Modelling electricity futures prices using seasonal path-dependent volatility

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Abstract

The liberalization of electricity markets gave rise to new patterns of futures prices and the need of models that could efficiently describe price dynamics grew exponentially, in order to improve decision making for all of the agents involved in energy issues. Although there are papers focused on modelling electricity as a flow commodity by using the Heath et al. (1992) approach in order to price futures contracts, the literature is scarce on attempts to consider a seasonal volatility as input to models. In this paper, we propose a futures price model that allows looking into observed stylized facts in the electricity market, in particular stochastic price variability, and periodic behavior. We consider a seasonal path-dependent volatility for futures returns that are modelled in the Heath et al. (1992) framework and we obtain the dynamics of futures prices. We use these series to price the underlying asset of a call option in a risk management perspective. We test the model on the German electricity market, and we find that it is accurate in futures and option value estimates. In addition, the obtained results and the proposed methodology can be useful as a starting point for risk management or portfolio optimization under uncertainty in the current context of energy markets.

Keywords: electricity futures price, forecast, seasonal path-dependent volatility, Heath-Jarrow-Morton model, option pricing

1. Introduction

During the past two decades we have seen comprehensive electricity sector liberalization and deregulation in all EU countries. The electricity market, once monopolistic, has become a competitive market where electricity prices are derived by the interaction of supply and demand. This new context, joined with the physical characteristics of electrical power, has generated new price patterns,

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never seen before, neither in financial markets, nor in commodity markets. Electricity is a flow commodity characterized by its very limited storability. The lack of economic storage opportunity makes the supply completely inelastic to price changes. Prices show extremely high volatility and sudden consistent jumps in their levels, called “spikes”. Market participants, both producers and consumers, are dramatically exposed to uncertainties in electrical power prices, and risk management techniques play a fundamental role in quantifying and mitigating price risk. Several studies have been addressed to the analysis of electricity prices, because they are generally used as reference for decisions done in energy trading. Therefore, well-performing forecast methods for day-ahead electricity prices are essential for energy traders and supply companies. In Keles et al. (2016), a methodology based on artificial neuronal networks is presented to forecast electricity prices. The forecasting reveals being an important instrument for dealer generally in all the commodity markets: Baruník and Malinska (2016) explain the term structure of crude oil prices using the dynamic Nelson-Siegel model and propose to forecast oil prices using a generalized regression framework based on neural networks.

García-Martos et al. (2013) extract common features in the volatilities of the prices of several commodity prices: the common volatility factors obtained are useful for improving the forecasting intervals and the results obtained and methodology proposed can be useful as a starting point for risk management or portfolio optimization under uncertainty in the current context of energy markets.

Since the beginning of liberalization, researchers and practitioners have been focusing their attention on studying electricity price evolution by setting up mathematical models able to capture the main features of price behaviour, in order to allow both derivative pricing and risk hedging. With the emerging of the electricity wholesale, a lot of interest has grown in electricity financial instruments in order to manage price risks: most of electricity futures and options on futures are traded on the New York Mercantile Exchange (NYMEX), while a large variety of electricity derivatives is traded in the OTC markets. Call and put options are the most effective tools available to merchant electrical power plants or power marketers for hedging price risk because electrical power generation capacities can be essentially viewed as call options on electricity, in particular when generation costs are fixed. Electricity is referred to as a flow commodity: all contracts guarantee the delivery of an established amount of electricity (MWh) continuously over a specific future time period (1 hour, 1 month, 1 quarter, 1 year). It can be settled with physical delivery or simply financially. The energy market is made of two segments: a market for spot trading and a derivative market. In the spot market, electricity is traded in an auction system for standardized contracts, and every day hourly contracts for each of the 24 hours of the coming day are evaluated. This is called the day-ahead market, characterized by physical delivery. In the derivative market, electricity forward contracts and futures contracts are settled either financially or with physical delivery. They usually have a monthly, quarterly, or yearly delivery period. Recently, European and Asian options with electricity futures as underlying have been launched in the more developed Electricity Markets (Nord Pool, Scandinavia and NYMEX, New York) and are becoming extremely important in the commodities markets, offering advantages to both the consumer (buyer) and the producer. The literature on electricity price modelling has rapidly developed in the last few years because of the growing need for obtaining models that describe electricity price behaviour in a realistic and accurate way. Indeed, from a risk management point of view, robust forecasts for electricity prices lead to proper hedging strategies being defined through the accurate pricing of financial derivatives, like options. According to Weron (2014), there are various approaches that have been developed to analyse and predict electricity prices, and in particular we can distinguish five groups of models: *i) Multi-agent models*, that consider the interaction among heterogeneous agents and build the price process by matching demand and supply in the market; *ii) Fundamental (structural) methods*, which model the impact of important physical and economic factors in order to determine electricity price dynamics; *iii) Reduced-form (quantitative, stochastic) models*, which investigate the statistical properties of electricity prices over time in order to describe their dynamics with the ultimate objective of derivatives evaluation and risk management; *iv) Statistical approaches*, which consist in applying statistical techniques of load forecasting or implementing econometric models; *v) Computational intelligence techniques*, which use the neural network approach to study the complex dynamic system. Obviously, there can be models that contemplate hybrid solutions, combining techniques from two or more of the groups. Although the classification of Weron allows us to characterize the research field according to the modelling approach, we can identify two macro-areas of research: *a)* the traditional one that concentrates on modelling electricity spot price dynamics; *b)* the alternative approach that describes and represents directly electricity futures prices. According to this partition, we have reviewed some of the existing literature.

With regard to the spot price modelling, Lucia and Schwartz (2002) model the natural logarithm of the spot price by assuming a mean reverting process estimated by using spot price data in the Nordic market. The price evolution of a futures contract is then determined by applying expected value under an appropriate martingale measure equivalent to the objective one. Other authors, such as Pilipovic (2007) and Eydeland and Wolyniec (2003) suggest a two factor model, in order to take into account the influence on the spot price given both by a short term and a long term source of randomness. According to Clewlow and Strickland (2000), and Eydeland and Wolyniec (2003), the introduction of a jump diffusion process appears the natural way to account for spikes, even if market incompleteness is introduced. Skantze et al. (2000) develop a model of electricity prices taking into account the important characteristic of seasonality, by studying the load and supply behaviour. Other papers focus directly on peak price dynamics by specific volatility settings: in particular regime switching models are used in order to predict price spikes (Mount et al. (2006)), by switching between the high-price regime and the low-price regime, according to the two transition probability functions. Huisman and Mahieu (2003) and Deng (2000), among others, suggest Markovian regime switching models for electricity price, characterized by the occurrence of stable and turbulent periods. Cuaresma et al. (2004) investigate the forecasting abilities of a battery of univariate models on hourly electricity spot prices, using data from the Leipzig Power Exchange. They find that an hour-by-hour modelling strategy for electricity spot prices improves significantly the forecasting abilities of whole time-series models, and that the inclusion of simple probabilistic processes for the arrival of extreme price events can also lead to better forecasts. Diongue et al. (2009) propose an approach based on the k-factor GIGARCH process for investigating conditional mean and conditional variance forecasts and modelling electricity spot market prices. They apply the proposed method to the German electricity prices market providing forecasting prices up to one month ahead. Higgs and Worthington (2008) propose a mean-reverting model and a regime-switching model to capture some features of the Australian national electricity market: high price volatility, strong mean-reversion and frequent extreme price spikes. Tan et al. (2010) define a day-ahead electricity price forecasting method based on wavelet transform combined with ARIMA and GARCH models. This method is examined for MCP prediction in the Spanish market and LMP prediction in the PJM market. Huisman and Kiliç (2013) examine the development of day-ahead prices in five European markets through a regime switching model. In particular, they distinguish between prices under normal market conditions and under non-normal market conditions. Aid et al. (2013) develop a structural risk-neutral model for electricity spot price that is particularly well-suited for spread options on the spot price since it is based on the economic relation that holds between fuel prices and electricity spot prices. Finally, an interesting approach is used by Nowotarski et al. (2013) for estimating the seasonal components of electricity. They show that wavelet-based models outperform sine-based and monthly dummy models. The major disadvantage of spot price models is that forward/futures prices are given endogenously from spot price dynamics. Therefore, the obtained dynamics of futures prices are most of the time not consistent with the observed market prices.

Regarding the futures prices modelling literature, Clewlow and Strickland (2000) have been among the first researchers to introduce the futures price curve modelling approach to the energy market in the framework of Heath et al. (1992). They use only few stochastic factors and the initial price curve as given in order to model futures prices under some equivalent martingale measure in a no-arbitrage environment. Koekebakker and Ollmar (2005), and Bjerksund et al. (2010) model a continuum of instantaneous-delivery forward contracts under risk neutral probability measure. Benth et al. (2008) proposes a model for electricity forwards that are frequently referred to as swaps since they represent an exchange of a fixed for floating electricity price. They model these swaps using the Heath-Jarrow-Morton (hereafter HJM) model. Hinz and Wilhelm (2006) face the not trivial problem of evaluating energy-related financial contracts written on prices of flow commodities. Starting from Gaussian HJM interest rate models, and following an axiomatic approach, which provides a connection to interest rate theory, flow commodity markets have been canonically constructed. With futures price models described, explicit formulae for caps, floors, collars, and cross commodity spreads are deduced by applying change-of-numeraire techniques. Barth and Benth (2014) model energy forward prices according to an infinite-dimensional approach. Similar to the HJM framework in interest-rate modelling, a first-order hyperbolic stochastic partial differential equation is used to model the dynamics of the forward price curves.

All of the authors in the literature on futures price models summarized above try to provide pricing models that address electricity features, and the Heath et al. (1992) framework is widely used to describe flow commodity dynamics. However, to the best of our knowledge, and to date, there is no published paper that deals with numerical price simulation which uses the main advantage of the Heath et al. (1992) model, i.e. the possibility of a customized volatility function. In fact, this approach gives the opportunity to simulate the daily electricity price taking into account the current precise calendar day, the part of the year and the obtained return level. In Table 1 we refer to the existing literature concerning the electricity futures modelling in the HJM framework and we highlight our main contributions in comparison with previous works.

Table 1: Futures price modeling literature in the HJM framework

Authors	Season.	Path dependence	Stochastic volatility	Calendar day forecast	Derivative pricing
Clewlow and Strickland (2000)	-	-	✓	-	✓
Koekebakker and Ollmar (2005)	-	-	-	-	-
Hinz and Wilhelm (2006)	-	-	-	-	✓
Benth and Koekebakker (2008)	-	✓	-	-	✓
Bjerksund et al. (2010)	-	-	-	-	✓
Barth and Benth (2014)	-	-	-	✓	-
Fanelli et al. (2015)	✓	✓	✓	✓	✓

In this paper, our purpose is to fill this literature gap by providing a futures price model which allows looking into observed stylized facts in the electricity market, in particular stochastic price volatility, and periodic behavior. This paper is part of the group of the reduced-form models, developed for risk management. The contribution to the existing literature is twofold. Firstly, we focus on seasonality by observing prices and obtaining the calendar-varying volatility parameters for each month of the year. The Heath et al. (1992) model gives us the opportunity to simulate the daily evolution of the forward curve in the free-arbitrage environment and therefore determine the daily futures price according to the volatility seasonal parameter of each specific day (see Chiarella et al. (2011), Fanelli (2016), Fanelli et al. (2016)). Secondly, the volatility we set has also a contingent component depending also on the spot return level. In this way the futures price is affected by the spot market realization, reflecting the fact that during turbulent periods high spot prices are more likely to occur, enhancing even futures price volatility. We build on the existing literature and propose a forecasting method that, by assuming a seasonal, path-dependent price volatility in the HJM framework (Heath et al. (1992)), provides a futures price estimation. This price is then used as underlying of a call option in a risk hedging perspective. We test the model on the German electricity market, and we find that it is accurate in futures and option value estimates. The referring framework of our model is given by Hinz and Wilhelm (2006). The electrical energy is not economically storable and, therefore, hedging by commodity storage is impossible (Hinz et al. (2005)). It is important to take into account the electricity production process: although electrical energy cannot be stored, a hedging is still possible by production capacity investments. This perspective allows us to use a pricing methodology in which the underlying is assumed to be storable Hinz and Wilhelm (2006). The producer always has the ability to produce electricity, creating a sort of "electricity storability". From this point of view, the true underlying of contracts becomes the physical ability to produce electrical power. According to this perspective, the electricity market becomes more complex and has to be considered as composed of both power electricity and agreements on power production capacities. The market reaches the equilibrium and determines the price process for all tradable assets that are both physical (production capacity agreements) and financial (forward/futures prices). There is a probability measure Q, equivalent to the market probability measure P, such that equilibrium asset prices are given by their future revenues, expected with respect to Q. By using the production portfolios, it is possible to replicate any financial contracts. By considering the market in this more general perspective, electricity cannot be stored, but hedging is still possible by using production capacity investments. We can consider the market as composed of an infinite number of futures contracts with instantaneous delivery period, and it makes sense to transform the futures price to a bond price and apply interest rate theory. The equilibrium in the production capacity market guarantees the existence of the measure Q, such that electricity asset prices are given by the expected value of future revenues of certain production portfolios. The equilibrium concept represents a way to price all contracts by the same measure "chosen by the market". The asset dynamics is therefore described directly under Q.

In this paper we model the future price dynamics assuming a virtual fixed income market linked to the real electrical power market, following the Hinz and Wilhelm (2006) approach. Our original contribution is given by assuming a seasonal path dependent volatility for futures returns that we model according to the Hinz (2003). The implementation of the model gives us the opportunity to simulate the daily evolution of the forward curve in the free-arbitrage environment and therefore determine the daily futures price according to the volatility seasonal parameters of each specific day. The chosen volatility has a seasonal component and a contingent component, so that, implementing the model, each specific day volatility is affected both by the part of the year the day is in, and the spot return level realized in that particular moment. We estimate option values according to different strike prices by applying the Monte Carlo approach, and comparing them with the EEX observed prices.

An interesting possible evolution of this research could be the comparison of the model performance in different EU/US electrical power markets. Besides, it would be interesting to investigate the opportunity of using the seasonal volatility model for futures prices in order to estimate spot price long term components, as in Conejo et al. (2010).

The remainder of the paper is organized as follows: Section 2 outlines the proposed model, Section 3 presents numerical results of the simulations, in Section 4 we describe a practical application of the proposed model, and finally Section 5 provides some relevant conclusions.

2. Futures Price Model

We consider the Electricity Market (hereafter EM) given on the time horizon $[0, T]$. Let $(\Omega, \mathcal{F}, P)$ be a filtered probability space where the filtration $\mathcal{F} = (\mathcal{F}_t)_{t\in[0,T]}$ is generated by a Brownian motion $(W_t)_{t\in[0,T]}$. All processes are assumed to be progressively measurable. At each time $t \in [0, T]$, price dynamics of a futures contract supplying 1 MWh at a time $\tau \in [0, T]$ are given by $(F_t(\tau))_{t\in[0,\tau]}$, which is a positive valued adapted stochastic process[3]. At time $t = 0$ we observe prices $(F_0^*(\tau))_{\tau\in[0,T]}$ for all future delivery times. According to Hinz and Wilhelm (2006), we assume that in the model the following axioms are valid:

(a) $(F_t(\tau))_{t\in[0,\tau]}$ is almost surely continuous for each $\tau \in [0, T]$;
(b) There is a risk neutral measure Q^F equivalent to P such that for each $\tau \in [0, T]$, $(F_t(\tau))_{t\in[0,\tau]}$ is a Q^F-martingale;
(c) Futures prices start at observed values $F_0(\tau) = (F_0^*(\tau))_{\tau\in[0,T]}$;
(d) Terminal prices $(F_t(t))_{t\in[0,T]}$ form a continuous spot price process.

A currency change is the key instrument making this model similar to a fixed income market model: the electricity futures price is expressed in units of the electricity price just in front of the delivery. In this new currency, the electricity price, $(p_t(\tau))_{t\in[0,\tau]}$, $\tau \in [0, T]$, behaves like a zero coupon bond, in the sense that its value converges to 1 when t coincides with τ:

$$p_t(\tau) = \frac{F_t(\tau)}{F_t(t)}, \qquad t \in [0, \tau], \qquad \tau \in [0, T]. \tag{1}$$

If we introduce the risky process $(N_t)_{t\in[0,T]}$, given by the reciprocal of the electricity price at delivery,

$$N_t = \frac{1}{F_t(t)}, \qquad t \in [0, T], \tag{2}$$

we call Currency Change Electricity Market (hereafter CCEM) the market consisting of the following processes:

- $(p_t(\tau))_{t\in[0,\tau]}, \quad 0 \le t \le \tau \le T,$
- $(N_t)_{t\in[0,T]}$.

[3] A complete list of the used symbols is in the Appendix A.

As we have already said, in the CCEM the two processes behave respectively like a zero coupon bond and a risky asset. Note that this base market has a structure similar to a money market. In fact, $p_t(t) = 1$ for every $t \in [0, T]$ and, if we define the compounding process $(B_t)_{t\in[0,T]}$ as the price at time t of the zero coupon bond with delivery T

$$B_t = p_t(T), \tag{3}$$

it can be proved (see Hinz (2006) and Hinz and Wilhelm (2006)) that in the CCEM there exists a risk neutral measure Q^M equivalent to P, under which the model is arbitrage-free. Thus, processes

$$\left(\frac{N_t}{B_t}\right)_{t\in[0,T]} \quad \text{and} \quad \left(\frac{p_t(\tau)}{B_t}\right)_{t\in[0,\tau]} \tag{4}$$

are Q^M-martingales, and Q^F is obtained from Q^M by using the change of numeraire

$$dQ^F = \frac{N_T}{B_T}\frac{B_0}{N_0}dQ^M. \tag{5}$$

Finally, the initial values of processes are

$$p_0^*(\tau) := \frac{F_0^*(\tau)}{F_0^*(0)}, \quad N_0^* := \frac{1}{F_0^*(0)}, \tag{6}$$

so that $F_0^*(\tau) = \frac{p_0^*(\tau)}{N_0^*}$. Moreover, in the CCEM the following reverse currency change holds

$$F_t(\tau) = \frac{p_t(\tau)}{N_t} \quad 0 \le t \le \tau \le T, \tag{7}$$

and it is applied to determine the electricity price. The CCEM is a virtual market, allowing for its characteristics to be modelled with instruments usually used for the fixed income market. In particular, we aim at modelling the futures return curve in order to obtain price dynamics. So, we model the price $(p_t(\tau))_{0\le t\le\tau\le T}$, that by construction is modelled like a bond, in the HJM framework, and use relation (7) to obtain the risk-neutral electricity futures prices. According to the HJM model, we assume that futures rate dynamics $f_t(T)$, $t \le T$, are given by the stochastic differential equation

$$df_t(T) = \alpha(t, T, \cdot)dt + \sigma(t, T, \cdot)dW(t), \tag{8}$$

where $\alpha(t, T, \cdot)$ represents is the instantaneous futures rate drift function, and $\sigma(t, T, \cdot)$ is the instantaneous futures rate volatility function. The process is driven by a one-dimensional Brownian motion. The third argument in the brackets $(t, T, \cdot)$ indicates the possible dependence of the futures rate on other path-dependent quantities, such as the spot rate or the futures rate itself. In order to avoid arbitrage opportunities, function $\alpha(t, T, \cdot)$ is determined in function of $\sigma(t, T, \cdot)$:

$$\alpha(t, T) = \sigma(t, T) \int_t^T \sigma(s, T) ds, \tag{9}$$

so that the integral equation for the futures rate process assumes the following form:

$$f_t(T) = f_0(T) + \int_0^t \sigma(s, T) \int_s^T \sigma(s, u) du ds + \int_0^t \sigma(s, T) dW(s). \tag{10}$$

The zero coupon bond price for all $\tau \in [0, T]$ is given by

$$p_t(\tau) = e^{-\int_t^\tau f_t(s) ds}, \tag{11}$$

so that the stochastic differential equation is

$$dp_t(\tau) = p_t(\tau) f_t(t) dt - \left(\int_t^\tau \sigma(t, s) ds \right) dW_t, \tag{12}$$

with $p_0(\tau) = F_0(\tau)/F_0(0)$.

Let the additional risky asset N_t have the following dynamics:

$$dN_t = N_t [f_t(t) dt + v(t) dW_t], \tag{13}$$

where $v(t)$ is a deterministic volatility and we can notice that the drift is also deterministic, being the spot rate $f_t(t)$ available on the market. One of the HJM approach advantages is that the fundamental inputs of the forward curve model are the initial curve futures rate curve, $f_0(T)$, and the volatility function $\sigma(t, T)$. Initial futures rates are obtained by using the initial futures price curve observed in the market, such that

$$f_0(t) = -\frac{\partial}{\partial t} \log F_0(t) \quad \text{for all } t \in [0, T]. \tag{14}$$

With regard to the volatility function, the Heat-Jarrow-Morton model gives the opportunity to choose a functional form suitable for capturing main features of price behaviour. As we observe in Figure 1, where we plot the Phelix EEX [4]one month futures time series (German market), apart from the anomalous price level during the year 2008 caused by macroeconomic factors (financial crisis, renewable supply, drop in demand, etc.), two main characteristics of electricity prices are evident: the stochastic variability and the seasonality.

[4]https://www.eex.com

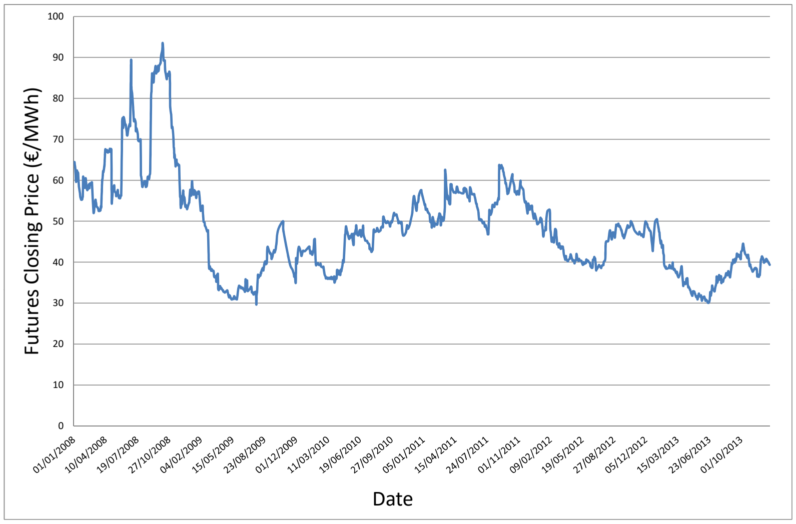


Figure 1: Phelix EEX 1 month Futures Time Series

Our contribution to the existing literature, above reviewed, is to implement the model by considering a seasonal path-dependent volatility function. Therefore, the following function is defined:

$$\sigma(t, T) = S(t, T) + X(t, T), \tag{15}$$

where $S(t, T)$ is the seasonal term and we assume $X(t, T) = [f_t(t)]^u$, with $u \in \mathbf{R}$, as the path-dependent term. The seasonal term has the following form

$$S(t, T) = \sum_{i=1}^{12} (\sigma_2^i + \sigma_1^i e^{-\lambda_i (T-t)}) D_i, \tag{16}$$

where dummy variables D_i, $i = 1..., 12$, allow estimation of the coefficients σ_1^i and σ_2^i for every month of the year. Parameter σ_1 represents the short term volatility coefficient, σ_2 the long term volatility coefficient, and λ the time decay. Thereby, the volatility function describes the monthly seasonality. In the present work we use a Reuters data-set on Phelix EEX futures contracts (€/MWh), consisting of daily futures closing prices spanning from 01.01.2008 to 12.31.2013, to estimate parameters of function (15) by using OLS non-linear regression. For each day we have a futures price term structure consisting of 6 prices, representing the future price of 1 MWh continuously delivered over the following periods (maturities): first month (M1), second month (M2), third month (M3), second quarter (Q2), third quarter (Q3) and one year (Y). In order to estimate the parameters referred to each month we form 12 panels, one for each month, consisting of a sequence of time series from year 2008 to 2013. So, the price observed at the $i-th$ day of the $j-th$ month of the year, for a futures contract delivering over the period k, called maturity, is

$$p_{ij}^k \qquad i = 1,2,...,30 \quad j = 1,2,...,12 \quad k = M1, M2, M3, Q2, Q3, Y. \quad (17)$$

Then return levels are obtained as

$$r_{ij}^k = \log \frac{p_{ij}^k}{p_{(i-1)j}^k}. \quad (18)$$

If we consider the panel referring to a specific month, we calculate the time series of the return volatility term structure according to different years and maturities. Therefore, we can estimate the short term volatility coefficient, σ_1, the long term volatility coefficient, σ_2, the time decay, λ, and the path-dependent coefficient u according to (15). By calibrating the volatility model, we find a common value for the path-dependent coefficient, that is $u = \frac{3}{2}$. Table 2 contains the other estimated coefficients.

Table 2: Volatility Parameter Estimation

Volatility Parameters				
Month	σ_1	σ_2	λ	St.Err.
January	0.00964956	0.0151517	2.72024	0.012600
February	0.00497714	0.00608459	2.74608	0.003762
March	0.00	0.0102933	0.9381311	0.005041
April	0.00432978	0.00873531	4.76766	0.003838
May	0.00178587	0.0116556	2.18629	0.004218
June	0.00	0.0115167	0.277421	0.005896
July	0.00380614	0.0122640	3.00701	0.003597
August	0.00374202	0.0134820	4.73199	0.003547
September	0.00	0.00900843	0.270790	0.005218
October	0.00722492	0.0114431	7.33136	0.007515
November	0.00480292	0.0122898	3.59743	0.005174
December	0.00954558	0.00261133	9.04186	0.005532

In order to show some empirical evidences of the volatility behaviour, we plot, as example, the volatility term structure of three months, February, August, and October, in Figures 2, 3 and 4.

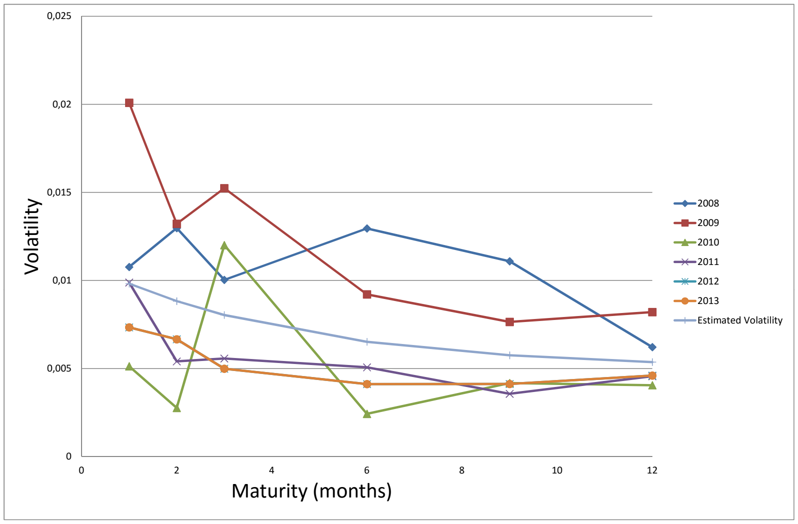


Figure 2: February volatility term structure

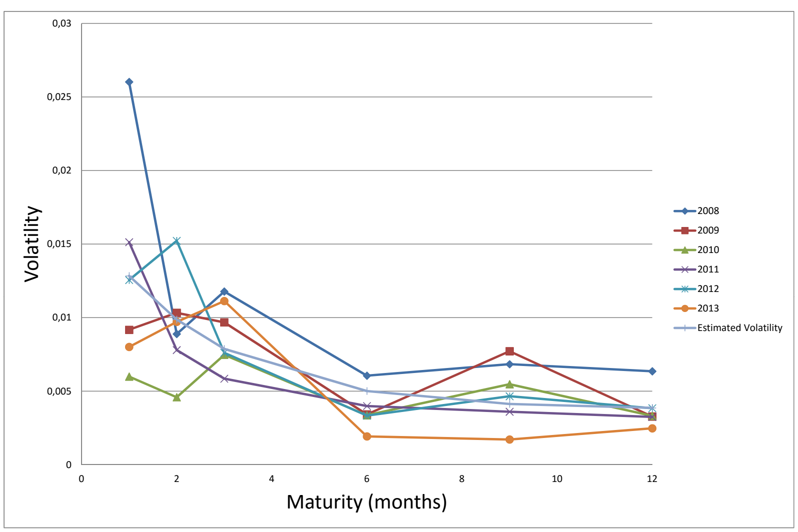


Figure 3: August volatility term structure

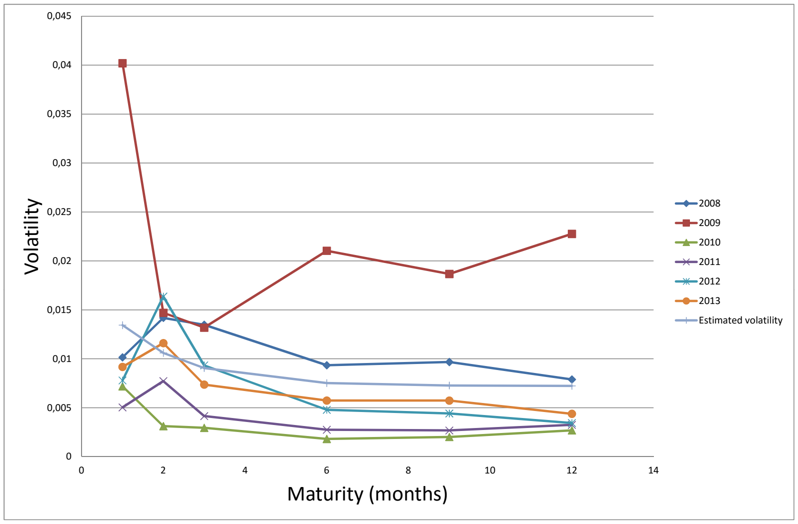


Figure 4: October volatility term structure

On one side, it is possible to notice a quite similar shape as short term futures prices are more volatile than the long ones. On the other side, looking at the scale on the y axis, it is evident how curve levels vary considerably through months.

3. The Numerical Implementation

In this Section, first we develop the numerical implementation of the described model in order to simulate the evolution of the futures rate curve over the time horizon of one year, $[0, T]$, given the initial futures rate curve $f_0(T)$. Then, we obtain the electricity futures price as a function of the return curve. Finally, we apply the Monte Carlo simulation approach in order to estimate the price of a European call option on the electricity futures price. The numerical scheme requires the approximation in discrete time of all the formulas defined in the previous section.

We divide the time horizon $[0, T]$ into N intervals of length $\Delta t = T/N$, so that any time t, $0 \leq t \leq T$, can be expressed as $t = n\Delta t, n \in \mathrm{N}$, any time τ, $t \leq \tau \leq T$, can be expressed as $\tau = h\Delta t, h \in \mathrm{N}$, and $f(n\Delta t, N\Delta t)$ is the discretized form of $f_t(T)$. We consider the values that, at time zero, the futures rate curve $f_0(T)$ assumes at the extremities of each interval, i.e. $f(0, 0)$, $f(0, \Delta t)$, $f(0, 2\Delta t)$, ..., $f(0, N\Delta t)$. We approximate the stochastic integral equation (10) according to the Euler-Maruyama scheme. Hence we obtain the generic recursive scheme for the futures curve evolution

$$f((n+1)\Delta t, m\Delta t) = f(n\Delta t, m\Delta t) + \sigma(n\Delta t, m\Delta t, \cdot) \sum_{i=n}^{m-1} \sigma(n\Delta t, i\Delta t)\Delta t + \sigma(n\Delta t, m\Delta t, \cdot)\Delta W(n+1) \qquad n < m \quad (19)$$

where $\Delta W(n) = \xi\sqrt{\Delta t}$, with ξ as a standard normal random draw, represents the discrete time form of a Brownian motion. The evolution covers one year time, and we obtain the futures rate curve for each day, using different volatility coefficients according to the different months. The volatility function (15) becomes in its dicretized form

$$\sigma(n\Delta t, m\Delta t) = \sum_{i=1}^{k} (\sigma_2^i + \sigma_1^i e^{-\lambda_i(m\Delta t - n\Delta t)}) D_i + [f(n\Delta t, m\Delta t)]^u \qquad i = 1, 2, ..., 12. \quad (20)$$

The futures rate curve at time t, $f_t(s)$, $t \leq s \leq \tau$, is used to calculate the bond value, $p_t(\tau)$, according to formula (11). The discretized form of (11) is given by applying the Euler approximation of the integral

$$\int_t^\tau f_t(s)\, ds = \sum_{k=n}^{h-1} f(n\Delta t, k\Delta t)\, \Delta t, \quad (21)$$

so that

$$p(n\Delta t, h\Delta t) = e^{-\sum_{k=n}^{h-1} f(n\Delta t, k\Delta t)\, \Delta t}. \quad (22)$$

Finally, the electricity futures price is

$$F(n\Delta t, h\Delta t) = \frac{p(n\Delta t, h\Delta t)}{N(n\Delta t)}, \quad (23)$$

where $N(n\Delta t)$ is obtained applying the recursive formulation of (13) and considering a constant diffusion term $v = 0.2$:

$$N((n+1)\Delta t) = N(n\Delta t) + N(n\Delta t)\left[f((n)\Delta t, (n+1)\Delta t) + v\Delta W(n+1)\right], \quad (24)$$

where $\Delta W(n) = \xi\sqrt{\Delta t}$, with ξ as a standard normal random draw.

In electricity markets, for risk management purposes, an option on futures price is usually combined with other assets in financial portfolios in order to hedge price risk. In this context, the described model for futures prices, implemented through the discussed numerical scheme, is used to price a European call option in an accurate and proper way.

Let us consider, at time $t = 0$, a European call option with the futures price $F_t(\tau)$, $0 \leq t \leq \tau \leq T$, as underlying, and payoff at maturity s, $0 \leq s \leq \tau \leq T$, given by $\Phi_s = (F_s(\tau) - E)^+$, where E is the strike price. The call price is obtained by discounting, at a risk free rate δ, the expectation of the payoff at maturity with respect to the risk-neutral measure as follows:

$$C(0, s, \tau) = e^{-\delta s} \mathbb{E}_Q \left[\Phi_s \left| \mathcal{F}_0 \right. \right], \tag{25}$$

where $E_Q \left[\cdot \left| \mathcal{F}_0 \right. \right]$ represents the expectation under Q and conditional to the information available at time $t = 0$, $\mathcal{F}_0$. The option value (25) is estimated by applying the Monte Carlo approach. We made Π simulations, with $\Pi = 100.000$, such that for each path we simulate the term structure of futures rates over the year according to equation (19), and we obtain the bond price (22) and the risky asset price (24). Consequently, putting prices (22) and (24) into (23) we obtain the electricity futures price. We use the Π futures prices to calculate Π call payoffs, $\Phi^j(H\Delta t)$, $j = 1, ..., \Pi$, at maturity $s = H\Delta t$. These payoffs are used in the following formula in order to evaluate the call option C_{MC} according to the Monte Carlo method:

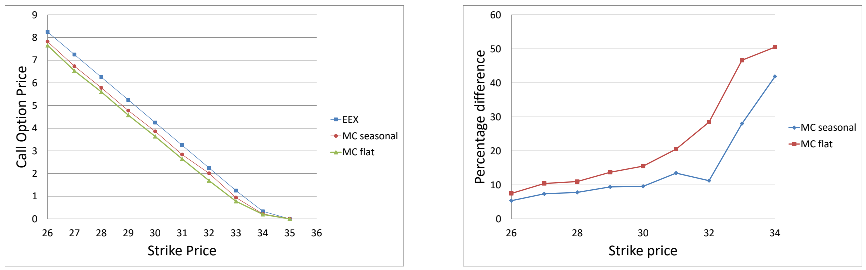
$$C_{MC}(0, n\Delta t, H\Delta t) = e^{-\delta H \Delta t} \frac{1}{\Pi} \Sigma_{j=1}^{\Pi} \Phi^j(H\Delta t). \tag{26}$$

In Table 3we show simulated prices of a call option with maturity one year, on an electricity futures contract with one month delivery. For each strike price E we indicate in the second column the quoted price as resulting on the web site *www.eex.com* on August 26th, 2014. In the third and fourth columns we show respectively the price obtained with Monte Carlo simulations and its Standard Error. The initial electricity futures price observed on August 26th, 2014, is 34.2 €/MWh.

Table 3: Actual Prices vs. Simulated Prices

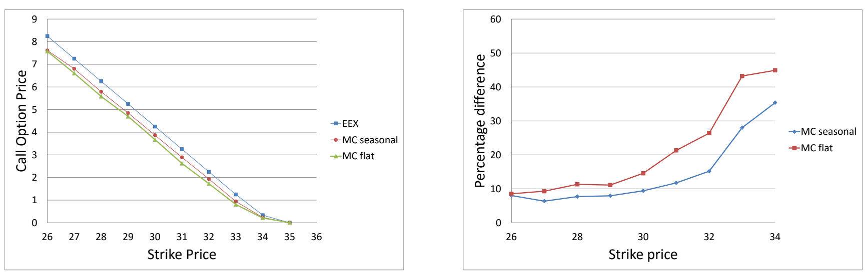
Call Option Prices							
		10.000 paths		50.000 paths		100.000 paths	
Strike price E	EEX *price*	MC *price*	*St. Err.*	MC *price*	*St. Err.*	MC *price*	*St. Err.*
26	8.250	7.819	0.012	7.613	0.007	7.762	0.004
27	7.250	6.734	0.012	6.803	0.005	6.721	0.004
28	6.250	5.781	0.010	5.786	0.004	5.756	0.003
29	5.250	4.778	0.009	4.849	0.004	4.828	0.003
30	4.250	3.861	0.008	3.868	0.003	3.827	0.003
31	3.250	2.839	0.007	2.889	0.003	2.892	0.002
32	2.250	2.010	0.006	1.932	0.003	1.907	0.002
33	1.250	0.942	0.006	0.942	0.002	0.960	0.001
34	0.333	0.217	0.003	0.232	0.001	0.229	0.001
35	0.001	0.012	0.000	0.012	0.000	0.001	0.000

In Table 3 we report the option prices obtained with 10.000 (column 3 and 4), 50.000 (column 5 and 6) and 100.000 paths (column 7 and 8). As we can observe, simulated prices in all cases underestimate the EEX price: in our opinion these results reveal an overestimate of the established derivative EEX prices. Our most precise estimates are obtained with 100.000 paths, revealing the good performance of the model in its Monte Carlo implementation. Results are precise even with the lower number of simulations, because the error we make is significant at worst at the third decimal place, already with 50.000 simulations. In order to verify the seasonality hypothesis, we have implemented the model by using the volatility parameters of Table 2 in a “flat”mode, i.e. by averaging the monthly volatility estimates. In Figures (5), (6) and (7) are visualized the simulated prices: EEX market prices, the prices obtained with the seasonal volatilities (MC seasonal) and the prices obtained with the “flat”volatility (MC flat). In all cases the seasonal volatility estimates reveals to be a valid instrument for the derivative pricing, because the prices obtained are more accurate in respect to the ones obtained with a flat volatility (Figures 5a, 6a and 7a). The seasonal model allows us to reach a better accuracy in the estimates, as we can observe in Figures 5b, 6b and 7b: these pictures represent the percentage differences, for the various strike prices, by comparing the EEX price both with the seasonal volatility price estimates (MC seasonal), and with the flat volatility



(a) Actual prices vs Simulated Prices. (b) Percentage Differences.

Figure 5: Monte Carlo simulation with 10.000 paths.



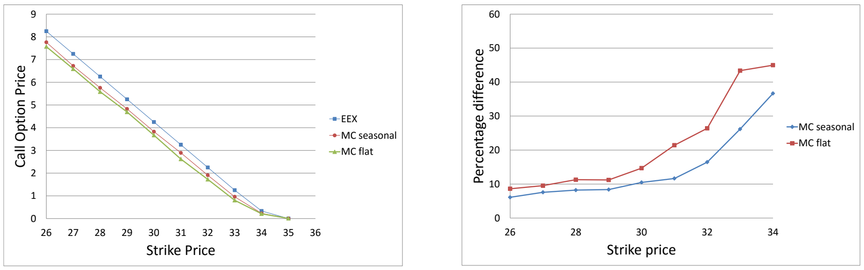
(a) Actual prices vs Simulated Prices. (b) Percentage Differences.

Figure 6: Monte Carlo simulation with 50.000 paths.

price estimates (MC flat). If we focus on the seasonal volatility model, the simulated prices have a percentage difference lower than 10% for strike prices from 26 €/MWh to 30 €/MWh, and lower than 20% for the strike price 32 €/MWh, while for the flat volatility model, the percentage difference results to always be greater, and for exercise prices greater than 31 €/MWh, the percentage error in the simulated price is more than 20%. The best accuracy of the estimated prices reveals that the seasonal model is a more adequate method for pricing derivatives on electricity futures, in comparison with a model that doesn't take into account the seasonality of volatilities. Results demonstrate that the developed model can be a valid tool for risk management,because it allows us to simulate and forecast futures prices and financial derivatives in an accurate way.

4. Hedging strategy

In this section, we show with an example how the developed model finds very relevant and useful applications in the risk management field. Trading



(a) Actual prices vs Simulated Prices. (b) Percentage Differences.

Figure 7: Monte Carlo simulation with 100.000 paths.

companies in electricity markets are among the most active users of financial derivatives. This is mostly due to the high volatility of electricity prices that leads companies to require rigorous hedging, but it is also due to the fact that increased competition within the industry forces companies to find new sources of profit, such as arbitrage and speculation. A great deal of recent literature discusses the role of financial derivatives in risk management strategies in electricity sector (see Brown and Toft (2002), Bruno and Fanelli (2016), Sanda et al. (2013), Stulz (1996)). Trading companies are used to assume a position on electricity futures contracts that can be adjusted with an option position in order to capture futures price features and to achieve an optimal risk exposure.

A natural candidate for selective hedging strategies in electricity markets is a covered call. This strategy assumes a short position on a European call option and a long forward position on the underlying of the call, that in our case is the futures contract. Trading companies could use a covered call strategy when they believe that the futures price will not rise much during the investment period, and it is therefore willing to sell the upside potential of the forward contract for the premium of the call option.

The trading company can choose how much upside potential it is willing to sell by selecting a suitable strike price for the call option. Thus, the strike price of the option is chosen according to company risk aversion or perceived return potential. It is known that higher strike prices allow the investor to hold more upside potential for itself, but, on the other hand, it receives a lower premium from the sale of the option.

We implement a covered call strategy that consists of a long forward position on one month futures price that has forward price F and maturity s, and a short position on a European call option on the same futures with the same maturity s and strike price E. The cash flows of the strategy are summarized in Table 4.

Table 4: **Covered call strategy cash flows.**
The strategy consists of a long forward position on a futures contract with maturity one year and forward price F and a short position on a European call option with the same maturity s. V_0 indicates the call premium and E the strike price. $F_s(\tau)$ is the futures price at maturity s. $\Phi_s = (F_s(\tau) - E)^+$ is the call payoff at maturity s.

Time	Forward	Call Option
0	0	$+V_0$
s	$F_s(\tau) - F$	$-\Phi_s$

As we show in Table 4, at inception, the value of the forward contract is null, whereas the call option value is V_0, that represents the premium, given by formula (25). At maturity s the value of position on the forward is given by the difference between the market actual futures price $F_s(\tau)$ and F, whereas the payoff of the call is $\Phi_s = (F_s(\tau) - E)^+$.

We implement the trading company strategy by assuming that F is equal to 32 €/MWh, s is one year, and that company believes that the futures price will not rise much during the investment period, so that it sells a call with 33 €/MWh strike price. According to Table 3, the estimated call premium is $V_0 = 0.96$ €.

The net profit structure of the strategy at maturity is given by Figure 8.

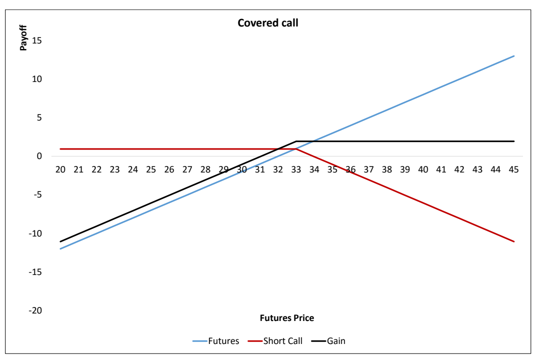


Figure 8: Net profit strategy at maturity as function of the electricity futures price at maturity.

Looking at Figures 8, we observe that the company has a low risk profile and it expects a decrease of futures prices because it prefers to limit its potential gains in the case of price increase in order to reduce losses in the event of price decrease.

5. Conclusions

In this paper, we have presented a futures price model which allows us to look into observed stylized facts in the electricity market, in particular stochastic price variability, and periodic behaviour. We have built on the existing literature (Hinz and Wilhelm (2006)) and we have modelled the futures price assuming a virtual fixed income market linked to the real electricity market. We have considered a seasonal path-dependent volatility for futures returns that have been modelled according to the Heath et al. (1992) approach. On the one hand, we have focused on seasonality by observing prices and obtaining the calendar-varying volatility parameters for each month of the year. The Heath et al. (1992) model gives us the opportunity to simulate the daily evolution of the forward curve in the free-arbitrage environment and therefore determine the daily futures price according to the volatility seasonal parameter of each specific day. On the other hand, the chosen volatility has even a contingent component depending also on the spot return level. In this way the behaviour of the spot price affects the futures price allowing to distinguish between a normal market and a turbulent one.

We have calibrated the model and estimated volatility parameters on one month futures prices in German electricity markets, in order to forecast futures prices. These prices have been used as underlying of a call option in a risk management perspective. We have therefore estimated option values according to different strike prices by applying the Monte Carlo approach and we have obtained results that are very accurate and coherent with the actual quotations.

We have implemented an algorithm which is able to capture one of the main characteristics of the electricity market, i.e. the seasonality, interpreting it as the accurate study of the volatility patterns along historical data: our contribution to the existing literature is to allow past observation to definitely model the price forecasting, affecting it with the daily definition of the volatility level, according to the calendar day (calendar day forecast), the reached spot level (stochastic volatility), and the developed price path (path dependence).

We are confident that this approach could be a valid instrument for supporting risk management in the electricity field, because it gives us the opportunity to study the price evolution obtainable with different volatility parameters over different time periods, letting the model adequately reflect the electricity market seasonality.

Appendix A. Nomenclature

$(\Omega, \mathcal{F}, P)$	Filtered probability space
$(W_t)_{t\geq 0}$	Brownian motion
$(F_t)_{t\geq 0}$	Electricity futures price
Q^F	Risk neutral measure in the electricity market
Q^M	Risk neutral measure in the CCEM
$(p_t)_{t\geq 0}$	Electricity price in the CCEM
$(N_t)_{t\geq 0}$	Risky asset in the CCEM
$(B_t)_{t\geq 0}$	Spot zero coupon bond in the CCEM
$(f_t)_{t\geq 0}$	Futures interest rate in the CCEM
$\alpha(t, T, \cdot)$	Instantaneous futures rate drift function
$\sigma(t, T, \cdot)$	Instantaneous futures rate volatility function
$v(t)$	Volatility of N_t
$S(t, T)$	Seasonal term of the futures interest rate volatility
$X(t, T)$	Path-dependent term of the futures interest rate volatility
σ_1	Short term futures interest rate volatility coefficient
σ_2	Long term futures interest rate volatility coefficient
λ	Time decay futures interest rate volatility coefficient
D_i	Dummy variable
r_{ij}^k	Return level of the futures contract p_{ij}^k
p_{ij}^k	The futures price observed at the $i-th$ day of the $j-th$ month of the year with delivery period k
$C(0, t, T)$	Time-zero price of a European call on the electricity futures price $F(t, T)$
$C_{MC}(0, t, T)$	Monte Carlo price of a European call on the electricity futures price $F(t, T)$
Φ_T	Time-T payoff of a European call on the electricity futures price $F(t, T)$
E	Strike price of a European call on the electricity futures price $F(t, T)$
Π	Simulation number

Appendix B. The Heath-Jarrow-Morton model

In the Heath et al. (1992) model instantaneous forward rates for fixed maturity T are modelled by the stochastic process

$$df(t,T) = \alpha(t,T)\,dt + \sigma_f(t,T)\,dW(t) \tag{B.1}$$

where

$f(t,T)$ is the forward rate at time t applicable to time T, $t < T$;
$f(0,T)$ is the initial forward rate curve observable at time $t = 0$;
$\alpha(t,T,\cdot)$ is the instantaneous drift function;
$\sigma(t,T,\cdot)$ is the instantaneous volatility function;
$W(t)$ is a Brownian motion generated by the probability measure P.

The symbol "·" as third argument in the drift and volatility function represents the possible dependence on path dependent quantities as the spot rate $r(t)$ or the forward rate $f(t,T)$. Integrating B.1 we obtain the instantaneous forward rate expressed in integral form:

$$f(t,T) = f(0,T) + \int_0^t \alpha(v,T,\cdot)\,dv + \int_0^t \sigma_f(v,T,\cdot)\,dW(v). \tag{B.2}$$

From the integral form we can obtain the spot rate process, simply by substituting $T = t$

$$r(t) = f(0,t) + \int_0^t \alpha(v,t,\cdot)\,dv + \int_0^t \sigma_f(v,t,\cdot)\,dW(v). \tag{B.3}$$

The price of a pure discount bond maturing at time T results therefore defined as:

$$P(t,T) = \exp\left(-\int_t^T f(t,s)\,ds\right). \tag{B.4}$$

By applying the natural logarithm:

$$lnP(t,T) = -\int_t^T f(t,s)\,ds.$$

By the use of the Fubini's Theorem and rearranging, we have:

$$\ln P(t,T) = \ln P(0,T) + \int_0^t \left[r(v) - \int_v^T \alpha(v,s)\, ds \right] dv \\ - \int_0^t \int_v^T \sigma_f(v,s)\, ds\, dW(v).$$

By defining

$$a(v,t) := -\int_v^t \sigma_f(v,s)\, ds,$$

$$b(v,t) := -\int_v^t \alpha(v,s)\, ds + \frac{1}{2} a^2(v,t), \tag{B.5}$$

applying Ito's Lemma, the stochastic differential equation for the bond price stochastic process is derived:

$$dP(t,T) = [r(t) + b(t,T)]\, P(t,T)\, dt + a(t,T) P(t,T)\, dW(t)$$

Applying the no arbitrage condition, the following relation has to hold:

$$b(t,T) + a(t,T)\Phi(t) = 0, \tag{B.6}$$

where $\Phi(t)$ is the market price of interest rate risk. By substituting B.6 in B.5 we obtain:

$$\begin{aligned} \alpha(t,T) &= \sigma_f(t,T) \int_t^T \sigma_f(t,s)\, ds - \sigma_f(t,T)\Phi(t) \\ &= -\sigma_f(t,T) \left[\Phi(t) - \int_t^T \sigma_f(t,s)\, ds \right]. \end{aligned} \tag{B.7}$$

This is the celebrate forward rate drift restriction obtained by HJM. We define a new Brownian motion:

$$\widetilde{W} = W(t) + \int_0^t (-\Phi(s))\, ds \tag{B.8}$$

and

$$d\widetilde{W}(t) = dW(t) - \Phi(t)\, dt \Rightarrow dW(t) = d\widetilde{W} + \Phi(t)\, dt. \tag{B.9}$$

By substituting B.7 and B.9 in B.1 we obtain:

$$\begin{aligned} df(t,T) &= \alpha(t,T)\,dt + \sigma_f(t,T)\,dW(t) \\ &= \left[\sigma_f(t,T)\int_t^T \sigma_f(t,s)\,ds - \sigma_f(t,T)\Phi(t)\right] dt \\ &\qquad + \sigma_f(t,T)\left[d\widetilde{W}(t) + \Phi(t)\,dt\right] \\ &= \sigma_f(t,T)\int_t^T \sigma_f(t,s)\,ds\,dt - \sigma_f(t,T)\Phi(t)\,dt + \sigma_f(t,T)d\widetilde{W}(t) \\ &\qquad + \sigma_f(t,T)\Phi(t)\,dt \\ &= \sigma_f(t,T)\int_t^T \sigma_f(t,s)\,ds\,dt + \sigma_f(t,T)d\widetilde{W}(t) \end{aligned}$$

which expressed in its integral form is:

$$f(t,T) = f(0,T) + \int_0^t \sigma_f(v,T)\int_v^T \sigma_f(v,s)\,ds\,dv + \int_0^t \sigma_f(v,T)d\widetilde{W}(v) \tag{B.10}$$

From B.10 we derive the spot rate process $r(t) = f(t,t)$:

$$r(t) = f(0,t) + \int_0^t \sigma_f(v,t)\int_v^t \sigma_f(v,s)\,ds\,dv + \int_0^t \sigma_f(v,t)d\widetilde{W}(v). \tag{B.11}$$

Choosing the money market account:

$$B(t,T) = e^{\int_0^t r(s)\,ds}$$

as numerarire, relative bond prices are given by:

$$Z(t,T) = \frac{P(t,T)}{B(t)} = P(t,T)\,e^{-\int_0^t r(s)\,ds}. \tag{B.12}$$

Application of Ito's Lemma to B.12 gives:

$$\begin{aligned} dZ(t,T) &= \left[-r(t)Z(t,T) + (r(t) + b(t,T))\,P(t,T)\,e^{-\int_0^t r(s)\,ds}\right] dt \\ &\qquad + a(t,T)P(t,T)e^{-\int_0^t r(s)\,ds}\,dW(t) \\ &= \left[-r(t)Z(t,T) + r(t)Z(t,T) + b(t,T)Z(t,T)\right] dt \\ &\qquad + a(t,T)Z(t,T)\,dW(t) \\ &= b(t,T)Z(t,T)\,dt + a(t,T)Z(t,T)\,dW(t). \end{aligned} \tag{B.13}$$

By Girsanov's Theorem the process B.13 can be written in terms of the Brownian motion B.8 resulting generated by an equivalent martingale probability measure $\widetilde{P}$:

$$\begin{aligned} dZ(t,T) &= -\Phi(t)a(t,T)Z(t,T)\,dt + a(t,T)Z(t,T)\,\left[d\widetilde{W}(t) + \Phi(t)\,dt\right] \\ &= -\Phi(t)a(t,T)Z(t,T)\,dt + \Phi(t)a(t,T)Z(t,T)\,dt + a(t,T)Z(t,T)\,d\widetilde{W}(t) \\ &= a(t,T)Z(t,T)\,d\widetilde{W}(t). \end{aligned}$$

The principal characteristic of the HJM model is that in this new formulation in both the bond price process B.14 and the spot rate process B.11, the market price of risk drops out. Thus derivative securities can be calculated independently of the market price of risk. Equation B.14 shows that under the probability measure $\widetilde{P}$, $Z(t,T)$ is a martingale and the bond value at any time t can be calculated simply as expected value which respect to the probability measure $\widetilde{P}$:

$$P(t,T) = \mathbb{E}_{\tilde{P}}\left[e^{-\int_t^T r(s)\,ds}\,|\mathcal{F}_t\right]. \tag{B.14}$$

$$\tag{B.15}$$

Analogously the value of the option on the bond can be obtained by taking the expectation with respect to the risk adjusted measure $\tilde{P}$ of the discounted payoff. Suppose we wish to price at time 0 a European call option with maturity time T_c, $0 \leq T_C \leq \overline{T}$ and exercise price E, its value is given by

$$C(0,T_C,T) = \mathbb{E}_{\tilde{P}}\left[e^{-\int_0^{T_C} r(s)\,ds}(P(T_c,T)-E)^+\,|\mathcal{F}_0\right]. \tag{B.16}$$

If the model has a Markovian representation, by applying of the Feynman-Kac Theorem, the derivative prices can be obtained by solving the partial differential equation. Otherwise numerical simulations as Monte Carlo simulation are used to evaluate derivatives.

References

René Aid, Luciano Campi, and Nicolas Langrené. A structural risk-neutral model for pricing and hedging power derivatives. *Mathematical Finance*, 23 (3):387–438, 2013.

Andrea Barth and Fred Espen Benth. The forward dynamics in energy markets - infinite-dimensional modelling and simulation. *Stochastics An International Journal of Probability and Stochastic Processes*, 86(6):932–966, 2014.

Jozef Baruník and Barbora Malinska. Forecasting the term structure of crude oil futures prices with neural networks. *Applied Energy*, 164:366–379, 2016.

Fred Espen Benth and Steen Koekebakker. Stochastic modeling of financial electricity contracts. *Energy Economics*, 30(3):1116–1157, 2008.

Fred Espen Benth, Jurate Saltyte Benth, and Steen Koekebakker. *Stochastic modelling of electricity and related markets*, volume 11. World Scientific, 2008.

Petter Bjerksund, Heine Rasmussen, and Gunnar Stensland. Valuation and risk management in the Norwegian electricity market. In *Energy, natural resources and environmental economics*, pages 167–185. Springer, 2010.

Gregory W. Brown and Klaus Bjerre Toft. How firms should hedge. *Review of Financial Studies*, 15(4):1283–1324, 2002.

Luana Bruno and Viviana Fanelli. Modelling the Italian electricity spot price in a risk management perspective. *Working paper*, 2016.

Carl Chiarella, Viviana Fanelli, and Silvana Musti. Modelling the evolution of credit spreads using the Cox process within the HJM framework: A CDS option pricing model. *European Journal of Operational Research*, 208(2):95–108, 2011.

Les Clewlow and Chris Strickland. *Energy derivatives: pricing and risk management*. Lacima Publ., 2000.

Antonio J. Conejo, Francisco J. Nogales, Miguel Carrión, and Juan M. Morales. Electricity pool prices: long-term uncertainty characterization for futures-market trading and risk management. *Journal of the Operational Research Society*, pages 235–245, 2010.

Jesús Crespo Cuaresma, Jaroslava Hlouskova, Stephan Kossmeier, and Michael Obersteiner. Forecasting electricity spot-prices using linear univariate time-series models. *Applied Energy*, 77(1):87–106, 2004.

Shijie Deng. *Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes*. Citeseer, 2000.

Abdou Kâ Diongue, Dominique Guegan, and Bertrand Vignal. Forecasting electricity spot market prices with a k-factor gigarch process. *Applied Energy*, 86(4):505–510, 2009.

Alexander Eydeland and Krzysztof Wolyniec. *Energy and power risk management: New developments in modeling, pricing, and hedging*, volume 206. John Wiley & Sons, 2003.

Viviana Fanelli. A defaultable HJM modelling of the Libor rate for pricing Basis Swaps after the credit crunch. *European Journal of Operational Research*, 249 (1):238–244, 2016.

Viviana Fanelli, Lucia Maddalena, and Silvana Musti. Asian options pricing in the day-ahead electricity market. *Working paper*, 2016.

Carolina García-Martos, Julio Rodríguez, and María Jesús Sánchez. Modelling and forecasting fossil fuels, CO 2 and electricity prices and their volatilities. *Applied Energy*, 101:363–375, 2013.

David Heath, Robert Jarrow, and Andrew Morton. Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica: Journal of the Econometric Society*, pages 77–105, 1992.

Helen Higgs and Andrew Worthington. Stochastic price modeling of high volatility, mean-reverting, spike-prone commodities: The Australian wholesale spot electricity market. *Energy Economics*, 30(6):3172–3185, 2008.

Juri Hinz. Modelling day-ahead electricity prices. *Applied mathematical finance*, 10(2):149–161, 2003.

Juri Hinz. Valuing virtual production capacities on flow commodities. *Mathematical Methods of Operations Research*, 64(2):187–209, 2006.

Juri Hinz and Martina Wilhelm. Pricing flow commodity derivatives using fixed income market techniques. *International Journal of Theoretical and Applied Finance*, 9(08):1299–1321, 2006.

Juri Hinz, Lutz Von Grafenstein, Michel Verschuere, and Martina Wilhelm. Pricing electricity risk by interest rate methods. *Quantitative Finance*, 5(1): 49–60, 2005.

Ronald Huisman and Mehtap Kiliç. A history of European electricity day-ahead prices. *Applied Economics*, 45(18):2683–2693, 2013.

Ronald Huisman and Ronald Mahieu. Regime jumps in electricity prices. *Energy Economics*, 25(5):425–434, 2003.

Dogan Keles, Jonathan Scelle, Florentina Paraschiv, and Wolf Fichtner. Extended forecast methods for day-ahead electricity spot prices applying artificial neural networks. *Applied Energy*, 162:218–230, 2016.

Steen Koekebakker and Fridthjof Ollmar. Forward curve dynamics in the Nordic electricity market. *Managerial Finance*, 31(6):73–94, 2005.

Julio J. Lucia and Eduardo S. Schwartz. Electricity prices and power derivatives: Evidence from the Nordic power exchange. *Review of Derivatives Research*, 5(1):5–50, 2002.

Timothy D. Mount, Yumei Ning, and Xiaobin Cai. Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters. *Energy Economics*, 28(1):62–80, 2006.

Jakub Nowotarski, Jakub Tomczyk, and Rafał Weron. Robust estimation and forecasting of the long-term seasonal component of electricity spot prices. *Energy Economics*, 39:13–27, 2013.

Dragana Pilipovic. *Energy Risk: Valuing and Managing Energy Derivatives: Valuing and Managing Energy Derivatives*. McGraw Hill Professional, 2007.

Gaute Egeland Sanda, Eirik Tandberg Olsen, and Stein-Erik Fleten. Selective hedging in hydro-based electricity companies. *Energy Economics*, 40:326–338, 2013.

Petter Skantze, Andrej Gubina, and Marija Ilic. Bid-based stochastic model for electricity prices: the impact of fundamental drivers on market dynamics. In *Energy Laboratory Publications MIT EL 00-004, Massachusetts Institute of Technology*. Citeseer, 2000.

René M. Stulz. Rethinking risk management. *Journal of applied corporate finance*, 9(3):8–25, 1996.

Zhongfu Tan, Jinliang Zhang, Jianhui Wang, and Jun Xu. Day-ahead electricity price forecasting using wavelet transform combined with arima and garch models. *Applied Energy*, 87(11):3606–3610, 2010.

Rafa Weron. Electricity price forecasting: A review of the state-of-the-art with a look into the future. *International Journal of Forecasting*, 30(4):1030 – 1081, 2014. ISSN 0169-2070.