Chapter 1 – Introduction

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First and foremost, the topic *Mathematics Education in the Digital Age* encompasses the whole field and any single edited volume on the subject cannot possibly do justice to the cumulative knowledge of over 50 years of mathematics education research. Consequently, we aim to privilege the context of the *Digital Age* by offering a comprehensive text that highlights the impacts this has, and will continue to have, on the aspects of learning and teaching mathematics in formal education systems and settings. In this editorial, we focus one-by one on the four main themes addressed within the book, which are:

- Mathematics teacher education and professional development in the digital age
- Mathematics curriculum development and task design in the digital age
- The assessment of mathematics in the digital age
- Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

We highlight how each of these themes is addressed by the chapter authors, either explicitly or more implicitly, to draw out the important inter-connections within and between the respective texts. By doing so, we aim to highlight not only the complex nature of the field but also the advancements in theoretical and practical knowledge that is enabling the mathematics

education community to continue to learn in this increasingly digital age. The chapters are presented alphabetically by lead author.

1.1 Mathematics teacher education and professional development in the digital age

Since the 1980s and 1990s and the introduction of the early mathematical computer applications to mathematics classrooms (i.e. LOGO, spreadsheets, dynamic graphing and dynamic geometry software), teachers of mathematics have been challenged to explore how these new tools impact on the curriculum and its pedagogy. In the intervening years, the mathematics education research field has expanded to consider the teacher in all of his/her roles within the context of both pre-service and in-service teacher education, with increasing emphasis on how to prepare and support teachers to conduct their professional lives in this fast-moving digital age.

The chapter by *Faggiano, Rocha, Sacristán and Santacruz-Rodríguez* adopts a historical approach to the analysis of major national or regional professional development initiatives in Colombia, Italy, Mexico and Portugal to aim to make explicit the "pragmatic theories" that have underpinned their design, implementation and (in some cases) evaluation. Their resulting methodological frame develops and expands five interconnected themes: the vision and goals of the initiative; its focus; the strategies and methods employed; constraints and incentives; and the approaches to evaluation. This frame offers a useful analysis tool to inform the design of teacher professional development interventions that address digital technology use in school mathematics education – and take account of important international differences.

Sacristán, Rahaman, Srinivas and Rojano provide the important perspectives from developing countries in their chapter, which offers insights from Mexico and India on technology integration in mathematics education. Within this, they consider the contextual factors that influence teachers' attitudes to and uptake of technology, alongside the nature of associated professional development design.

Gueudet, Pepin, Courtney, Kock, Misfeldt and Lindenskov Tamborg also adopt an international perspective as they compare the affordances and constraints of digital education platforms offered as resource banks for teachers of mathematics to support them to design lessons. Drawing on the documentational approach, the authors conclude a number of factors that differentiate such platforms, which link to national education policies and perspectives on the nature of teachers' work, in particular the important aspect of the role of the teacher as a designer of lessons. Their chapter highlights the developing complexity of this role in the digital age.

Aspects of the role of the teacher are addressed less explicitly in the Chapters by *Jankvist, Dreyøe, Geraniou, Weigand and Misfeldt* (concerning teachers' roles as formative assessors of students' learning), *Albano, Kondratieva and Telloni* (reflecting the multiple roles of undergraduate mathematics' lecturers as designers, teachers and researchers of students' digital experiences) and *Leung and Donewska-Todorova* (offering pedagogical insights into how the pragmatic to abstract continuum is addressed within digital task design).

Finally, three chapters produce tools and frameworks that may prove useful for teachers in aspects of their roles as curriculum and lesson designers.
Donewska-Todorova, Trgalova, Schreiber and Rojano compare the analysis of selected digital mathematics tasks to develop a set of criteria that might indicate a sense of their quality.
Koch, Confrey, Clark-Wilson, Jameson and Suurtamm describe and analyse three digital maps of the mathematics curriculum, which offer different ways to conceive its design and construction. The resulting maps offer opportunities to reflect on the design and enactment of the mathematics curriculum in dynamic ways.
Skott, C. Psycharis and Skott, J. adopt a social perspective to help understand how the use of digital resources affects one teacher's professional identity, resulting in the *Patterns of Participation* framework.

Research on aspects of initial teacher education and teachers' ongoing professional learning is evolving rapidly to incorporate aspects of digital technological tools beyond only training activities with particular digital tools. However, the geographical and cultural diversity of technology integration within mathematics education, as brought to the fore by several chapters in this book, indicates that the dissemination of existing research, with further validation of the theories and methods adopted may support more research-informed approaches. However, as new mathematical technological tools emerge, alongside the increased interoperability of the existing tools, there will always be the continued need for related research. There is still much to be understood about the components of more effective teaching practices involving technological tools and how these relate to the evolution of teachers' knowledge.

Mathematics curriculum development and task design in the digital age

This theme concerns issues related to digital curriculum materials, resources (including those using digital technology) and e-textbooks with a focus on their design, appropriation, use, and wider dissemination.

Task design is explicitly addressed in chapters by *Albano, Kondratieva and Telloni*; *Leung and Donevska-Todorova*, and *Donevska-Todorova, Trgalová, Schreiber and Rojano*.

The chapter by *Leung and Donevska-Todorova* explores the potential of digital technology to allow students experiencing mathematical ideas and concepts in learning situations where discursive mathematical practices supported by digital artefacts gradually evolve into formal mathematical knowledge pragmatic-abstract continuum. They call for designing technology-based tasks addressing the *pragmatic-abstract continuum*, which they illustrate on examples from primary to university levels.

Albano et al. suggest using a tetrahedron model, considering the didactic system in the e-learning context composed of a mathematical knowledge to be taught, a learner, a tutor and an author, to design tasks for blended learning. The authors present and discuss examples of online resources to be used in face-to-face learning at the university.

Donevska-Todorova et al. question the quality of digital resources (e.g., apps or audio podcasts) and technology-based mathematical tasks (e.g., including dynamic geometry or spreadsheet). The authors suggest a set of criteria that might help teachers select the most adequate ones or serve as guidelines in task design.

The use of existing digital resources and their impact on students' learning is dealt with in chapters by *Jankvist, Dreyøe, Geraniou, Weigand and Misfeldt* and *Rezat, Schacht and Häsel-Weide*.

Rezat et al. focus on students' interactions with digital textbooks and highlight challenges students face with two characteristic features of these digital resources, namely interactive answering formats and automated feedback. Implications the authors draw from their study point to the need of a careful use of such a resource in the classroom, which requires teacher's awareness of the goals of the tasks and of possible effects of automated feedback on students' mathematical activity.

Jankvist et al. analyze CAS-supported assessment tasks to highlight benefits from using technology that allows for formative assessment or self-assessment, and that in some cases shifts the focus on mathematical concepts rather than on complex calculations. On the other hand, the authors show examples in which technology cuts away the relevance of the mathematical task.

The chapter by *Gueudet, Pepin, Courtney, Kock, Misfeldt and Lindenskov Tamborg* focuses on digital educational platforms available for mathematics teachers. The authors analyze affordances and constraints of the platforms as tools supporting teacher curriculum design.

In their chapter, *Koch, Confrey, Clark-Wilson, Jameson and Suurtamm* present three initiatives of mathematics curriculum digital mapping. The three different approaches share a common goal to enhance mathematics teaching and learning by offering another view of the curriculum conveying connections between mathematical concepts and providing access to related instructional resources or assessment tasks.

These contributions open new avenues for further research. Interactivity appears as the foremost affordance of technology-supported tasks and feedback is therefore a critical issue in task design. It requires a deep didactic and cognitive analysis. Personalized and adaptive feedback, feedback helping students become autonomous learners or scaffolding that gradually fades away are some of the task design features calling for future developments. Further research is also needed to explore teachers' use of new resources such as educational platforms, e-textbooks, digital curricular maps or resource quality criteria, and their effects of mathematical teaching and learning.

Assessing mathematics in the digital age

Evolution in curriculum and task design influenced by the use of digital technologies has to mirror in evaluation and assessment in mathematics education. The quantity and diversity of approaches and tools for this aim evolve together with concerns about access, individualization and securing efficacy in teaching and learning.

In this sense, *Jankvist, Dreyøe, Geraniou, Weigand and Misfeldt* discuss the use of CAS for the purpose of summative and formative assessment and the ways in which CAS alter or augment these. They also offer examples of examination tasks from secondary school, upper secondary school and university level in Denmark, Germany, and the United Kingdom, which have been applied in different contexts. As a result the authors conclude that CAS is a challenge to the traditional assessment scheme and allow for new types of assessment situations to be implemented, because of the increased computational power that CAS provide. However, there is on the one side the danger that these new types are too complex and too difficult. On the

other side these new types could end up promoting pragmatic strategies for solving standard tasks with CAS due to such training being aligned with the assessment. There seems to be no general solution for summative assessment. Maybe, mixed examinations, one part of the examinations with technology, the other part without, might be a good compromise. Another possibility is emphasizing formative assessment. This type of assessment with digital technologies can support both the students and the teacher. Teachers will get summative results on students' performance, and this information enables teachers to make informed decisions about next steps for their students.

Fahlgren, Brunström, Dilling, Kristinsdóttir, Pinkernell and Weigand set out a framework for analysing technology-rich assessment systems, especially for formative assessment. This framework is about assessment through and with technology, the change of the content, the functionality and aspects of feedback. It is used to analyse three distinctive and innovative formative assessment systems: STEP (Seeing The Entire Picture) from Israel, SMART (Specific Mathematics Assessments that Reveal Thinking) from Australia, and STACK (System for Teaching and Assessment using a Computer algebra Kernel) from the UK. As a result, the framework allows a rough comparison of the systems, however, a more fine-graded framework is necessary to pinpoint some characteristics and differences between technology-rich assessment systems. One important conclusion of this analysis is the guideline that formative assessment has to be integrated into a learning process and the use of digital assessment systems might be a part of this process.

Interesting questions about automated feedback for self-assessment can be brought into connection with digital textbooks. For example, *Rezat, Schacht and Häsel-Weide* discuss feedback in relation to student's activation when engaging with digital textbooks.
Stepping outside of a comfort zone of already known practices and taking responsibilities for changes when applying digital media in mathematics education is challenging, in particular in the period of a pandemic crisis. Such changes are required not only in the teaching methods, techniques and tools, but also in assessment. This is an intensively growing issue, which is to be adequately addressed during the follow up ERME Topic Conference on Mathematics Education in the Digital Age (MEDA 2) opening many new questions for further research.

Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

Research in mathematics teaching and learning, especially in the digital age, deals with complex phenomena involving multiple theoretical perspectives and different kinds of methodological approaches. The chapters in this book refer to many different theories, e.g. design theories, prescriptive theories, theories linking research and practice, theories addressing the transfer of learning arrangements to other learning conditions etc. that, eventually combined/networked in a synergic way, shape research in mathematics education in the digital age.

For example, the chapter by *Gueudet, Pepin, Courtney, Kock, Misfeldt and Tamborg* concerning the impact of digital platforms in terms of teachers' curriculum design as aspect/s of teachers' documentation work, combines specific theoretical tools developed within the Documentational Approach to Didactics (the instrumentation and instrumentalization processes; and the concept

of connectivity) to investigate the affordances and constraints of digital platforms in three different national contexts.

Aspects of theoretical perspectives and methodological approaches are explicitly addressed in the chapters by *Faggiano et al.*, *Psycharis et al.,* and *Skott et al.*

Faggiano, Rocha, Sacristán and Santacruz-Rodríguez offer an empirically-informed and research-based approach to support the design of mathematics teachers' professional development to exploit technological affordances and prepare mathematics teachers for present and future digitally-enhanced contexts. By exploring the evolution of and implications from mainly large-scale national professional development initiatives, they aim to arrive at a more theoretical grounding for the topic, supported by case examples that exemplify the most pertinent aspects. To do this, they have identified a set of aspects (and sub-aspects) that emerged in relation to five major themes and reveal a set of 'pragmatic theories' alongside a consideration of the interconnections between these aspects.

Psycharis, Kafetzopoulos and Lagrange present a framework for analysing classroom situations by networking Connected Working Spaces (CWS) and Abstraction in Context (AiC). Their perspective allows a deeper look at students' cognitive evolution as they experience functions in a plurality of digital and non-digital settings (physical context, geometry, measures, algebra). Furthermore, they give insight into the relationship between uses of technology and the development of students' mathematical knowledge. With the attempt to focus on how the combination of AiC and CWS can help to make sense of students' conceptualisation of function as covariation, they also underline that the networked analysis may sensitize task designers, teachers and researchers to the complexity of students' conceptualising processes.

Skott, C, Psycharis and Skott, J bring new perspectives and understandings of teacher's development in the use of digital resources. They use a social practice perspective (Patterns of Participation – PoP) to investigate teachers' roles in technology-based learning environments and the influence of culture and society on the way digital resources are used in the classroom. Comparing the findings with a previous study in which a different theoretical approach was used, the authors claim that different frameworks and theoretical approaches – including networking and double analyses – are needed to develop balanced understandings of the integration of digital resources in mathematics classrooms that acknowledge the significance of teachers' professional identities.