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Information in correlation detection for plenoptic imaging purposes

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Abstract: In the present work, we quantify the information content of intensity correlation measurements as compared to conventional intensity detection. In particular, we focus on the problem of estimating the separation between two point-like emitters and evaluate Fisher information for the two types of measurements. We consider two different scenarios: intensity correlations are measured, in the first case, in the image plane, and, in the second case, between the image and the Fourier plane of the object. Both detection schemes are shown to benefit from measuring correlations between the detected intensities, a result that indicates the advantages offered by both quantum and quantum-inspired (i.e., correlation-based) imaging protocols over conventional, intensity based, approaches.

1. Introduction

Correlation plenoptic imaging (CPI) is a newly developed 3D imaging technique that exploits photon number correlations on two detectors to retrieve the *light-field* [1–4]. CPI has been shown to be characterized by an advantageous imaging performance compared to standard single sensor light-field imaging (LFI) designs, which collect the light field by inserting a lenslet array into the optical path [5–7]. In fact, the microlenses impose a substantial trade-off between resolution and 3D capability of LFI [8]; such a trade-off can be completely overcome in the lenslet-free design of CPI [3,9,10]. Interestingly, when the image of a defocused object is reconstructed through CPI, the resulting lateral resolution is independent of the numerical aperture (NA) [2,10]; in fact, the NA defines the lateral resolution only at focus (i.e., the Rayleigh limit), where no reconstruction is typically needed. This feature has been recently associated with the properties of spatial coherence of light [11]. However, a large NA is still very beneficial for CPI, both because it entirely defines the tomographic capabilities of the technique (namely, its axial resolution) [10] and because it affects the noise performance [12].

The procedure currently used to extract images from the four-dimensional correlation function measured in CPI is the so-called *refocusing algorithm* [13], which recovers a specific axial section of the scene of interest based on purely geometrical arguments. Refocusing, however, might not be the most efficient way of extracting the information about the sample that is encoded within the measured correlation function. In this paper, we characterize the unprocessed correlation function in terms of its Fisher information content and address the problem of analyzing the information content of the plenoptic image processing.

The mathematical formulation of our problem hinges upon the description of the propagating signal described by the complex amplitude u. We will formulate the problem advantageously by adopting quantum formalism where this amplitude is described, in the x-representation, by the

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quantum state $u(x) = \langle x|u\rangle$. Propagation is described by the generic integral transformation

$$u_{out}(x) = \int dq'\, u_{in}(q')\, h(x-q'), \tag{1}$$

where h is the propagator. Notice that the measurement is done on the output amplitude u_{out}, while the input amplitude u_{in} depends on the sample unknown parameter(s). The measured intensity and second-order correlation function are given, respectively, by

$$I(x) \equiv \left\langle u^*_{out}(x)\, u_{out}(x)\right\rangle = \int dq'dq''\, h^*(x-q')\, h(x-q'')\left\langle u^*_{in}(q')u_{in}(q'')\right\rangle, \tag{2}$$

$$\begin{aligned} G(x_1,x_2) &\equiv \left\langle u^*_{out}(x_2)\, u^*_{out}(x_1)\, u_{out}(x_2)\, u_{out}(x_1)\right\rangle \\ &= \int dq_1'\, dq_2'\, dq_1''\, dq_2''\, h_1^*(x_1-q_1')\, h_2^*(x_2-q_2')\, h_1(x_1-q_1'')\, h_2(x_2-q_2'') \\ &\quad \left\langle u^*_{in}(q_1')\, u^*_{in}(q_2')\, u_{in}(q_1'')\, u_{in}(q_2'')\right\rangle. \end{aligned} \tag{3}$$

In this work, we shall always assume a fully incoherent input signal, as described by the relationship

$$\left\langle u^*_{in}(q')\, u_{in}(q'')\right\rangle = \delta(q'-q'')\, I_{in}(q'). \tag{4}$$

In this hypothesis, the read-out signal reduces to

$$I_{out}(x) \equiv \left\langle u^*_{out}(x)\, u_{out}(x)\right\rangle = \int dq'\, |h(x-q')|^2\, I_{in}(q'), \tag{5}$$

$$G(x_1,x_2) = I_{out}(x_1)\, I_{out}(x_2) + \left|\int dq'\, h_1^*(x_1-q')\, h_2(x_2-q')\, I_{in}(q')\right|^2. \tag{6}$$

The critical problem for information processing of correlation measurements is to exploit the full information content of the acquired data. Actually, measuring intensity correlations in CPI amounts to extracting information from the intensity statistics detected by two separate spatially-resolving sensors (see, e.g., [14,15]). Thus, this measurement is expected to provide an increase in Fisher information compared to the conventional measurement of the mean intensity. The underlying hypothesis is that the employed illumination is characterized by spatio-temporal correlations (i.e., it is a thermal or an entangled photon source), giving rise to either purely quantum or HBT-like photon bunching phenomena.

Notice that, as this study is confined to resolving the image plane, the utilization of detection schemes employing microlens arrays, which merely diminishes the resolution, is not considered in our analysis.

2. Detection model

Inspired by the work reported in Ref. [16], we shall now formulate the model for detection with correlated and non-correlated light. As reported in Fig. 1, we shall consider the signal, which depends on the unknown parameter s, to be split by a beam splitter into two channels while being mixed with a vacuum. Two detectors placed in the output ports of the beam splitter detect the signal in the transverse x-y plane, where only 1D profiles are processed to keep the discussion simple. The signal is assumed to be faint enough for the detectors to click separately, and correlations in the given detection window appear rarely.

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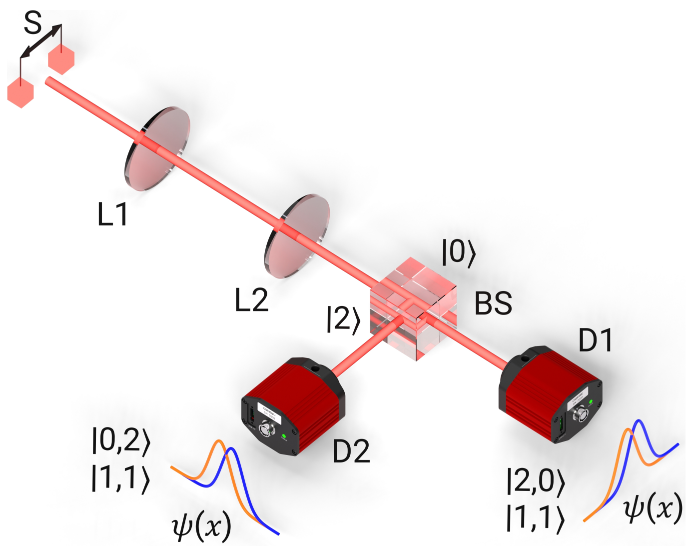

Fig. 1. Two emitters, separated by a distance s, are imaged by a lens system L1 and L2 with magnification $m = 1$. The beam splitter (BS) directs the 2-photon input state $|2\rangle$ to the spatially resolving detectors D_1 and D_2, which project the output states of the BS into the detected mode $\psi(x)$, and mix it with the vacuum state $|0\rangle$. The 2-photon input state gives rise to two possible outcomes for each port: $|2,0\rangle$ and $|1,1\rangle$ or $|0,2\rangle$ and $|1,1\rangle$.

Let us define the 1-photon and 2-photon states as

$$|1\rangle = \sum_i \psi_i^* a_i^\dagger |0\rangle, \tag{7}$$

$$|2\rangle = \left(\sum_i \psi_i^* a_i^\dagger\right)\left(\sum_j \psi_j^* a_j^\dagger\right)|0\rangle, \tag{8}$$

and write the input modes of the beam splitter, a_i, in terms of the output modes c_i, d_i, as given by transformation

$$a_i = \frac{1}{\sqrt{2}}(c_i + d_i). \tag{9}$$

The detection is done by projecting the output states of the beam splitter into detected modes, which we describe as wave packets corresponding to a localization of the detected photons. Obviously, the 1-photon state can only contribute to the detection of uncorrelated events (1,0) and (0,1), which are characterized, in the x-representation of the input quantum state, by equal probabilities $\frac{1}{2}|\psi(x)|^2$. Let us now focus on the case of two-point emitters with the relative phase φ, as described by the wave function

$$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi^{(1)}(x) + e^{i\varphi}\psi^{(2)}(x)\right). \tag{10}$$

When the relative phase φ is fluctuating, the detected signal reads

$$S_1 = \left\langle |\psi(x)|^2 \right\rangle_\varphi = \frac{1}{2}\left(\left|\psi^{(1)}(x)\right|^2 + \left|\psi^{(2)}(x)\right|^2\right). \tag{11}$$

This signal was analyzed using super-resolution protocols inspired by the quantum estimation theory [17–19].

Similarly, the 2-photon state can contribute to the correlated events (2,0), (1,1), and (0,2) detected in positions x_1, x_2 with the total probability density given by $|\psi(x_1)|^2, |\psi(x_2)|^2$. Finally, the contribution averaged over phase reads

$$\begin{aligned} S_2 &= \left\langle |\psi(x_1)|^2 \, |\psi(x_2)|^2 \right\rangle_\varphi \\ &= \frac{1}{4}\left(\left|\psi^{(1)}(x_1)\right|^2 + \left|\psi^{(2)}(x_1)\right|^2\right)\left(\left|\psi^{(1)}(x_2)\right|^2 + \left|\psi^{(2)}(x_2)\right|^2\right) \\ &+ \frac{1}{4}\left[\psi^{(1)*}(x_1)\,\psi^{(2)}(x_1)\,\psi^{(1)}(x_2)\,\psi^{(2)*}(x_2) + cc.\right]. \end{aligned} \tag{12}$$

Depending on the statistics of the signal, these contributions S_1 and S_2 should be taken with the probabilities of occurrence of a particular n-photon state in the signal. The problem can be straightforwardly extended for detection in arbitrary plane after the beam splitter, e.g., Fourier plane [20], or a combination of different planes on different output ports.

3. Fisher information in correlated measurements

3.1. Fisher information for a single parameter estimation

Let us briefly review the statistical theory of Fisher information for single parameter estimation. To this end, we shall re-derive the form that is typically used in physical applications. Let us assume the data detected in the $i-$th detector are random variables n_i fluctuating with a probability $p(n_i|s)$, which depends on the unknown parameter s. The likelihood function is defined as

$$\mathcal{L} = \Pi_{\{n_i\}}\, p(n_i|s), \tag{13}$$

such that $\sum_{n_1,n_2,n_3\cdots} \mathcal{L} = 1$ represents the normalized multidimensional probability distribution in the data space $\{n_i\}$. Fisher information ($\mathcal{F}$) provides the limit for any unbiased estimator of parameter s denoted here as $\hat{s}$ according to the Cramér-Rao inequality

$$\mathrm{Var}(\hat{s}) \geq \frac{1}{\mathcal{F}}, \tag{14}$$

$$\mathcal{F} = \left\langle (\partial_s \ln \mathcal{L})^2 \right\rangle. \tag{15}$$

Averaging is assumed with respect to the likelihood, and explicit form depends on statistics of detected data. We will assume a multinomial distribution of detected events n_i under the condition $\sum_i n_i = N$. In this case

$$(\partial_s \ln \mathcal{L}) = \sum n_i \frac{\partial_s p_i}{p_i}, \tag{16}$$

$$\mathcal{F} = \sum_{i,j} \left\langle n_i n_j \right\rangle \frac{\partial_s p_i}{p_i} \frac{\partial_s p_j}{p_j}. \tag{17}$$

Since variance and covariance of multinomial distribution is given as

$$\mathrm{Var}(n_i) = N p_i (1 - p_i), \tag{18}$$

$$\mathrm{Cov}(n_i, n_j) = -N p_i p_j, \; i \neq j, \tag{19}$$

we get finally

$$\mathcal{F} = N \sum_i \frac{(\partial_s p_i)^2}{p_i}. \tag{20}$$

This particular form is also valid for other distributions, e.g., the Poissonian distribution, and is adopted for optimization over all possible measurements yielding quantum Fisher information.

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Notice that the variance of the estimator scales with a number of detected events N, which appears in the product with Fisher information. Statistical theory is valid both for experiments with one photon, which must be repeated N times, as well as for experiments with a strong field, where the total intensity is distributed immediately between the detection channels. Since our approach is related to parameter estimation read out from partially coherent signals, we normalize resolution to equal resources by multiplying the Fisher information by the strength of the detected signal. In other words, Fisher information quantifies the resolution per one detected event, but signals may differ in strength. Both cases will be plotted in investigated correlation schemes.

3.2. Chain rule for Fisher information

The main goal of this article is to evaluate the information in intensity correlated measurements and compare it with those for standard (uncorrelated) intensity measurement. To this end, we shall use a fundamental theorem, the so-called chain rule for Fisher information [21], which reads:

$$\mathcal{F}_{X,Y}(\theta) = \mathcal{F}_X(\theta) + \mathcal{F}_{Y|X=x}(\theta), \tag{21}$$

where $\mathcal{F}_{Y|X=x}(\theta)$ is the Fisher information of Y relative to θ calculated with respect to the conditional density of Y given a specific value $X = x$.

Let us summarize how to apply the chain rule in the context of optical signals and their statistical properties, particularly when considering signals composed of contributions from different photon numbers. These contributions are detected independently by different detectors; hence, the signals generated by different photon numbers are not directly correlated with each other. As discussed in this context, the chain rule applies to the correlated probability distributions that arise from contributions with the same photon number, but does not apply across different photon numbers since contributions from different photon numbers are independent and uncorrelated.

The implications of the above considerations for the specific states of interest are the following: The state $|1\rangle$, which appears in the signal with probability a_1, contributes only to the uncorrelated signal, while the state $|2\rangle$, which appears with probability a_2, contributes both to correlated and the uncorrelated components of the signal. The argument can clearly be extended to contributions from higher photon numbers: Each distinct photon number contributes to the overall signal independently, and the analysis can be generalized accordingly. Fisher information, which measures the amount of information that an observable random variable can carry about an unknown parameter, is given by the sum of the contributions from all mutually exclusive events (i.e., each event characterized by a different photon number). This sum allows quantifying of the contributions to Fisher information from each distinct photon number.

In essence, this emphasizes that when analyzing the Fisher information for parameter estimation in optical signals, one must consider the independent contributions from different photon numbers separately and then sum them to get the total Fisher information. This approach ensures that all contributions, whether correlated or uncorrelated, are properly accounted for the analysis.

4. Estimate of two points separation

We shall now consider the signal coming from two equal intensity emitters placed in a transversal plane to the optical axis, separated by a distance s and assuming a priori information about the emitters centroid. We study parameter s estimation through correlation measurements performed in two scenarios: detection in the image plane (S) and detection in the image and the Fourier (F) planes according to plenoptic imaging scheme. Both situations are depicted in Fig. 1 and 2, respectively.

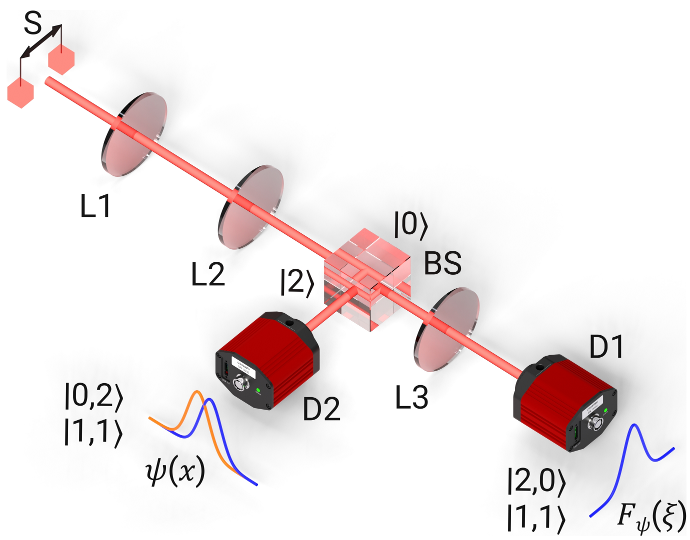

Fig. 2. Two emitters, separated by a distance s, are imaged by a lens system L1 and L2 with a magnification $m = 1$. Lens L_3 in the transmitted arm of the beam splitter (BS) performs a Fourier transform. BS directs the 2-photon input state $|2\rangle$ to position resolving detectors D1 and D2, which project the BS output states into the detected modes $F_\psi(\xi)$ and $\psi(x)$, respectively, and mix it with the vacuum state $|0\rangle$. Because of the 2-photon input state, there are two possible outcomes for each port: $|2, 0\rangle$ and $|1, 1\rangle$ or $|0, 2\rangle$ and $|1, 1\rangle$.

4.1. Correlation detection in the image plane

For the problem of detection in the image plane, we denote the system point spread function (PSF) in the two arms of the plenoptic system as

$$\psi^{(1,2)}(x) = \psi(x \pm s/2), \tag{22}$$

and assume for simplicity ψ to be real and Gaussian

$$\psi = \sqrt[4]{\frac{1}{2\pi}}\sqrt{\frac{1}{\sigma}}e^{\frac{-x^2}{4\sigma^2}}, \ \sigma = 1. \tag{23}$$

The detected signal for one-photon contributions is

$$S_1 = \frac{1}{2}\Big(\psi^2(x - s/2) + \psi^2(x + s/2)\Big), \tag{24}$$

and the term tends to classical Fisher information for intensity measurement

$$\mathcal{F}_1(s) = \int dx \, \frac{e^{-\frac{1}{8}(s+2x)^2}\left(e^{sx}(s - 2x) + s + 2x\right)^2}{32\sqrt{2\pi}\,(e^{sx} + 1)}. \tag{25}$$

Notice that when 2-photon correlations are measured, see Eq. (12), the first term of the detected signal is given by $S_1(x_1)S_1(x_2)$, which follows the trends of the Fisher information of intensity measurement [18]. However, correlations also give rise to an additional term

$$G_2(x_1, x_2) = \frac{1}{2}\,\psi(x_1 - s/2)\,\psi(x_1 + s/2)\,\psi(x_2 - s/2)\,\psi(x_2 + s/2), \tag{26}$$

and two-photon correlations are thus given by the signal

$$S_2(x_1, x_2) = S_1(x_1)\,S_1(x_2) + G_2(x_1, x_2), \tag{27}$$

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with the strength

$$N_2 = \frac{1}{2}e^{-\frac{s^2}{4}} + 1. \tag{28}$$

Using this notation, the Fisher information reads

$$\mathcal{F}_2(s) = \int dx_1 dx_2 \frac{Q1Q2}{32\pi Q3} \tag{29}$$

$$Q1 = \exp\left(\frac{1}{4}\left(-s^2 - 2s(x1 + x2) - 2\left(x1^2 + x2^2\right)\right)\right) \tag{30}$$

$$Q2 = \left(-Q3s + x1\left(e^{sx1} - 1\right)\left(e^{sx2} + 1\right) + x2\left(e^{sx1} + 1\right)\left(e^{sx2} - 1\right)\right)^2 \tag{31}$$

$$Q3 = 2e^{\frac{1}{2}s(x1+x2)} + e^{s(x1+x2)} + e^{sx1} + e^{sx2} + 1. \tag{32}$$

To fairly compare Fisher information for correlated and non-correlated measurements, we compute the signal coming from the marginal distribution of S_2

$$S_{2M}(x_1) = \int dx_2\, S_2(x_1, x_2) = \frac{e^{\frac{1}{4}(-s^2-2sx1-2x1^2)}\left(e^{\frac{s^2}{8}} + e^{\frac{sx1}{2}} + e^{\frac{1}{8}s(s+8x1)}\right)}{2\sqrt{2\pi}}, \tag{33}$$

which preserve the strength of the signal S_2 but describes only non-correlated detection events. The strength of the signal is the same as in the case of S_2

$$N_{2M} = N_2, \tag{34}$$

and Fisher information per one detection is

$$\mathcal{F}_{2M}(s) = \int dx_1 dx_2 \frac{e^{\frac{1}{4}(-s^2-2sx1-2x1^2)}\left(e^{\frac{s^2}{8}}(s + 2x1) + 2se^{\frac{sx1}{2}} + e^{\frac{1}{8}s(s+8x1)}(s - 2x1)\right)^2}{32\sqrt{2\pi}\left(e^{\frac{s^2}{8}} + e^{\frac{sx1}{2}} + e^{\frac{1}{8}s(s+8x1)}\right)}. \tag{35}$$

The corresponding Fisher information for signals S_2 and S_{2M} are summarized in Fig. 3. It is apparent that the information in correlated distribution is always larger than the information in the corresponding marginal distribution. This result is consistent with the so called data refinement inequality, which is a direct consequence of the chain rule [21].

4.2. Correlation detection between the image and the Fourier planes

We shall now consider the plenoptic detection scheme reported in Ref. [20], and develop an analogous theory for correlation measurements performed in the image and the Fourier planes. Since the separation s gives rise to a phase shift in the Fourier plane, the complex amplitude of the signal reads in this case

$$f(\xi) \propto F_\psi\left(e^{i\xi s/2} + e^{-i\xi s/2+i\varphi}\right), \tag{36}$$

where F_ψ is the Fourier transform of the PSF. It is evident that correlating two signals detected in the Fourier plane would not be instructive, since the detection is parameter insensitive. In

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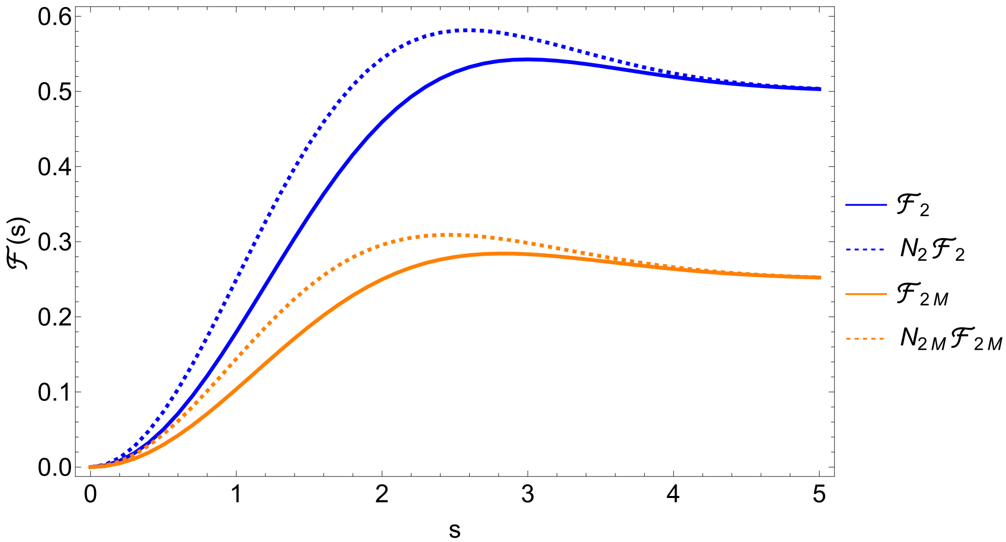

Fig. 3. Fisher information for estimating the separation s between two point emitters, as computed for the full correlation S_2 (blue) and the marginal S_{2M} (orange) distribution, which corresponds to non-correlated measurement. The curves, computed for a PSF defined by Eq. (23), show the Fisher information per one detection $\mathcal{F}$ (solid) and the overall Fisher information (dashed), as given by the product of the signal strength N and the Fisher information per one detection.

analogy with Ref. [20], we shall correlate intensities in the image and the Fourier planes, which gives the signal

$$\begin{aligned} S_F(x,\xi) &= \left\langle \left|\psi(x-s/2)+e^{i\varphi}\psi(x+s/2)\right|^2 \left|F_\psi\left(e^{i\xi s/2}+e^{-i\xi s/2+i\varphi}\right)\right|^2\right\rangle_\varphi \\ &= F_\psi^2\left(|\psi(x-s/2)|^2+|\psi(x+s/2)|^2+\psi(x-s/2)\,\psi(x+s/2)\cos(\xi s)\right). \end{aligned} \tag{37}$$

The Fisher information per detection associated with this signal comes out to be

$$\mathcal{F}_F = \int dxd\xi \frac{F_\psi^2\xi^2|\psi(x-s/2)|^2\,|\psi(x+s/2)|^2\sin(\xi s)^2}{|\psi(x-s/2)|^2+|\psi(x+s/2)|^2+\psi(x-s/2)\,\psi(x+s/2)\cos(\xi s)}, \tag{38}$$

and for our particular Gaussian PSF we receive

$$\mathcal{F}_F = \int dxd\xi \frac{e^{-2\xi^2-\frac{1}{8}(s-2x)^2}\left(4\xi\sin(\xi s)+s\cos(\xi s)-4x\sinh\left(\frac{sx}{2}\right)+2s\cosh\left(\frac{sx}{2}\right)\right)^2}{16\pi\left(e^{\frac{sx}{2}}\cos(\xi s)+e^{sx}+1\right)}. \tag{39}$$

An essential characteristic is the S_F strength, which is twice as strong in comparison to the S2

$$N_F = e^{-\frac{s^2}{4}}+2. \tag{40}$$

The results summarized in Fig. 4 clearly show the advantage offered by correlation plenoptic imaging (correlation of S and F) over conventional imaging detection schemes (correlations in S). Although Fisher information per one detection is comparable in both scenarios, the strength of the plenoptic signal gives rise to a significant difference for the overall Fisher information, as expected from Eq. (21).

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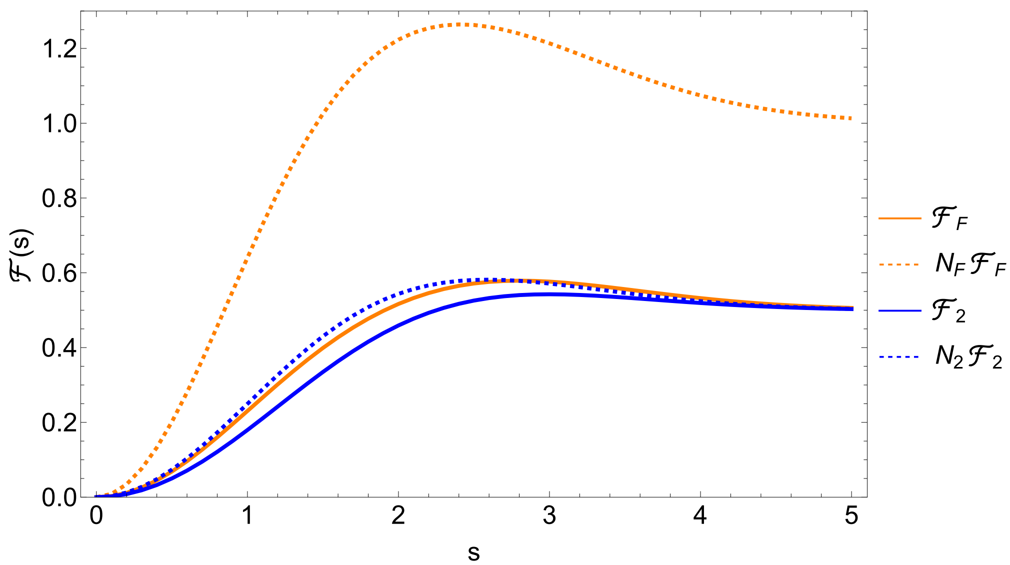

Fig. 4. Fisher information for estimating the separation of two point emitters s, as computed for the signals associated to correlations measured in the image planes S_2 (orange) and in the image and the Fourier planes F (blue). The curves, computed for a PSF defined by Eq. (23), show both Fisher information per one detection (solid) and overall Fisher information (dashed), which is given by the product of signal strength and Fisher information per one detection.

5. Discussion

The Fisher information computed in the case of correlated measurements and its comparison with the case of uncorrelated detection offer interesting insights into the information content carried by an optical signal. In fact, we have found that the considered optical schemes involve a coherent signal which is not resolved by uncorrelated (phase insensitive) detection, but is partially resolved by correlated detection. The results also show the importance of considering the different strength of the detected correlated and uncorrelated signals, when comparing the two scenarios, in line with the discussion reported in Ref. [22].

The relevance of this aspect is clearly illustrated by comparing the different estimation strategies. Let us first consider the case in which the parameter is estimated after accumulation of the same amount of detected events. Since Fisher information quantifies the information content per detected event, it is certainly a relevant quantity to be considered, as reported by the continuous lines in Fig. 3 and Fig. 4. Of course, when the strength of the signals varies, such observations are extended during different time windows; this is due to the fact that a weak signal requires more time to record the same amount of events.

However, a different estimation strategy can be based on detection performed during a fixed time window. In this case, the strength of the signal plays a role in the estimate of precision, and Fisher information needs to account for the overall strength of the signal, as reported by the dashed lines in Fig. 3 and Fig. 4.

In the plenoptic scheme where detection in the image plane are correlated with those in the Fourier plane, the normalized Fisher information is found to significantly increase with respect to the case of conventional detection in the image plane. This effect can be attributed to the role played by the strength of the signal, as demonstrated by the fact that the comparison of Fisher information per detection event gives similar results in the two cases. The effect of the strength

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of the signal should therefore be taken into account also in cases where the structure of the signal is more complex.

The non-monotonic behavior of Fisher information, as reported in Figs. 3 and 4, represents a subtle point that warrants further investigation. The presented results align with expectations for small separations and remain constant for large separations, but the behavior in the intermediate region, where the Fisher information reaches its maximum and slightly decreases, is not intuitive, although the differences are small. In fact, it is reasonable to expect monotonicity for distinguishing smooth objects such as Gaussian PSFs (Eg. (23)). However, in our case, we are dealing with partially coherent signals giving rise to correlation functions characterized by a richer structure. Although we do not currently have a precise explanation for this behavior, we notice that it appears to be associated with the square of the $S1$ signal structure, which appears in the expressions of the Fisher information reported in Eq. (35) and Eq. (39).

6. Conclusions

By using the example of the superposition of the optical signal from two separated point sources, we have studied how Fisher information is distributed in the detected optical signal among correlations of different orders. The contributions from a single-photon state appear as an incoherent mixture, while the correlations from a two-photon contribution form a partially coherent superposition. We have detailed how the contributions to Fisher information combine, with a particular focus on the fact that a single-photon state detected in the image plane manifests as an incoherent mixture of point spread functions. For signals originating from a two-photon state, we have illustrated the chain rule for Fisher information, showing that the information from correlated detection exceeds the one of the marginal distribution.

Moreover, we have emphasized the critical importance of properly normalizing Fisher information relative to the signal strength. This effect is particularly significant in the case of the plenoptic configuration, where the image plane is correlated with the Fourier plane, enabling the achievement of super-resolution. Our findings are bridging classical optics expressed in terms of intensities and correlations with quantum information processing and represent an important step towards the development of optimal detection schemes. The obtained results could be applicable in CPI experiments utilizing correlated photon pairs, similar to the one detailed in [3]. Such experiments employ single-photon detection and statistical properties of pseudo-thermal light, which facilitate straightforward application of the proposed theory.

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References

1. M. D'Angelo, F. V. Pepe, A. Garuccio, *et al.*, "Correlation plenoptic imaging," Phys. Rev. Lett. **116**(22), 223602 (2016).
2. G. Massaro, D. Giannella, A. Scagliola, *et al.*, "Light-field microscopy with correlated beams for high-resolution volumetric imaging," Sci. Rep. **12**(1), 16823 (2022).
3. G. Massaro, P. Mos, S. Vasiukov, *et al.*, "Correlated-photon imaging at 10 volumetric images per second," Sci. Rep. **13**(1), 12813 (2023).
4. C. Abbattista, L. Amoruso, S. Burri, *et al.*, "Towards quantum 3d imaging devices," Appl. Sci. **11**(14), 6414 (2021).
5. E. H. Adelson and J. Y. A. Wang, "Single lens stereo with a plenoptic camera," IEEE Trans. Pattern Anal. Machine Intell. **14**(2), 99–106 (1992).
6. R. Ng, M. Levoy, M. Brédif, *et al.*, "Light field photography with a hand-held plenoptic camera," Stanford University Computer Science Tech Report CSTR 2005-02 (2005).
7. R. Ng, "Fourier slice photography," ACM Trans. Graph. **24**(3), 735–744 (2005).
8. T. Georgiev and A. Lumsdaine, "Resolution in plenoptic cameras," in *Frontiers in Optics 2009/Laser Science XXV/Fall 2009 OSA Optics & Photonics Technical Digest* (Optica Publishing Group, 2009), paper CTuB3.

Optics EXPRESS

9. F. Di Lena, G. Massaro, A. Lupo, *et al.*, "Correlation plenoptic imaging between arbitrary planes," Opt. Express **28**(24), 35857–35868 (2020).
10. G. Massaro, "Assessing the 3d resolution of refocused correlation plenoptic images using a general-purpose image quality estimator," Eur. Phys. J. Plus **139**(8), 727 (2024).
11. G. Massaro, B. Barile, G. Scarcelli, *et al.*, "Direct 3d imaging through spatial coherence of light," Laser Photonics Rev. **18**(8), 2301155 (2024).
12. G. Massaro, F. Di Lena, M. D'Angelo, *et al.*, "Effect of finite-sized optical components and pixels on light-field imaging through correlated light," Sensors **22**(7), 2778 (2022).
13. G. Massaro, F. V. Pepe, and M. D'Angelo, "Refocusing Algorithm for Correlation Plenoptic Imaging," Sensors **22**(17), 6665 (2022).
14. M. Cassano, M. D'Angelo, A. Garuccio, *et al.*, "Spatial interference between pairs of disjoint optical paths with a single chaotic source," Opt. Express **25**(6), 6589–6603 (2017).
15. M. D'Angelo, A. Mazzilli, F. V. Pepe, *et al.*, "Characterization of two distant double-slits by chaotic light second-order interference," Sci. Rep. **7**(1), 2247 (2017).
16. M. Parniak, S. Borówka, K. Boroszko, *et al.*, "Beating the rayleigh limit using two-photon interference," Phys. Rev. Lett. **121**(25), 250503 (2018).
17. M. Paúr, B. Stoklasa, Z. Hradil, *et al.*, "Achieving the ultimate optical resolution," Optica **3**(10), 1144–1147 (2016).
18. M. Tsang, "Resolving starlight: a quantum perspective," Contemp. Phys. **60**(4), 279–298 (2019).
19. J. Řehaček, Z. Hradil, B. Stoklasa, *et al.*, "Multiparameter quantum metrology of incoherent point sources: towards realistic superresolution," Phys. Rev. A **96**(6), 062107 (2017).
20. D. Giannella, G. Massaro, B. Stoklasa, *et al.*, "Light-field imaging from position-momentum correlations," Phys. Lett. A **494**, 129298 (2024).
21. R. Zamir, "A proof of the fisher information inequality via a data processing argument," IEEE Trans. Inf. Theory **44**(3), 1246–1250 (1998).
22. Z. Hradil, J. Řeháček, L. Sánchez-Soto, *et al.*, "Quantum fisher information with coherence," Optica **6**(11), 1437–1440 (2019).