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THE BORDER AS A SEMIOTIC CROSS-CULTURAL SPACE: SEMIOTICS AND PRAGMATICS ASPECTS OF INTERDISCIPLINARITY IN MATHEMATICS EDUCATION

This essay aims to advance the debate on the conceptualization of interdisciplinarity in mathematics education by presenting the idea of the border as a dynamic semiotic cross-cultural space. We refer to three theories, that originate in different research fields and use specific languages: De Luca Picione and Valsiner's idea of the border as a metaphor, Lotman's idea of the semiosphere, and Habermas's construct of rationality. An interdisciplinary activity with high school students based on Ceva's theorem is analysed to show how the complementarity of these theories can provide a key to characterize interdisciplinarity in mathematics education as semiotic mediation among different cultures and as a comparison among rationalities. We conclude with possible developments to shed light on the educational potential of interdisciplinary activities in emerging fields.

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THE BORDER AS A SEMIOTIC CROSS-CULTURAL SPACE: SEMIOTICS AND PRAGMATICS ASPECTS OF INTERDISCIPLINARITY IN MATHEMATICS EDUCATION

Roberto Capone and Eleonora Faggiano

This essay aims to advance the debate on the conceptualization of interdisciplinarity in mathematics education. It asks how the connection between mathematics and other disciplines can be set up on a theoretical basis, considering semiotics and pragmatic aspects of the processes in mathematics education.

We believe that students' attitude to generalize mathematical knowledge to new contexts of application is naturally encouraged in interdisciplinary activities. On the other hand, activities that favour interdisciplinarity and interrelatedness could provide students with opportunities to discover how a topic or concept can link and connect to other topics or concepts within and across disciplines and also within their lives outside of school. Since the 1970s, interdisciplinary education has undergone many recent advances mainly related to the interest in STEM disciplines (Apostel, 1972). But what do we mean by interdisciplinarity? In this work, we refer to the following Klein's (1990) definition of interdisciplinarity:

Interdisciplinarity is a means of solving problems and answering questions that cannot be satisfactorily addressed using single methods or approaches. Whether the context is a short-range instrumentality or a long-range reconceptualization of epistemology, the concept represents an important attempt to define and establish common ground (p.196).

In addition to the numerous studies in interdisciplinary contexts, in recent years some researchers have also recognized the need to set up theoretical bases for interdisciplinarity. Most of these proposals (e.g. Leung, 2020; Levrini, Tasquier, Branchetti, and Barelli, 2019) are based on the idea of boundary crossing (Akkerman & Bakker, 2011) which focuses on the useful notion of boundaries, as constitutive of what counts as expertise or as central participation, offering an understanding of them as dialogical phenomena. The model of boundary crossing is based on a spatial metaphor and, thus, interdisciplinarity is described in terms of intersection between areas. In agreement with Tytler and colleagues (2019), we recognize that the spatial metaphor has some limitations, for example in assuming that disciplines have fixed characteristics, essential methods, concerns, distinct boundaries, and it is not enough to describe and characterize the nature of the interactions between the disciplines.

Our perspective is in line with the socio-cultural approach, and the novelty lies in stressing that the borders of the disciplines are dynamic semiotic spaces and that to conceptualize interdisciplinarity we need to pay attention to the processes. To this aim, we consider the perspectives of cultural semiotics and linguistic pragmatics.

To conceptualize interdisciplinarity, in this theoretical contribution, we will introduce the notion of border as a semiotic cross-cultural space. Our arguments are framed by the idea of the *border* as a metaphor of interdisciplinarity (De Luca Picione, Marsico, Tateo and Valsiner, 2022), Lotman's (1984) idea of the *semiosphere* and Habermas's (1981) construct of *rationality*. These theories originate in different disciplinary fields (psychology, semiotics and pragmatics, respectively) and use specific languages that require in-depth study by the layman, for which we refer to the sources. Our aim with this essay is to show how the complementarity of these theories can provide a key to characterize interdisciplinarity in mathematics education.

In what follows, we first present an interdisciplinary teaching activity based on Ceva's theorem, designed and experimented with 10th-grade students. Then, we describe our theoretical perspective, and finally, to exemplify it, we provide an empirical analysis of the activity that allows us to interpret interdisciplinarity as semiotic mediation among different cultures and as a comparison among rationalities.

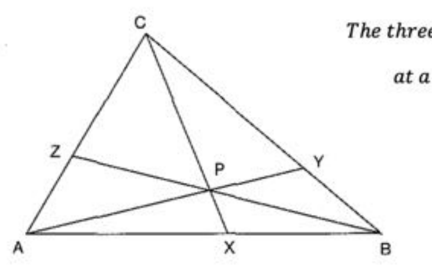
INTERDISCIPLINARITY IN MATHEMATICS EDUCATION: THE EXAMPLE OF CEVA'S THEOREM

In this section we present an interdisciplinary teaching activity in which students are encouraged to assume a stereoscopic view that is, at the same time, looking broadly and in opposite directions, sometimes even crossing eyes or trying to hold multiple planes of vision together.

The design of the activity was based on the following principles: allow students to work in a fixed disciplinary contest; foster the connection between mathematics and other disciplines; promote linguistic practices that can be meaningful to connect the different activities and build up to a gradual conceptualization (in the sense of Vergnaud's Theory of Conceptual Fields, 2013), developing a more precise language and promoting argumentation; recognize the other's point of view as a possible alternative to one's point of view and not in opposition to it; recognize that there are different ways to the solution of a problem.

As a topic, we focused on a theorem that is mostly known as Euclidean plane geometry theorem: Ceva's theorem, so called in honour of the Italian mathematician Giovanni Ceva (1647–1734).

In Euclidean geometry, Ceva's theorem is a necessary and sufficient condition for the concurrence of three-line segments, called cevian, which joins a vertex of a triangle with a point on the opposite side (or its extension). The theorem states that in a triangle the sides are cut by three cevians that are concurrent in a single point if and only if the product of the three (oriented) length ratios along each side equals 1 (Figure 1).



$$\frac{\overline{AX}}{\overline{XB}} \cdot \frac{\overline{BY}}{\overline{YC}} \cdot \frac{\overline{CZ}}{\overline{ZA}} = 1$$

Figure 1.: Euclidean representation of the Ceva's theorem

The theorem was proved already in the XI century, but it bears Ceva's name due to the proof included in his booklet, published in 1678, 'De Lineis Rectis se invicem secantibus statica constructio' (Figure 2). In his work on the statical construction of straight lines cutting each other, Ceva attacked problems of geometry purely through the use of static and successfully solved challenges that were previously deemed insurmountable (Hanna & Jahnke, 2002). In the design of the teaching activity, we intended to exploit Ceva’s idea to ‘replace lines by weights’ instead of applying geometrical constructions by ruler and compass, as geometers usually do.

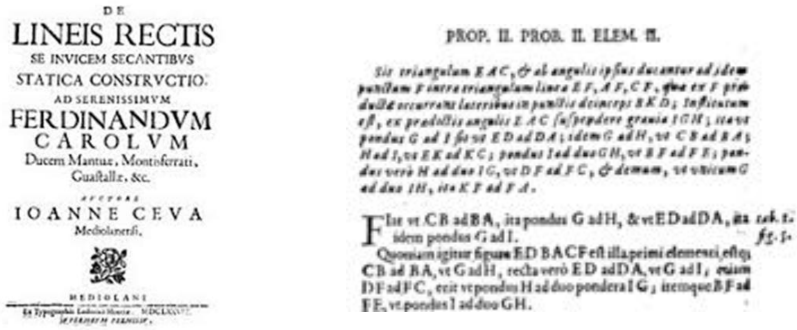


Figure 2.: The cover of ‘De Lineis Rectis se invicem secantibus statica constructio’ and the original Latin text of the (Ceva’s) theorem

The teaching activity was developed according to the following phases:

Phase 1: Students were provided with the original Latin text of the theorem and divided into small groups, they were requested to translate the text into their mother language (Figure 3a)[1].

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[1] Students in grade 10 are used to translating from Latin to their language. The level of Latin language of the text was adequate to the competencies of the students at their level of studies. Moreover, since grade 9, they were introduced to Euclidean Geometry and its theorems and proofs.

Phase 2: Guided by the teacher, students were invited to discuss whether the problem they are faced with is a physics problem, a mathematics problem or both, by answering the following questions: 1) which elements of the text can be related to a mathematics problem? 2) which elements of the text can be related to a physics problem? They were then asked to identify the strategies and the tools that could be used to solve the problem. Figure 3b shows a wordcloud, created by the answers of students: they mostly referred to proof ("dimostrazione") and proportions as possible strategies/tools.

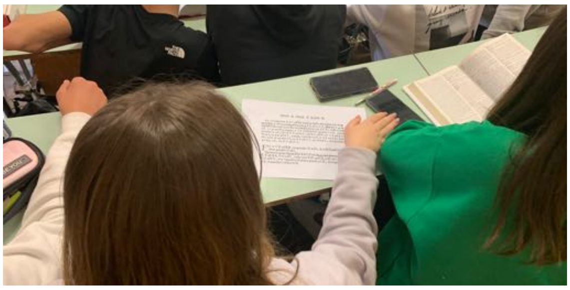

Figure 3a.: Ceva's theorem translation from the original text (Phase 1)

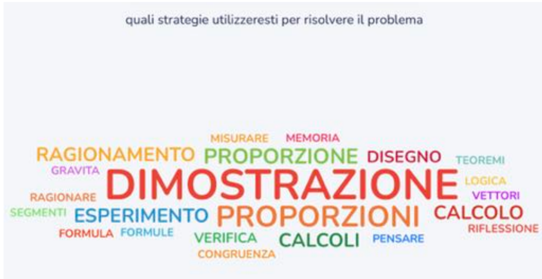

Figure 3b.: Students' wordcloud regarding possible strategies to prove the theorem (Phase 2)

Phase 3: Students were given a real problem situation: find the ideal condition for a triangle to keep itself balanced on a stud. To solve the problem, they could use cardboard, pencils, scissors, a precision scale, and some small tabular pasta as weights (Figure 4).

Figure 4.: Students working on the equilibrium problem and one of the product of their work (Phase 3)

Phase 4: Students were asked to prove the theorem with the classical approach and tools of the axiomatic Euclidean geometry (Figure 5a). A discussion was then orchestrated by the teacher to bring students' attention to the analogies between the original text of the theorem, the experimental activity, and the geometrical proof of the theorem.

Phase 5: Students were asked to construct and verify Ceva's theorem using a dynamic geometry software (Figure 5b).

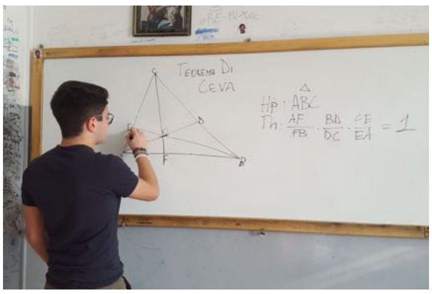

Figure 5a: Students proving Ceva's theorem with the tools of Euclidean geometry (Phase 4)

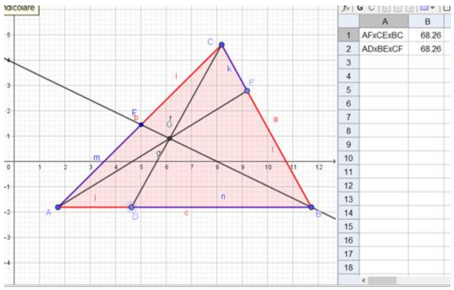

Figure 5b: Students' construction and verification of the Ceva's theorem in GeoGebra (Phase 5)

Phase 6: Students were invited to choose one of the three approaches to present and prove the theorem. Then, according to their preference, they were divided into three groups and asked to prepare themselves for a debate concerning the reasons for their choice. Students in each of the three groups were finally invited to argue the reasons why they considered the chosen approach as the best to solve Ceva's problem. The debate was followed by a final collective discussion orchestrated by the teacher.

Through the analysis of video and audio recordings, reports of the observation, and students' materials collected by experimenting with this interdisciplinary activity, we have found that encouraging students to study the same topic from different disciplinary points of view allows us the discover of information about students' conceptualization (as also in Adesso, Capone, Fiore and Tortoriello, 2020). However, the interest in understanding the meaningfulness of this interdisciplinary activity in mathematics education made us face the need to develop a deeper analysis of the students' conceptualizing processes at the borders of the disciplines (Capone, 2022).

In this paper, we attempt to discuss the results of this interdisciplinary activity with the aim to show how the nature of the interactions between mathematics and other disciplines (such as physics in this case) can be described and characterized considering semiotics and pragmatics aspects.

To do this we now need to introduce some theoretical aspects.

THE BORDER AS A METAPHOR OF INTERDISCIPLINARITY

In this study, we refer to De Luca Picione and colleagues' (2022) idea of border from a psychological perspective, as a semiotic tool that enables dynamic processes of development and psychic elaboration. The human experience is characterized by a continuous process of semiotic construction of meaning through operations such as classification, categorization, and interpretation. These operations facilitate human behaviour, thinking, and interpersonal relations. In this sense, borders allow us to construct a symbolic field in which semiotic processes take place. These processes of signification and interpretation commence with their differentiation from the external environment.

Why the relevance of the concept of borders is significant in a semiotic perspective? This is because the organization and transformation of meaning involve dynamic cultural processes that occur in transitional areas and stages defined by borders. We refer to these dynamic cultural processes using the notion of *liminality* introduced in anthropology as the quality of disorientation that occurs when we stand at the threshold between our previous way of structuring our identity, time, or community, and a new way (Turner, 1974). Furthermore, when information is transmitted across borders, and there is an exchange between various structures and substructures with semiotic 'irruptions' of specific structures into unfamiliar territory, new information is generated.

Interdisciplinarity, from the point of view of mathematics education, can foster this exchange of information, outlining the trajectories of cultural transformations. Disciplinary cultures, in fact, are only recognized through their projections of otherness, and this otherness can only be grasped through the function of the border interposed between the spaces that characterize a certain disciplinary domain. Borders metaphorically represent a dynamic balance between maintaining the internal stability of a semiotic system and the possibility of transformation, between entropic dissolution and the development of new structures. Although the notion of border conveys the common-sense meanings of separation, definition, closure, and demarcation between distinct disciplines, the flip side of the border's coin is precisely its ability and function to create dynamic topological relations and structures, allowing for comparison, exchange, and dialogue.

To summarize, this psychological perspective suggests seeing the border as a semiotic cross-cultural space that separates and combines mathematics from other disciplines offering a relational possibility and thus representing a metaphor of interdisciplinarity. However, the idea of border as a semiotic cross-cultural space does not alone provide insights into educational phenomena. In our view, it needs to be enriched by semiotics and pragmatics aspects which allow us to characterize interdisciplinarity in mathematics education as semiotic mediation among different cultures and as a comparison among rationalities. To address this aim, in what follows we present Lotman's concept of semiosphere and Hamermas's construct of rationality.

THE CONCEPT OF SEMIOSPHERE

The existence of a border where educational phenomena occur also requires answers to questions such as: how does what is on the border interact with what is around it? What opportunities does the work on the border offer with respect to working in a specific discipline?

The concept of semiosphere, developed by Lotman (1984) in the cultural semiotic research field, allows us to investigate interdisciplinary educational phenomena, focusing on the dialogic relationship between the disciplines, and give answers to the questions above. We thus take up Lotman's idea that interdisciplinarity is the way through which a complex view of reality can be obtained.

Lotman (1984) identifies the border, not as a limit that separates/combines but as a place where a frame acquires meaning. According to Lotman, borders mainly realize processes of differentiation, at the temporal level, between a before and an after (past/future), at the topological, interspatial level, between an inside and an outside (subject/object), at the topological, intra-spatial level, between a Me and a non-Me (subject/alterity). Furthermore, the *semiotization* of borders thus enables the organization of time, space, and relations between subjects. To characterize the border, Lotman introduces the idea of the semiosphere as a space enclosed within a porous border in which the different sign systems and communicative codes of culture (language, art, science, mathematics, etc.) can exist and new information has been taken up and shared.

The borders also delineate the disciplinary domains: they are semiotic borders that have a paradoxical function because, at the same time, they restrict and put together, differentiate and create relations, diversify and homologate, close and open, define identities and favour the translation processes that arise from the recognition of otherness.

> *The border is a bilingual mechanism, translating external communications into the internal language of the semiosphere and vice versa. [...] The function of any border or film [...] comes down to a limitation of penetration, filtering and the transformative processing of the external to the internal. [...] At the level of the semiosphere it represents the division of self from other, the filtration of external communications and the translation thereof into its own language, as well as the transformation of external non-communication into communications, i.e. the semiotization of incoming materials and the transformation of the latter into information (Lotman, 1984, p. 60-61).*

If we consider the disciplines as semiotic systems, it is not a matter of a simple process of encoding and decoding signals. What characterizes a semiotic system? In our view, it is the interpretative and constructive function of meaning. Indeed, as also Umberto Eco (1976) shows, the threshold between an informative process and a semiotic process comes from interpretation. Lotman (2009) argues that what allows a structure to be a thinking system is a dialogical process with the outside (precisely through the organization of borders). The passages between inside and outside are processes of translation and interpretation.

To understand how what is on the border interacts with what is around it, and thus how semiospheres interact with each other, we refer to the notion of enantiomorphism. According to Lotman, all sense-generating mechanisms start from an early state of symmetry, that is, of equilibrium and staticity, which is progressively complicated through the production of a mirror symmetry. In other words, a first semiotic system, called culture C1, in a state of symmetrical equilibrium, receives one or more texts belonging to a second semiotic system, called culture C2; through the interpretation –which is in fact a disintegration/reorganization– of the texts received, the culture C1 'pulls out' an image of the culture C2. This produced image is a kind of mirror symmetry elaborated by C1, defined by Lotman as an enantiomorphic model, against which culture C1 can mirror itself by finding similarities and differences (Figure 6).

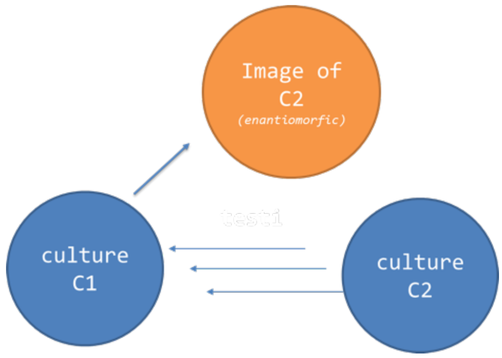

Figure 6: Scheme of the enantiomorphism between cultures

For example, let's say that Physics is a culture C1 and Mathematics is a culture C2. Physics might receive a mathematical formula or theorem from Mathematics and analyse it to create a mirror image of what it thinks Mathematics is like. This image of Mathematics created by Physics is like a model that Physics can use to compare itself to Mathematics and see what similarities and differences they have in terms of their approaches, tools, and theories. By doing so, Physics could gain a better understanding of Mathematics and how it can be applied to solve real-world problems in Physics. On the other side, on the border also Mathematics might receive from Physics examples that actualize mathematical formulas or theorems and this generates a mirror image of Physics, gaining a better understanding of how Mathematics can be applied to solve real-world problems in Physics. The exchanges on the border, hence, do not only allow to solve problems from different points of view but also contribute to the internal reflection of each discipline through the comparison with the other. In this sense, the work on the border offers more opportunities with respect to working in a single specific discipline.

HABERMAS'S CONSTRUCT OF RATIONALITY

How could we understand and interpret the way the communication between disciplines can take place and how interdisciplinarity contributes to constructing meaning? In this attempt, we refer to pragmatic linguistics and, in particular, to Habermas's theory of communication (1981).

According to Habermas, the adjective rational can be attributed to a person who carries out a discursive activity intending to account for his choices according to criteria of validity. Rationality consists of three interrelated components: epistemic rationality (an opinion is rational when it is possible to justify it in a given context.), teleological rationality (one acts rationally when one acts according to a purpose by implementing intentionally chosen means), and communicative rationality (you aim to make the interlocutor share your communication content by choosing effective means of communication). Comparing different points of view leads to affirming all points of view as acts of truth through communicative interaction.

Habermas's linguistic pragmatics suggests us the idea that rationality can be associated with each semiosphere as a characteristic feature of the communication process between disciplines, as a social action oriented towards understanding and sharing meanings.

SEMIOTICS AND PRAGMATICS ASPECTS OF INTERDISCIPLINARITY IN THE EXAMPLE

In what follows we come back to our data, analysing and discussing them to consider the overall development of the students' conceptualization of Ceva's theorem from the semiotic point of view (according to Lotman) and the pragmatic point of view (according to Habermas).

During the first phase, students started to translate the text assuming it was a mathematics theorem, as it begins considering a triangle and some particular segments. At a certain point, however, they encounter the words *gravia* and *pondus* –meaning weights in Latin–, which cause the following kind of exclamation:

Student1: Cosa hanno a che fare queste due parole con un problema di matematica? Non riesco a capirne il significato.

Following De Luca Picione and Valsiner, as long as students recognize that they are within a disciplinary border, they move easily and use the tools of the discipline. When they find themselves in an interdisciplinary situation, the border confronts them with a crisis, a break between what is on this side and what is on the other side, creates a situation of liminality. When students are asked to identify the words in the text that make them think it is a mathematics problem, they refer to the words: triangle, side, line, proportion, angle, extension, point. When they are then asked to identify the words in the text that make them think it is a physics problem, they refer to the words: weight, balance, heavy. When they are asked to say within which disciplinary borders the text can be identified, most students think it is a physics problem, and few students answer that it can be both a mathematics and a physics problem. However, when asked to choose a strategy for solving the problem, only a few students recognized that the problem could be traced back to a real situation and solved by the experimental method, in the Galilean sense. As said before, most of them answered that they would use proof, in the Euclidean sense.

The border as a metaphor for interdisciplinarity, however, it is not enough. An interpretation of the processes is needed to deepen the educational phenomena. In our view, Habermas's construct can give us a contribution in this sense.

According to the activity's design, phases 3, 4 and 5 aim to provide students with the specific tools of a single discipline at a time. Data analysis revealed that students successfully use the discipline's epistemology when asked to find solutions in each disciplinary domain. In student dialogues, elements of epistemic rationality can be discerned when students motivate an opinion in a given context. Moreover, teleological rationality is evident when most of the students intentionally use the rationality of a specific discipline. For example, they read and translate the original text in phase 1 to see in the original text which of the approaches is the best approximation of Ceva's intentionalities; in phase 3 they use the experimental-inductive method to find the condition for a triangle to keep itself balanced on a stud; they use the hypothetic-deductive method when asked to prove the thesis from the theorem's hypothesis in phase 4; they use dynamic geometry software in phase 5. Communicative rationality prevails in phases 2 and 6: each group of students aims to make the interlocutor share their communication content by choosing effective means of communication.

In the final phase, students are invited to take a stand in favour of the chosen (disciplinary) point of view. They were brought to work at the border, and they are in a mental situation of liminality (De Luca Picione, Marsico, Tateo, and Valsiner, 2022). It is in this phase that, as shown in the following excerpts, students reveal again their disorientation. It becomes evident that they stand at the threshold between their preferred way of solving the problem with the epistemology and the tools of a discipline, and the way to do it within the cognitive domains of another discipline:

Student 1: Il nostro gruppo ha scelto la dimostrazione matematica perché, se il ragionamento funziona, cioè una volta che la tesi è verificata, nessuno può dire di no, non lo fa.

Student 2: Sì, ma se devi risolvere un problema nella tua vita, non ti limiti a scrivere l'ipotesi e la tesi e poi fare la dimostrazione. Cerchi un modo più concreto. [...] Ricostruendo il problema, si vede praticamente che funziona.

Student 3: Secondo me, la via più breve è quella di utilizzare un software: si inseriscono i dati, e il problema è risolto, fa tutto.

Student 1: Sì, ma la dimostrazione vale sempre, devi fare la verifica ogni volta. Certo, il computer risolve tutto, ma qualcuno deve averlo programmato prima.

Habermas's construct has helped us to analyse the processes that occur at the border, and in particular the role of comparison between rationalities. But why is this not enough? Because it does not help us understand how re-signification takes place, that is, how an object within one discipline acquires a specific meaning also within another discipline. This is why we need to consider the perspective of cultural semiotics.

At the border, recognition of the validity of statements occurs. This recognition occurs through a re-signification of what is other and what is foreign. What is other appears in one's semiosphere as an enantiomorphic form of the object of the original semiosphere. Students recognize the centre of gravity (in Physics) as an enantiomorphic form of the incentre (in Mathematics) that remains identified by the intersection of three Cevians by moving the cursor with the mouse in GeoGebra. Each element does not lose its original meaning but also acquires others, which correspond to as many enantiomorphic forms:

Student 1: Infatti, se prendiamo i punti incentro, baricentro e ortocentro, che coincidono in un triangolo ceviano, è lo stesso che considerare il punto di equilibrio.

Student 2: È vero, il punto di equilibrio coincide con l'incentro; Pertanto, se troviamo sperimentalmente il punto di equilibrio, significa che alcune condizioni geometriche sono soddisfatte e quindi abbiamo l'incentro.

Student 3: Certo, ma è comunque più facile verificarlo con GeoGebra; Non devi fare proporzioni e non devi perdere tempo a sperimentare. Basta muovere il mouse e il gioco è fatto.

Student 1: Quello, secondo me, può essere un metodo di verifica, ma non si possono dare giustificazioni razionali. Hai ancora bisogno di qualcuno che crei il programma per te. Ancora non lo escluderei.

Thus, students have learned to recognize the other's point of view as a possible alternative to their own point of view and not in opposition to it; they have recognized that there are different ways to arrive at problem resolution. Furthermore, it is worthy of note that interdisciplinarity does not exclude disciplinarity but rather reinforces it.

The empirical analysis of this activity has been provided to exemplify how the opportunity to assume different perspectives and to compare them can promote the work at the border that in this way becomes a semiotic cross-cultural space. The description and interpretation of the students' conceptualization processes during the interdisciplinary activity were developed by considering the semiotics and pragmatic aspects.

CONCLUDING REMARKS

This paper discusses how interdisciplinarity in mathematics education can be characterized by considering both semiotic and pragmatic aspects. With this aim, we introduced the concept of border as a semiotic cross-cultural space. From a psychological perspective, the border is seen as a semiotic tool that enables dynamic processes of development and psychic elaboration. The relevance of the concept of borders is significant in a semiotic perspective because organizing and transforming meaning involves dynamic processes that occur in liminal areas defined by borders. Starting from this view of the border as a metaphor of interdisciplinarity (De Luca Picione, Marsico, Tateo, and Valsiner, 2022), in this paper, we have analysed an interdisciplinary teaching activity based on Ceva's theorem using Lotman's idea of the semiosphere (1984) and Habermas's construct of rationality (1981).

We explored the idea of the semiosphere, which is a space enclosed within a porous border in which the different sign systems and communicative codes of culture (language, art, science, mathematics, etc.) can exist and generate new information. These borders also delineate disciplinary domains: they are semiotic borders. One of the ideas that arise is that interdisciplinarity can give a propulsive boost to knowledge. Indeed, diversity between disciplines, understood as asymmetry, as highlighted by Lotman, is the starting point of a translational process, just as a water pump works if there is a difference in the level or a battery works if there are two poles at whose ends a potential difference is established. Addressing a problem in an interdisciplinary way means offering multiple points of view, sometimes distant sometimes closer, argued according to the characteristic tools of each discipline.

Linguistic pragmatics also gives us a key to understanding the activity at the border among several disciplines. Indeed, the use of several registers can foster the epistemic dimension. The different function of registers represents the teleological dimension (heuristic in the production of conjecture on the one hand, in the verification of equilibrium on the other). The communicative dimension is expressed in making the language of one's own semiosphere understandable to those belonging to the other semiosphere. Our idea is that the borders delineated between disciplines should not necessarily be crossed but recognized as a place where the frame gains meaning, a place where semiotic transformations take place.

We discussed our example to show how interdisciplinarity in mathematics education can be characterized as a semiotic mediation among different cultures and as a comparison of rationalities. From the educational point of view, the plurality of disciplinary perspectives, and their related learning methodologies, can contribute to providing students with opportunities to discover how a topic or concept can link and connect to other topics or concepts within and across disciplines, thus shaping citizens capable of choosing critically, to analyse problem situations in a poliscopic manner. The borders can trigger processes of signification across cultures, however, it remains to be explored how to deepen the educational potentiality of interdisciplinary activities. Topics connected with Artificial Intelligence, nanotechnology and climate change, for instance, cannot be grasped within the semiosphere of a single discipline, but require the mastering of different semiotic signs and different forms of rationality. Our approach opens up a way to explore these emergent research areas: on one hand, it can help to understand the way interdisciplinary activities can foster and strengthen disciplinary knowledge and competencies; on the other hand, it offers a frame to investigate how interdisciplinary teaching approaches allow students to acquire new competencies, beyond the disciplines, towards new frontiers.

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