teacher training university. Usually GeoGebra (MS and HS), sometimes Graphing Calculators (HS) and Programming (Scratch for lower levels; Python for upper levels), rarely sensors, virtual reality (PS), augmented reality (HS) and serious games.	Depends on level and teacher training university. Usually, Graphing Calculators (HS), Sensors (HS), Excel (all), GeoGebra (all), Programming (Scratch for lower levels; Python for upper levels).	Depends on level and teacher training university. In Normal schools, little DT. In universities, usually, Graphing Calculators (e.g. Desmos) (HS), Excel (MS & HS), GeoGebra (all). Programming (Scratch for lower levels; Python for upper levels).	For BS: Usually general courses on DT in education only. Since 2022, also courses on virtual & hybrid education & each state may have additional courses. No particular DT resources are specified.
DT for in-service teachers (CPD)	In-service teacher training is not nationally regulated. Is mostly provided by universities or private institutions and/or associations. No systematic programmes in the teaching and learning of mathematics with DT. Only episodic initiatives for DT integration training, with ministry of education support and EU funding.	CPD is mandatory for career advancement. Different institutions offer DT training courses. Usually in a specific DT resource. Training centres of groups of schools, get government support and offer free CPD DT courses. Other training centres are created by societies (e.g., Associação de Professores de Matemática -APM; or Sociedade Portuguesa da Matemática -SPM). These, and also universities, sometimes provide free courses, other times paid.	Free CPD for DT in teaching is done by national & local governments usually every year. Universities' master's degrees for teachers generally promote the use of DT.	For BS: Almost none by official institutions. Some online courses by Ministry of Education (but low attendance). Some universities' master's degrees for teachers may include some use of DT. For HS. For the new 2023 MCCEMS, general CPD is being provided and includes some DT.
Requirements to become teachers	A relevant master's degree + a qualification for teaching (acquired by training and exams). - For K & PS: 5-year programme in Primary Education Sciences (includes apprenticeships; ends in master's degree & teaching qualification). - For MS & HS: master's degree in maths + additional university courses in anthropology, psychology and pedagogy, as well as in methods and technologies for teaching contents, and pass qualifying exam.	- For Grades K-6: 3-year degree in Basic Education + master's degree (of at least 1 year). - For MS & HS (grades 7-12): 3-year degree in Mathematics + 2-year master's in Mathematics Education. In all cases, training includes apprenticeships where future teachers have supervised in-classroom experience.	- For Grades K-5 (PS): 5-year degree in Education and/or Mathematics Education + 3 years of teaching experience. - For MS & HS (grades 6-11): 5-years degree in Mathematics, Mathematics Education or Engineering* + 3 years of teaching experience. * Those with engineering degree, need an additional 1-year education course.	Since 2013 reform: -For PS: A bachelor degree in education - For MS: a degree in teaching of mathematics; or Mathematics, Engineering or Architecture + additional minimum 120h education course. - For HS: no specific requirements
Certification exam	Yes for all levels.	No (discussed but never implemented).	Yes (Prueba Saber Pro).	Yes (since 2013) for BS.

Public competition for teaching position	Yes (if needed, schools can temporarily hire additional teachers directly).	Yes (if needed, schools can temporarily hire additional teachers directly).	Yes (Concurso público docente) (If needed, local authorities can temporarily hire additional teachers directly).	No. Just a waiting list. HS hire directly, initially temporarily.
Induction period after recruitment	New teachers need to complete 1-year induction period with a positive assessment.	Legislation (MEC, 2012) proposes 1 year induction for new teachers, but has never been implemented, (Currently is being reconsidered).	New teachers need to complete a 1-year induction period with a positive assessment.	
Evaluations of teachers	No.	Yearly (needed for career advancement).	Yearly (needed for career advancement).	No. (There was an attempt between 2013-2017, but abandoned).

Codes in table: K: pre-school; PS: primary-school, MS: middle-school, HS: high-school, BS: basic school level, DT: digital technologies

In Italy, teachers in state-run schools are public employees with a national collective labour contract that regulates their conditions of service. According to that contract, CPD is both a right and a professional duty, as it contributes to the development of teachers' professional life. Thus, CPD is compulsory for all teachers in state-run schools. Some initiatives for CPD in the teaching and learning of mathematics with DT are developed with the support of the MIUR and of EU funding, but they are mainly episodic and not equally distributed in the Italian territory.

In Portugal and Italy, DT are part of the initial training for teachers. Depending on the institution offering the training, it can be addressed in a specific course or as part of Didactics of Mathematics courses. In Portugal, for CPD, different institutions provide technical knowledge as well as knowledge about how to use the resource to teach mathematics. Graphing calculators and GeoGebra are among the most used technologies. More recently there are also several courses (25-50h long) devoted to programming in Python or Scratch, which often require implementing some tasks with students. Currently, teachers are receiving training for general DT proficiency (e.g., the use of email, Moodle, etc.), attending to a government concern of their digital skills.

In Colombia and Mexico, it is worth noting that pre-service teacher training universities (called "Normal schools") do not offer much content related to DT in mathematics education (at most, a general pedagogical use of DT). Degrees in mathematics education are offered by universities and include didactics of mathematics (including DT). Particularly in Colombia, many primary teachers and mathematics teachers pursue a master's degree in mathematics education (for which scholarships are available); and many of those master's programmes have some emphasis on the use of DT for mathematics. In general, CPD in Colombia is mainly done by universities and higher education institutions: two significant programmes for training teachers in the use of DT for mathematics were the PINTCM (see section 3.2) and the 2002 "Teacher Training on the Use of New Technologies in the Mathematics" curriculum (MEN, n.d.).

For basic schools in Mexico, after an intensive teacher training phase in DT for mathematics in the early 2000s, attached to the EMAT and Enciclomedia programmes, there has been very little training in that area for the basic school level. This is despite the fact that in-service teachers are offered CPD through the Programme for Teachers' Professional Development (PRODEP), which offers general courses (very few on DT) created by different regional (state) governments and available online through a website (SEP, 2020). Thus, we note that training in math-oriented DT is scarce in pre-service courses and CPD is also limited.

We note in our analysis, differences in our countries' recruitment and professional development of teachers that may affect DT integration, even between Colombia and Mexico, which have similar cultures. Some of these differences are due to differences in educational conceptions and policies, but they also reflect access to resources and other socio-economic issues (as discussed in Sacristán, in press).

5 Textbooks and assessment

In this section, we examine some issues (see Table 6) from the two analysis categories "Textbooks" and "Assessment", which follow the national curricular guidelines and have an important influence on teaching practices and DT adoption.

In all of our countries, free textbooks are provided to students at certain levels. Some educational authorities issue their own textbooks (e.g., the ministries of education of Mexico and Colombia publish textbooks for primary-schools); but, in general, commercial publishing houses do so according to the national guidelines (in that case, the cost is covered either by the parents or by some governments –e.g., in Italy for primary-schools, in Portugal for students at public schools). At least for the basic school level, textbooks are mandatory; except in Colombia where their use is not compulsory. If there is a choice of textbooks, either each school, or each teacher (see discussion on autonomy in previous section), may decide which to adopt.

Table 6. DT (particularly math-oriented) in curricula and textbooks

		Italy	Portugal	Colombia	Mexico
	School level				
DT included official curricular guidelines	**BS**	Yes	Yes	Yes	No. But for 2023, STEM is proposed.
	HS	Yes	Yes	Yes	New curricula (MCCEMS) to begin in 2023, emphasises a "digital culture"
Types of recom-mended DT	**PS**	Calculators	calculators; robots; applets; DGS: GeoGebra; visual programing environment (Scratch); data bases (Pordata Kids, Dollar Street); spreadsheets.	Official national repository and applets.	N/A
	MS	Calculators, spreadsheets, dynamic geometry	Calculators; applets; DGS: GeoGebra; visual programing environment (Scratch); data bases (Pordata, INE, ALEA); spreadsheets; camera; Internet	Official national repository, applets, GeoGebra, Excel.	N/A
	HS	Graphing calculators, spreadsheets, CAS, dynamic geometry	Graphing software; CAS; graphing calculators (also used in grade 12 exam); sensors; spreadsheets; DGS (GeoGebra); applets; Python; smartphones; Internet.	Same as for MS + graphing calculators.	For the MCCEMS curricula, nothing specific is mentioned

		Italy	Portugal	Colombia	Mexico
Official Textbooks	**Requirement**	- Mandatory, - Free only for PS students	- Mandatory, - Free for students in state schools,	- Not mandatory, but available. - Free for students in state schools,	-Mandatory for BS only -Free for all (government issued) in PS - For MS, local governments sometimes provide them in state schools
	Selection	Teachers collective in each school select from authorized publishers' versions (for MS only, in case of Mexico)			
	Digital versions	All countries have digital versions			
	DT included in textbooks	Some content (especially for MS. and HS): e-textbook usually accompanied by interactive tasks for self-assessment	Some tasks propose the use of technology. The teachers e-textbook has links to some digital resources	Some content use of DT, and some interactive textbooks.	No content in PS
DT resources	**DT Repositories**	Developed in context of some projects (e.g. m@t.abel, PQM…); also in collaboration with the Italian Mathematical Union	Developed by the houses publishing the textbooks and in the context of some projects	Yes. There is a national repository	Mostly only videos
	Other resources available	Resources developed by the publishers of textbooks or within the University Didactical Research Groups	Teachers have access to applets, GeoGebra files, PPTs and platforms with a diversified set of resources developed by the authors of the textbooks or the houses publishing them	Yes. Some universities have their own digital repositories	Enciclomedia, EMAT, but no longer officially used

Codes in table: K: pre-school; PS: primary-school, MS: middle-school, HS: high-school, BS: basic school level , DT: digital technologies,

Textbooks have an important influence on teaching practices, often becoming models of instruction, as stated by Valverde et al. (2002) –who studied how textbooks interpret and translate policy into practice and pedagogy across countries. Valverde et al. (2002) also point out that textbooks interpret and reflect content in a way meant to be suggestive of the enactment of intention of curriculum policies. The use of textbooks particularly impacts the use of digital technologies.

Although the designers of the textbooks vary, often teachers write or participate in their design. This means that they may be limited by their own experience and thus may not consider novel approaches (such as how to integrate DT –see the Portugal example in section 7.2, which could be done with paper-and-pencil). Also, both publishers and teachers are used to certain types of traditional textbooks, so change is difficult. This prevents significant changes in textbooks (as well as in assessments); while the reliance on textbooks and assessments also prevents changes in classroom practices.

In most of our countries, digital resources do not appear in a prominent way in textbooks[1]. Even when the national guidelines or curricula suggest the use of DT, how this is integrated in

[1] This may change in Portugal with the newest reform.

textbooks is up to the designers. Unfortunately, we could not do a systematic analysis of the textbooks, but we can point to some observations: What is included varies in quantity (in general, there is, at most, a passing recommendation or example) and quality (e.g., some tasks are the same as pencil-and-paper tasks with DT as an add-on).

Some textbook authors create a symbol (often the image of a computer) with links to DT resources or video presentations. In Italy, even if it is possible to find tasks in the textbooks requiring the use of technology, most of the marks in textbooks only link to digitalised versions of the textbooks or additional exercises. In Portugal, these symbols refer to tasks requiring the use of DT (usually, GeoGebra, Excel and Scratch/Python); most are explorations or modelling tasks. At high-school level, in Portugal, the use of graphing calculators is mandatory (including in exams), so it is common to have tasks (or parts of them), requiring their use.

As said before, many teachers rely heavily on textbooks to structure their lessons; in particular, they are often used as "scripts". This is also true for examinations: the learning goals presented in the national guidelines become criteria to be assessed. In that sense, exams are prescriptive, and engage schools to attempt to achieve that each pupil can attain them. Assessments are of several kinds: regular school exams, regional or national exams, and participation in international evaluations.

In Italy, Colombia and Mexico, these are classical paper-and-pencil exams that generally do not use nor take into account digital resources; nevertheless, we mention these because they can affect teaching practices. In general, regular assessments (mid-term and/or end-of-year) evaluate the learning aims established in the national curricular guidelines; but schools may define the methods and criteria for carrying them out.

In Italy, the end of high-school (grade 13) exam for scientific schools includes a test in mathematics and/or physics, for which, since 2017, graphing calculators without CAS are allowed. In Portugal, as said before, for the national end-of-high-school exam (grade 12) the use of graphing calculators (without CAS and in exam mode) is mandatory with at least one question that cannot be answered without it.

Additionally, in all of our countries, students take part in other periodic national standardised tests (see Table 7), which aim to assess the efficacy and improve the quality of the educational system as a whole (as well as individual pupils' learning, in the case of Colombia and, previously, in Mexico) through students' learning attainments in mathematics, the mother tongue and/or science.

Table 7. Assessments

	Italy	Portugal	Colombia	Mexico
National examinations	"Prove Invalsi" for grades 2, 5, 8, 10 & 13 – https://www.invalsiopen.it/	"Prova de aferição", for grades 2, 5, 8 & "exame nacional" grades 9, 12 – https://iave.pt/provas-e-exames/provas-e-exames/	ICFES or "Pruebas Saber" for grades 3, 5, 7, 9 and 11 – https://www.icfes.gov.co/	Between 2005-2019: ENLACE & Excale, later PLANEA. Mostly abandoned by current administration, except in some high schools. – http://www.inee.edu.mx/evaluaciones/
Exams include tasks with DT	No	Yes: graphing calculators for grade 12	Recommended (up to the teachers)	No
Online assessments for math	"Prove Invalsi" are computer-based for grade 8 and 10 since 2018 and for grade 13 since 2019.	From 2024/2025, all online. A pilot program is ongoing.	Partially: a pilot program from 2020 "Evaluar para avanzar".	No

In Italy, standardised tests, called “prove INVALSI” are mainly considered by teachers as a duty rather than a didactical resource (i.e., teachers do not structure their lessons to cater for them; Faggiano et al., 2023). This is not the case in other countries, where teachers tend to structure their lessons so as to prepare students to pass tests. However, although teachers in Italy do not use standardised tests as teaching resources, their teaching in the scientific high-school is mostly devoted to preparing students to take the final grade 13 mathematics state examination. In Portugal, Colombia and Mexico, teachers teach according to exam requirements.

Also, in Mexico and Colombia, teachers are required to deliver evaluations regarding their teaching outcomes, so they use commercially-produced tests that follow the national curricular guidelines, and present their students’ results to do this. These tests have also often been used to train students for national and/or international standardised assessments (TIMSS or PISA), and also may determine how teachers structure their courses.

Thus, a common issue is that the content of classes is a preparation for tests, rather than aimed at improving (mathematics) learning. And if teachers are too busy following textbooks and/or preparing students to pass tests, then there are fewer opportunities for DT integration.

6 Digital technologies in official guidelines and repositories

In this section, also part of the “Textbooks and other resources” category, we focus on how each country recommends and provides for the use of digital technologies. We reflect also on technology use during the COVID-19 pandemic, as well as general challenges for integrating the recommendations in practice.

In general, there are two or three levels of technology policy and implementation (depending on the size of the country): The national level, the regional level and the school level. Schools are generally equipped with technological resources funded by governments through the ministries of education or regional/local administration. The national policy usually provides a national budget that may include technology provisions. In Colombia and Mexico, which are geographically larger countries, local governments (region/province/city) need to foster the implementation of national policies, making decisions based on their situations, buying equipment, allocating resources, etc. Then, at the school level, school administrators and teachers need to learn and implement technologies as encouraged and/or supported by national policies and local governments. Italy and Portugal, being smaller countries, have only two levels of policy and implementation: the national (non-regional) one and the local (school/teacher). But because they are European member states, although each country has its own educational policies, they can be involved in initiatives and receive a budget from the EU to improve the quality and accessibility of education (including some related to technology integration).

6.1 Educational consequences of the COVID-19 pandemic

Before continuing this section, we need to take into account some educational consequences of the COVID-19 pandemic, where students were isolated at home, and teachers had to rely on ICT as a means to conduct distance education. The pandemic caused governments to develop emergency policies to ensure some continuity in education, and an urgent need for ICT. It also revealed existing limitations of DT integration in schools, as there was much unpreparedness to deal with the necessary and sudden distance education (made worse by the lack of solid previous

digital integration in education). In this sense, it pushed the use of technology, although not towards a meaningful use for mathematical learning (Drijvers et al, 2021).

The pandemic situation put in evidence socio-economic diversities, the lack of Internet connections and the low availability of digital devices at home (for the case of Mexico, see Sacristán et al., 2021), which made it difficult to connect with teachers and others.

Mobile smartphones became an important tool, particularly in Colombia and Mexico. Teachers used tools like WhatsApp to communicate with their students and send them worksheets, and students/parents sent homework back (e.g., snapshots of solved printed worksheets) to the teachers through those same means. Also in Italy and Portugal, WhatsApp and email were relied upon at the beginning of the pandemic (see Rocha et al., 2021, for Portugal).

As said above, national (and local) governments promoted important emergency initiatives (often expanding previous programmes). Some efforts to bring computers and the Internet to the neediest students were made in some places, albeit insufficiently. To ensure that those without internet access could keep learning, TV channels (as described further below) were used. For students with Internet access, the use of online platforms such as Google Classrooms, was also promoted.

In all of our countries, TV lessons and on-demand videos were produced in cooperation with educational authorities, and given by teachers, shown on national/regional TV and generally available on the Internet. In Italy, short TV thematic lessons were produced mainly for high-school and given by experts (https://www.raiscuola.rai.it/tags/matematica). In Portugal, "*Estudo em casa*" (Study at home – https://www.rtp.pt/play/estudoemcasa/) was used for all school levels. In Mexico (with "*Aprende en casa*" –Learn at home, https://laescuelaencasa.mx/escuela-en-casa/niveles/index.html; https://www.youtube.com/c/aprendeencasa/playlists) and Colombia (with "*Profe en tu casa*" –Teacher in your home, https://www.senalcolombia.tv/programa/profe-en-tu-casa), these TV programmes became the main means of content delivery for primary and middle-schools.

In the following section we describe how digital technologies are presented in the official curricular guidelines for mathematics, with a focus at the end of primary-school and/or beginning of middle-school. We also mention some of the main repositories of digital resources that are recommended and/or supported by each government or by private organisations such as publishing houses or professional societies, and which include, for example, digital versions of the textbooks, as well as other accompanying resources.

6.2 DT guidelines and resources in each country

6.2.1 DT in Italy's curriculum and repositories

In the introduction of the Italian national guidelines for basic school (MIUR, 2007) that state the goals and aims for mathematics, there are some general considerations regarding the use of technologies. Among them, there is the following statement:

> *The conscious and motivated* **use of calculators and computers** *must be appropriately encouraged from the early years of primary-school, for example to verify the correctness of*

mental and written calculations and to explore the world of numbers and shapes. (p. 94, our emphasis)

The use of technology is considered again in the list of learning aims to be attained at the end of primary-school, for both the themes of "numbers" and "space and figures", as follows:

> *Carry out the four operations with confidence, considering whether to resort to mental calculation, written calculation or with a calculator, depending on the situation. Reproduce a figure based on a description, using the appropriate tools (squared paper, ruler and compass, squares, dynamic geometry software).* (p.95)

While at the end of middle-school the corresponding learning aims are:

- *Perform additions, subtractions, multiplications, divisions, sorts and comparisons between known numbers (natural numbers, whole numbers, fractions and decimal numbers), whenever possible in mind or using the usual written algorithms,* **calculators and spreadsheets and evaluating which tool may be more appropriate.**
- *Reproduce geometric figures and drawings, using appropriate tools (ruler, square, compass, protractor,* **geometry software***) in an appropriate way and with accuracy.* (p. 97, our emphasis)

Some interesting, but underused, repositories of teaching resources are available on official websites from national projects, such as of the already mentioned PQM (Progetto Qualità e Merito, 2015) and m@t.abel (Progetto m@t.abel, 2015). Some other resources can be found on the Italian Mathematics Union's website (UMI-CIIM: https://umi.dm.unibo.it/materiali-umi-ciim/) or that of the DiFiMa project (https://difima.i-learn.unito.it/).

As mentioned in section 3.2, there are also some official ministerial projects on coding and computational thinking in Italy (e.g., see https://www.indire.it/en/progetto/computational-thinking/); however, these are not curricular and mathematics is not mentioned at all.

6.2.2 DT in Portugal's curriculum, and resources

Since 2021, Portugal has been through a curricular change that includes computational thinking as part of the mathematics curriculum (ME, 2021; implemented in 2022), values mathematical literacy, and defines eight objectives that all students must be able to achieve, in particular:

> *Develop and mobilise* ***computational thinking****, a skill that has been gaining importance in Mathematics curricula in several countries. Computational thinking presupposes the integrated development of practices such as abstraction, decomposition, pattern recognition, analysis and definition of algorithms, and the development of debugging and process optimization. These practices are essential in mathematical activity and provide students with tools that allow solving problems, especially related to programming* (ME, 2021, p. 3; our translation and emphasis).

However, the inclusion of computational thinking is an area of concern for many teachers who do not know how to address it.

Besides computational thinking, the official curricular documents clearly value the use of DT. They start with a general idea of the role of DT, addressing what to do with it and which resources to use:

> *Technological tools must be considered as unavoidable and powerful resources for teaching and learning Mathematics. Students' digital literacy should include performing calculations, building graphs, performing simulations, collecting, organizing and analysing data, mathematical experimentation, research and modelling, sharing ideas. All students should be able to freely access* ***calculators, robots, applications available on the Internet*** *and* ***software for statistical treatment, geometry, functions, modelling****, and* ***visual programming environments****. [...] The integration of technology in mathematical activity must be understood as an instrument, not as an end in itself, to promote more meaningful learning and expand the contexts in which the student's action takes place and the diversity of perspectives on the mathematical objects studied…* (ME, 2021, pp. 6-7; our translation and emphasis).

They also include more specific information about DT use for each mathematical topic.

In the last 30 years, curricular changes have also included training programs to help teachers understand the ideas behind each reform, with the development of teaching resources (often available online). Currently, there is a ministry of education repository of tasks (http://aem.dge.mec.pt/pt/recursos) to support teachers, but most of them are paper-and-pencil; only in a few tasks, the use of a DT resource such as a smartphone and GeoGebra is suggested.

Nowadays, publishing houses are also producing teacher support materials: e.g., books or online resources (some as open access, others available when schools adopt the corresponding textbooks). These include books on computational thinking with tasks using, for instance, Scratch, GeoGebra and Excel.

6.2.3 DT in Colombia's curriculum and repositories

In Colombia, the national guidelines (MEN, 2006), as well as the official textbooks, recommend DT use to favour problem-solving and the communication of mathematical ideas in class, but it is not mandatory. For example, in the grade 8 textbook (MEN, 2015), GeoGebra is proposed in the study of different topics (e.g., function); however, its use is scarce for geometry.

On the other hand, the Colombian national government promotes an official teaching repository (www.colombiaaprende.edu.co) that was created, with the support of South Korea, for a project on ICT training. These resources include different kinds of resources for teaching, accompanied by didactical suggestions and complementary activities for students (e.g., lessons planned by teachers and shared with colleagues; other resources like videos, tests and specific apps). However, this repository does not include specific math-oriented software.

6.2.4 DT in Mexico's curriculum and repositories

As in the case of other countries, the introduction of the 2017 national mathematics guidelines (SEP, 2017b) for middle-school, give the recommendations given below (our emphasis). These highlight different digital resources, such as spreadsheets and GeoGebra, and their possible affordances for mathematical activities (although it is noted that this would only be possible if schools have the necessary resources):

> *Using activities that use technological tools it is possible to encourage students to explore mathematical ideas and concepts, as well analyse and model problematic phenomena and situations. The most frequently used tools in the design of activities for learning in*

*mathematics are **spreadsheets**, **symbolic manipulators, and graphing tools**. The free software **GeoGebra** combines the features of the previous programs, which allows you to work with different **dynamic representations** of concepts and situations, such as graphical, numerical and algebraic representation. One of **the didactic affordances of the aforementioned programs is that these representations are dynamically linked to each other.** Through properly selecting activities available on the **internet**, designed with those tools and with other digital applications, the teacher can incorporate their use in mathematics classes when the school has the necessary infrastructure.*
(SEP, 2017b, p. 303)
*It is convenient to **think of situations or activities** that encourage the application of different mathematical tools or **that involve the use of technology.***
(SEP, 2017b, p. 307)

Nothing else is stated regarding technology in the printed guidelines, but in the corresponding website (https://www.planyprogramasdestudio.sep.gob.mx/sec-ae-pensamiento-mate1.html), for middle-school there are a few examples of technology-based activities (called "ICT use"). For example, for the equivalent of 7th grade, for the topic of "Number", an activity to identify which fractions in a given set convert to decimal fractions and which are not possible to develop with the spreadsheet is given. Then, for the topics of "Proportionality", and "Patterns, geometric figures and equivalent expressions", there are references to several activities using Spreadsheets from the EMAT programme (no other technology-based activities are presented). And for the topic of "Figures and geometric bodies" for all three middle-school grades, the website states "It is suggested to work with dynamic geometry programs, such as GeoGebra, and with activities in LOGO". All these references are interesting and show the impact of the EMAT programme; on the other hand, it is striking that no links are given to any of the referenced resources, other than printed pages numbers from the EMAT activity books.

The new program announced in 2019 (see section 3) has a repository (https://nuevaescuelamexicana.sep.gob.mx) but does not include resources for mathematics beyond videos. Regional state governments do have other repositories, but with few digital resources for mathematics other than videos and digitised activities. There are also repositories of digitised versions of the compulsory primary-school textbooks, as well as of the commercial textbooks for middle-school (e.g., https://www.conaliteg.sep.gob.mx/secundaria.html), but it seems a backstep from what Enciclomedia attempted, particularly for mathematics, prior to 2006 (see Trigueros & Lozano, 2012).

6.3 Remarks on the guidelines and resources of our countries

We observe that although the official guidelines of our four countries recommend DT use and usually which type to be used (with DGS, spreadsheets and calculators standing out), only in the case of Portugal it is mandatory. Some repositories are also available in each of our countries, but the ones by official authorities seem limited and lacking mathematics-enhancing software (e.g., DGS) or tasks for those.

7 How a mathematics curricular topic is implemented with technology

To end this chapter, we identified and chose a school topic, present in all of our curricula at the end of middle-school, that we considered could be addressed using digital resources (in particular with dynamic geometry): *Congruence and similarity of figures.* To our surprise, we found scarce examples, in resources for teachers, on approaches to that topic with digital technologies (the most significant ones were from Italy, and Mexico a long time ago). We start with the Italian approach, which we consider the most comprehensive.

7.1 Italy: Teaching method and 8th grade tasks for introducing the concept of similar triangles using dynamic geometry

This example comes from the repository of the ministerial project PQM (Progetto Qualità e Merito, 2015), with the participation of several schools, and which includes many resources, designed by a group of expert teachers, for promoting learning in the logic-mathematics area by exploiting the potential of new technologies.

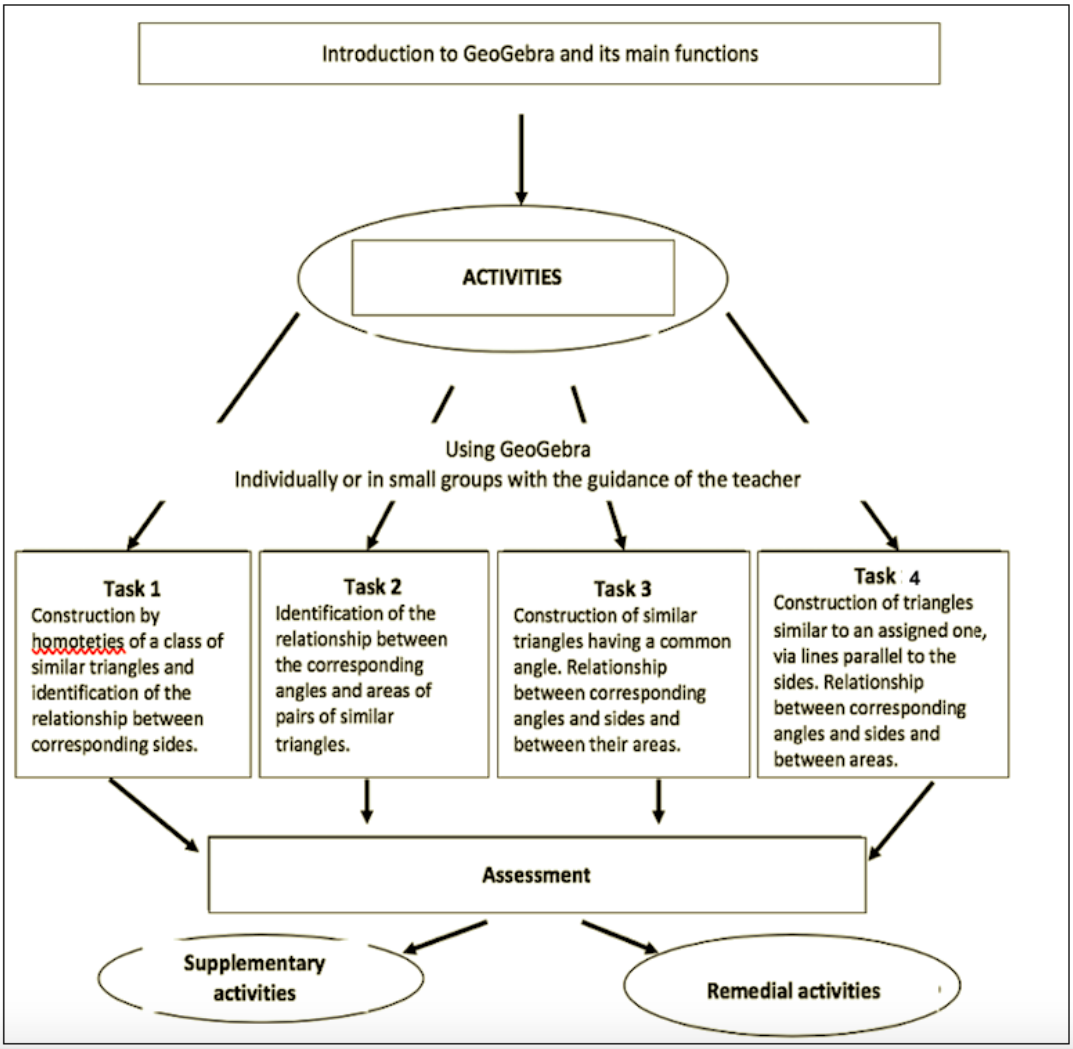


Figure 1. Structure of an Italian teaching proposal with GeoGebra on Proportionality in Geometry. (Translated from Progetto Qualità e Merito, 2015)

The teaching proposal (http://www.scuolavalore.indire.it/nuove_risorse/proporzionalita-in-geometria-con-geogebra/) that we present, uses dynamic geometry (GeoGebra) to introduce 8-grade students to the concept of Similar Triangles and their properties (in the area of proportionality in geometry). It is designed to be developed through five activities/tasks (including a brief introduction to relevant GeoGebra functionalities) (see Figure 1), lasting around twelve hours (including the final assessment and further remedial or supplementary tasks). As shown in Figure 2, it includes methodological notes.

To encourage initial peer discussion on what is proposed in the various activities, pupils will work in pairs (1 worksheet per pair). The activities are largely heuristic in nature and are, usually, organized according to the following scheme:

1. analysis/reproduction of a drawing.
2. structured reflection on the assigned task.
3. discussion, with the teacher, on the proposed solutions.
4. definition, by the teacher, of the concepts introduced.

The moments of discussion are usually placed at the end of each activity; sometimes they are anticipated to meet the needs of pupils who would find it more difficult to work independently.

In such a kind of activity, the teacher's role is crucial, as she needs to manage the reflections and comments from the pupils in an effective way, to arrive at sharing with them a definition of the concept to be introduced.

Figure 2. Teaching organisation and methodology of the Italian proposal. (Translated from Progetto Qualità e Merito, 2015)

The aim of the initial activity is to construct a class of similar triangles (those having a common angle) and identify the relationship between corresponding sides. Students are first asked to use the “dilation” tool and conjecture how it works. Next, through an explorative task, they are led to state their own conjectures about possible regular situations in the various pairs of constructed triangles with respect to the measurements of their sides. Then, through a collective discussion mediated by the teacher, they are expected to arrive at a correct definition of the conjectured property; the teacher will then introduce the term "similar". The activity concludes with the discovery of the properties of similar triangles, giving meaning to the notion of proportionality in geometry, and opening a window on Thales Theorem.

In this proposal, dynamic geometry software (DGS) is used to move directly and indirectly the constituent elements of the figures through drag-and-drop actions, in order to make a conjecture by interpreting the invariants and the relations between them. It is thus an interesting example that shows the essential role that the dynamism of the software can play to allow students to pose and test a conjecture.

7.2 Portugal: Curricular guidelines and a 7th grade school task on similar triangles with dynamic geometry

The Portuguese curriculum (ME, 2021) emphasise that “technological tools should be considered as unavoidable and powerful resources for teaching and learning Mathematics“ (p. 6). This use is reinforced in the context of geometry teaching: “The study of geometric objects must be

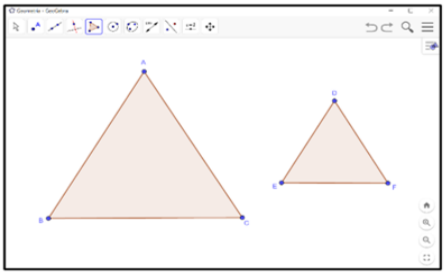
accompanied by experience (where technology plays a key role)" (pp. 10-11). And there are several references to technology use in relation to the teaching and learning of the concept of similarity:

> *Confront the meaning of similarity in real life with that of Mathematics, to promote students' understanding.*
> *Propose, in pairs or in groups, the representation and analysis of enlarged and reduced figures using DGS and other instruments (squared and isometric paper, pantograph, photocopies or digital image manipulation), to identify the invariant characteristics of similar figures.*
> *[...]*
> *Use DGS to encourage construction in pairs of students of polygon enlargements and reductions using the homothety method and dynamic enlargement or reduction factors.* (Translated from ME, 2021, pp. 40-41)

The task (Figure 3) presented in one of the grade 7 official textbooks, asks students to construct pairs of triangles meeting certain conditions. The students are then invited to analyse the pairs of triangles looking for similarity and to try with different pairs of triangles. From the results achieved in this analysis they are supposed to elaborate on the conditions two triangles should verify to be similar. The task promotes students to reflect on the relations between the length of sides of the triangles and the measure of the angles, giving them the opportunity to take decisions and elaborate on the triangle similarity criteria. DGS can be used to experiment with different triangles, offering students the opportunity to explore and formulate conjectures.

Similar triangles

1. Using a dynamic geometry program (such as Geogebra), construct non-congruent pairs of triangles [ABC] and [DEF], such that:



1.1. the sides of the triangle [ABC] measure three times the sides of the triangle [DEF]

1.2. $B\hat{A}C = E\hat{D}F, \overline{ED} = \frac{\overline{BA}}{2}$ and $\overline{DF} = \frac{\overline{AC}}{2}$

1.3. $B\hat{A}C = E\hat{D}F$ and $C\hat{B}A = F\hat{E}D$

1.4. $B\hat{A}C = E\hat{D}F, \overline{ED} = 4\overline{BA}$ and $\overline{EF} = 4\overline{BC}$

2. On each pair of triangles drawn, if necessary, measure sides and angle amplitudes.

Determine the ratio between the lengths of the sides of each pair of triangles represented.

Are the triangles similar?

Experiment with different triangles under the same conditions.

3. Based on the results obtained in the previous question, what conditions do you suggest should be verified when one wants to recognize whether two triangles are similar?

Translated and adapted from Almeida (2022, p. 130)

Figure 3. A Portuguese task on similar triangles from a 7th grade approved textbook (Translated and adapted from Almeida, 2022, p. 130)

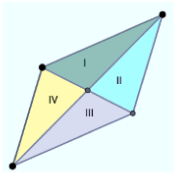
7.3 Mexico: Curricular guidelines and middle-school tasks with dynamic geometry, on the congruence of triangles and quadrilaterals

The 2017 Mexican mathematics curricular guidelines for the equivalent grade 9, for the topic of "Form, space and measure - Figures and geometric bodies", state as learning aim that the student "Builds similar polygons. Determines and uses similarity criteria of triangles" (translated from SEP, 2017b, p. 324); the corresponding website suggests the use of GeoGebra or LOGO. However, we couldn't find any recent activities on this topic. There are, however, two from the EMAT middle-school Dynamic Geometry activity book (Zubieta et al., 2000), that we present in Figure 4. These tasks direct students in the constructions with DGS and pose questions for reflections, explorations and for finding justifications (if not proofs) of why the figures are (or not) congruent.

Triangles and quadrilateral: Figures directly or inversely congruent

Aim: Distinguish when two figures are directly congruent or inversely congruent.

How are the triangles formed by the diagonals that cross the rhombus shown? Some are directly congruent, while others are inversely congruent.

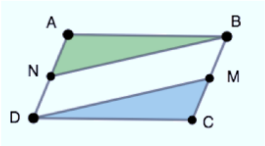


- If the point of intersection of the diagonals is the common vertex of the four triangles, what is the value of the angle, at this common vertex, in each of the four triangles?
- Therefore, to classify triangles as directly or inversely congruent, a respective rotation or reflection will suffice.
- Which triangles are directly congruent?
- Show the above using the ROTATE command and describe what happens.
- Which are the inversely congruent triangles?
- Show the above using the REFLECTION command and describe.

Triangles and quadrilateral: How to check the congruence of figures

Aim: To use translation and rotation to verify congruence between two figures.

The drawing shows the parallelogram ABCD and the midpoints of the opposite sides BC and AD.



- How are triangles ABN and CDM related to each other?
- Can there be a rotation that puts one triangle on top of the other?
- What point could be the centre and what is the value of the angle?
- To place one triangle on top of another, you could also perform a translation and then a rotation (or vice versa); in this situation it would be necessary to indicate with a vector the direction of the translation.
- Could you point out that vector in the drawing?
- Then, you would have to choose a point as the centre and the value of the angle for that rotation. What would it be?
- Try to do it with both methods and describe what happened.

Figure 4. Mexican middle-school dynamic geometry tasks on congruence of triangles and quadrilaterals (Adapted and translated from the EMAT book by Zubieta et al., 2000)

7.4 Colombia: Curricular guidelines and interactive apps for the teaching of similarity in 8th grade

In the Colombian mathematics curriculum, the teaching and learning of similarity is considered as a fundamental spatial thinking content, from the first to the last grades of basic school. In middle-school, for grades 8-9, the guidelines (MEN, 2006) state as learning aims: "*I conjecture*

and verify properties of congruence and similarity between two-dimensional figures and between three-dimensional objects in solving problems" and it is intended that students apply and justify the use of congruence and similarity criteria between triangles.

Looking for teaching resources with DT for such topic, it was noteworthy that the national official repository (https://www.colombiaaprende.edu.co/en/node/91283) does not include anything using DGS (nor for any other topic); this is because the ministry wanted to create its own DT resources. However, the repository does provide 8th-grade interactive apps for the topic (see Figure 5). However, these could be considered as computer-aided instruction (CAI) tasks, that translate what could be solved with paper-and-pencil, rather than provide any technology-enhanced mathematical thinking.

Figure 5. An activity on similarity of figures from the Colombian national digital resources' repository.

7.5 A reflection on the examples

Both the Italian and Colombian examples are resources for teachers that include didactical suggestions, although the Italian one highlights a complete methodology for technology integration. In contrast, Portugal and Mexico's examples are tasks for students only.

We observe that three of the examples propose explorations with DGS, but with very different approaches. In the case of Mexico, it is a guided construction with questions directed towards specific properties. In the Portugal example, the approach is more open and exploratory, and the questions are not only for identifying the conditions needed for triangles to be similar, but on creating an awareness of why they are true. Finally, the case of Italy also promotes constructions for identifying geometric properties; but the example stands out because it includes heuristics and explicitly promotes a working methodology (including collaborative work and discussion).

8 Final remarks

In this chapter, through a cross-national analysis of the situations in our countries, we identified factors that influence policies and implementations of DT in mathematics teaching-learning practices, that may go beyond differences in history and culture. The factors include: socio-political changes; educational systems; the needs to comply with national curricular guidelines and/or expected outcomes (e.g., assessments), which shape classroom practices; issues of DT access and time, as well as of school and teachers' autonomy (and freedom to innovate); teachers' PD (which affect their PTK).

As recognised by the OECD, there is a gap between the intention, and the implementation of a renewed curriculum (Gouëdard et al., 2020). Indeed, in our analysis, we found that there is a

discourse in policies to promote digital technologies' use, but in practice the availability and integration of such resources in mathematics classrooms is still scarce (even the efforts during the pandemic didn't change this, promoting general ICT use, rather than math-oriented DT resources e.g., for content-learning). Policies usually place more emphasis on providing access to technology rather than on providing support for DT integration (such as through PD) –Trouche's et al. (2012) access/support dimension. And, although the trends of society (socio-cultural factors), such as those placing importance on creating digital cultures (including that of computational thinking) are taken into account by policy makers with less or more emphasis, they are still subject to political changes. Different political forces have different views of education (e.g., conceptions of learning, the role of education actors, of assessments, of DT), which lead to changes in educational policies and curricula[2], and affect what happens in schools. We thus see a powerful influence of top-down decisions, whereas the mathematics education needs that may emerge have less leverage (in Trouche's et al., 2012, top-down/bottom-up dimension). Also, research tends to have little impact on policies and practice; indeed, there are factors that hinder bridging the gap between research and practice (Aguilar & Castaneda, 2022). The scarce inclusion of DT in three of our countries' national guidelines for mathematics, and in all of our countries' official repositories, is an example of this, since there are plenty of interesting examples of meaningful DT integration in the literature (including by some authors involved in large projects and/or PD in our own countries).

This contrasts with programmes launched in other countries. In particular, the relatively recent push for developing computational thinking and its uptake in mathematics curriculum policies (see chapter by Tamborg et al., in this handbook), is not taken into account in our countries for mathematics (except Portugal). In that respect, many countries, such as the UK, Canada, Australia, and nowadays also Portugal, to name a few, have developed or adopted curricula that integrate coding in mathematics.

Traditions have a hold (textbooks, methodologies) and there is often resistance to innovative approaches. The "inertia of classroom cultures" makes changes difficult and slow. New changes/policies take time to be assimilated by all the stakeholders, which often is not considered by policymakers, nor can endure government changes. Furthermore, for the uptake by teachers and schools of the novel ideas in national policies/recommendations, a change in the didactical contract and in the teachers' professional attitude is required. As Krainer (2021) says, there is a need to have a negotiation between the policy, practice and research communities.

Acknowledgement

We acknowledge the work of Chongyang Wang, from Beijing Normal University, China, in the discussions and initial drafts of this chapter.

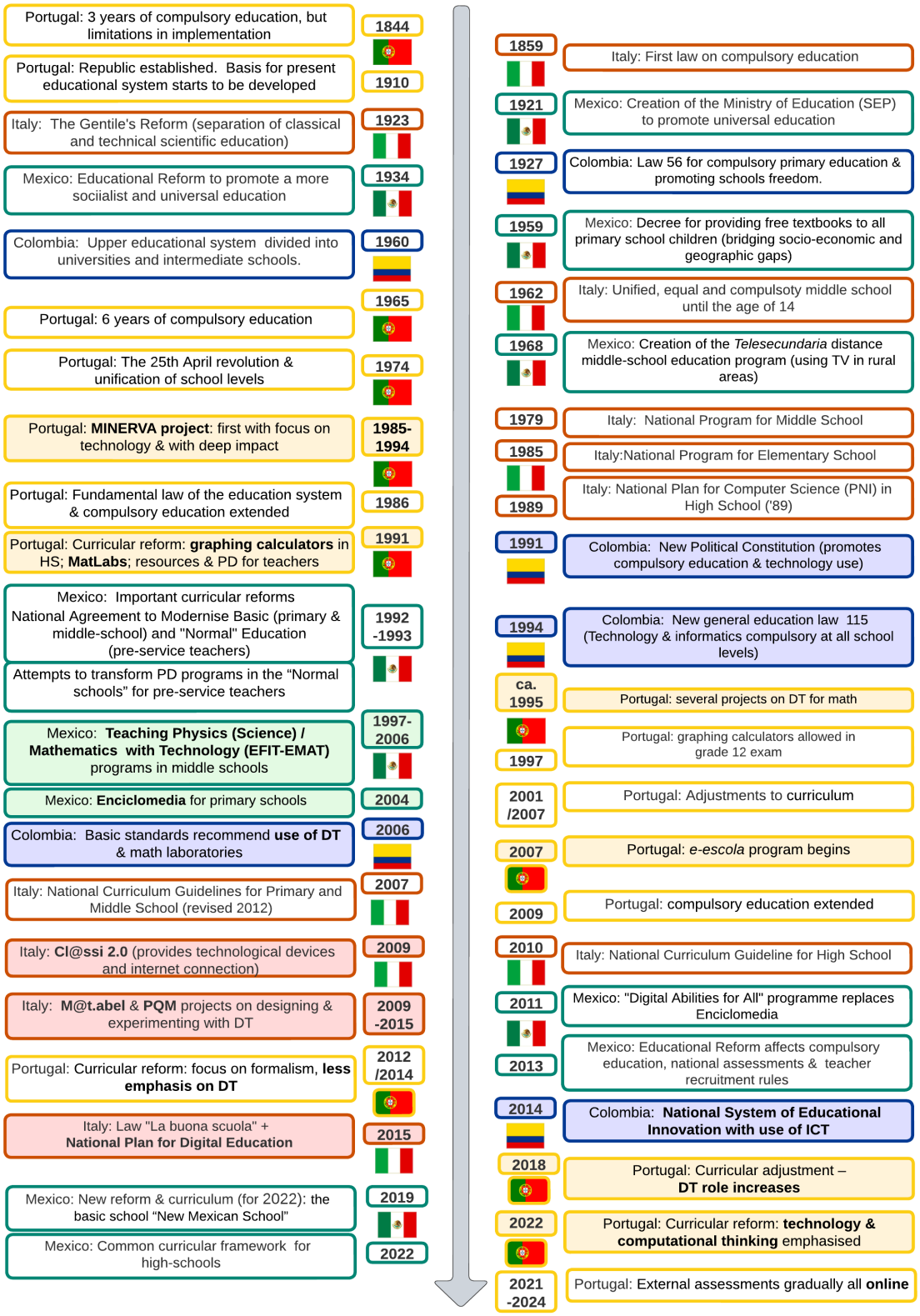
[2] Even during the writing of this chapter, many curricular changes occurred in three of our countries (all except Colombia).

Appendix: Timeline of historical educational turning points

Historical Turning Points in Educational Policies

in Italy, Portugal, Colombia and Mexico

Note: Timeline not to scale



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