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The Dominance-based Rough Set Method for analysing patterns of flexibility allocation and design-cost criteria in large-scale irrigation systems

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Abstract:

Water planners must provide end-users reliable and high-quality access to freshwater while complying with financial, institutional and water availability constraints. Over-investment in design can result in stranded assets of significant value, and often unwanted environmental implications. Under-investment can lead to supply restrictions affecting human health, economy, and environment. The present study uses the Multiple-Criteria Decision Analysis (MCDA) method to develop a balancing strategy concerning complexities encountered in water resources planning of irrigation systems. The methodology relies on a classification model that extracts posterior information regarding relevant combinations between flexibility allocation and design-cost criteria. The model delineates outcomes in the form of "*if..., then ...*" rules that translate decision possibilities facing water planners: "*if (the design is more flexible by this amount), then (we expect this range of cost increment)*". The results are refined using a confusion matrix that excludes incorrect and/or contradictive rules. The outcome reveals that cost is more subject to elasticity at the hydrant ($\boldsymbol{e_h}$) increment than the network's coefficient (r). In addition, the analysis shows that the operation parameter $P_{(q)}$ has only a minor effect on the cost and subsequently on final decision. Any elasticity ($\boldsymbol{e_h}$) less than 3 assigned with any given coefficient (r) becomes in a low cost increment. Cost increases as the coefficient (r) decreases for any given value of ($\boldsymbol{e_h}$). Elasticity from 4 to 5 with a network's coefficient (r) equal to or greater than 18/24 becomes a medium cost increment. Elasticity ($\boldsymbol{e_h}$) from 5 to 6 associated with an (r) equal or less than 16/24 becomes a very-high cost increment. Finally, rather than identifying one solution that

seems better than others, this approach provides an interactive schematic that helps identify the appropriate range of flexibility justified with the expense's criterion, which allows for debate and supports decision making.

Keywords: System flexibility; Cost efficiency; irrigation planning; Multiple Criteria Decision Analysis

1. Introduction

Numerous water supply and distribution systems (e.g., irrigation pipelines, canals, storage reservoirs, etc.) in many countries were constructed between 1950 and 1990 and thus require various expansions, upgrades, and rehabilitation as structures in the system reach the end of their design lifespan (Fletcher, Lickley, & Strzepek, 2019). This is further compounded by change impacts and uncertainties associated with various dimensions of social and hydrologic variability, such as extended periods of flood or drought (Rayer, Haustein, & Walton, 2021) and shifting social, environmental, economic, and political dimensions (Gosling & Arnell, 2016). For example, the Northern Colorado Water Conservancy District, in partnership with the U.S. Bureau of Reclamation, constructed the Colorado-Big Thompson Project to collect and store water from the mountainous region and convey it to farmers and municipalities in the populated eastern region. This project is being expanded currently by the Windy Gap and Chimney Hollow facilities to meet current and future water user demands. In addition to infrastructure and design upgrades, efficiencies and system resiliency could be improved through modern water management tools as the majority of these systems were developed prior to remote sensing and the various decision analysis and risk assessment methods that can take advantage of large data records of system performance (Frost et al., 2016; Lami & Moroni, 2020). The complexities of these systems and the objectives of shareholders can make the path of system improvement elusive. In addition, the upgrade alternatives are always associated with a multi-criterion problem, with associated uncertainties and tradeoffs to be made between capital expenditure, system performance, and design flexibility (Khadra & Lamaddalena, 2010). At times water experts argue on the probable scenarios of alternative policies dealing with uncertainties, or worse, reach an unjustified agreement through compromise that replaces acknowledgment of uncertainties and information gaps with policy priorities. To cope with uncertainty-based problems, several multiple criteria decision analysis or decision making (MCDA or MCDM) approaches have been developed (Pérez-Gladish, Ferreira, & Zopounidis, 2021) that are commonly used to solve two types of problems: well-structured and ill-structured problems (Zhao, Ho, & Chang, 2019).

Well-structured problems refer to the situation where the initial state, goal state, and constraints are clearly defined while completing the targeted objectives. The multiple-objective decision-making models (MODM), which require procedural knowledge of both goals and objectives and follow a completely defined step-by-step approach, are a good fit for well-structured problems (e.g., Global Criterion Method (GCM), Utility Function Method (UFM)) (Jonassen, 1997; Laureiro-Martínez & Brusoni, 2018; Ribeiro, Romão, Luz, Gomes, & Costa, 2020). Conversely, ill-structured problems refer to situations in which objectives are very complex or even conflicting, initial state and executable goals are not clear, and a well-defined set of operations is missing. The path forward requires a formalized method to assist decision-making in situations involving both multiple criteria and multiple uncertainties. The formalized method aims to (1) find optimal operating rules that meet various combinations of objectives, (2) set the interests and objectives of multiple actors since the input of both quantitative and qualitative information from every actor are considered as criteria, and (3) understand the complexity of the multi-actor setting by providing output information that is easy to communicate (Greco, Ehrgott, & Figueira, 2016). Thus, the Multi-criteria Analysis Models are appropriate for ill-structured problems that are often vaguely formulated with many uncertainties (e.g., Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), ELimination Et Choix Traduisant la REalité (ELECTRE), Rough Set Theory (RS)) (PAWLAK, 1998; Rahim et al., 2018; Wei, Gao, & Wei, 2018).

The most successful theoretical approach that refers to ill-structured problems is the Rough Set theory. The approach is mathematically based and concentrates on understanding the indiscernibility (similar) relation that characterizes objects that can be understood as elementary granules of knowledge about the objects.

As an extension to the Classical Rough Set Theory (RS), the “Indiscernibility relation” between objects is replaced by a “Dominance relation” within the rough approximation groups, yielding a new rough set model known as the Dominance Based Rough Set Approach (DRSA) (Skowron & Dutta, 2018; Skowron & Suraj, 2013). The DRSA has been successfully applied to several ill-structured problems such as developing an epidemiological surveillance decision-making system for an enhanced assessment of seasonal influenza risk (Younsi, Chakhar, Ishizaka, Hamdadou, & Boussaid, 2020), developing an auto loan fraud detection system (Błaszczyński, de Almeida Filho, Matuszyk, Szeląg, & Słowiński, 2021), providing survey-based insights about consumer attitude toward and acceptance of insect-based food (Roma, Palmisano, & De Boni, 2020), and developing a real-time recognition system that traces online assaults in 5G networks (Acharjya & Ahmed, 2021). The DRSA has also been applied within agricultural and environmental monitoring fields in policy assessment and decision support systems (DSS). Karami et al. (2014) used the approach to develop a study about analysing and evaluating water quality and its dynamics in the Latyan Watershed, north of Tehran, Iran . Boggia et al. (2014) developed a decision support system based on the DRSA to assess the level of rural sustainable development in the region of Umbria, Italy. Ahmadisharaf et al. (2015) applied the DRSA approach in assessing flood management options under multiple scenarios.

However, the authors know of no attempt within the field of water resources management to consider the design-cost criterion in irrigation systems with the uncertainties associated with upgrade investments.

Traditionally, planners develop a long-term supply and demand prediction and include a factor of safety to account for future uncertainties (Chadwick, Gironás, Barría, Vicuña, & Meza, 2021; Stakhiv, 2011). This traditional approach can lead to additional expenditures related to less-than-optimal schemes such as overdesign when demand is significantly less than supply or costly retrofits when actual demands significantly exceed forecasted demands (under-design). Meanwhile, society will have to face some severe economic and environmental consequences, such as unserved demand and additional expensive measures, if demand ends up higher than the forecast (Chadwick et al., 2021; Sawassi & Khadra, 2021). The challenge, in other words, is balancing under future uncertainties the trade-offs between risk of failure, cost, and system flexibility.

The present study uses the Multiple-criteria Decision Aiding Method to develop a balancing strategy concerning these complexities encountered in the water resources management of irrigation systems. We set up the Dominance-based Rough Set Approach in particular to define the best range of flexibility we can afford in designing an irrigation system, while accounting for a cost-effectiveness criterion.

2. Material and Methods

2.1 The design hypothesis in irrigation systems: The flexibility option

The model presented by Clément (1966) embedded the notion of system flexibility into empirical parameters that proved to be sufficiently sensitive in the design of water distribution networks. Clément's First model (1966) is stochastic, being one of the most applicable in the Mediterranean countries for the computation of peak discharges in on-demand pressurized networks. The empirical parameters included by Clément are defined as; the elasticity at the hydrant (e_h), the coefficient of utilization of the network (r), and the operation quality parameter ($P_{(q)}$). First, the Elasticity at the hydrant ($\boldsymbol{e_h}$) defines the freedom assigned to farmers to organize their irrigation, in which the nominal discharge of the hydrants ($\boldsymbol{d}$) has to be selected much higher than the duty ($\boldsymbol{D}$), each hydrant is assigned an elasticity ($\boldsymbol{e_h}$) (eq. 1) that allows users to satisfy the irrigation requirements in less than 24 hours.

$$\boldsymbol{e_h = \frac{Rd}{q_s\,A}} \qquad \textbf{(1)}$$

where R is the total number of hydrants, d is the nominal discharge of the hydrant (ls^{-1}), A is the area of the plot (ha) and, q_s is the specific continuous discharge ($l\ s^{-1}\ ha^{-1}$) calculated for an average hectare, thus representing the cropping pattern, during the peak period.

Since the likelihood of all hydrants operating simultaneously is unlikely to occur, the use of the probabilistic approach is justified. The values assigned to e_h typically range between 3.00, when the maximum irrigable area is considered, and 6.00, when the minimum irrigable area is considered (Labye, 1988).

Second, the coefficient of the utilization of the network (r) defines irrigation duration. During the peak period, the operating time of the network in hours is considered less than 24 hours, conferring elasticity of the network.

$$\mathbf{p} = \frac{\mathbf{t'}}{\mathbf{T'}} = \frac{\mathbf{t'}}{\mathbf{rT}} = \frac{\mathbf{q_{s\,A\,T}}}{\mathbf{Rd\quad rT}} \qquad \textbf{(2)}$$

Finally, the Operation quality parameter (P_q) defines the risk of exceedance of the upstream discharge during the peak irrigation period. This parameter value is specific to each project and the corresponding risk tolerances but typically is in the range of 95 to 99% (i.e., accepting a risk of exceedance for the upstream discharges of 5% to 1%, respectively) (Labye, 1988). The ranges of values to attribute to these parameters and their impact on the overall cost of the irrigation network are not often debated or field verified. Consequently, these values are empirically selected and frequently under-estimated (Sawassi, Khadra, & Lamaddalena, 2021).

To illustrate, let's first consider a set of attributes of an irrigation system in which (***e_h***) is the elasticity at the hydrant, (*r*) is the coefficient of utilization of the network, and (P_q) is the operation quality parameter, which represents the probability of exceedance of the peak discharge. Knowing that i, j, and z are the set of the indices related to each attribute respectively, $(\boldsymbol{e_h})_i = \{1, 2, 3, 4, 5, 6\}$, $r_j = \{16/24, 18/24, 20/24\}$ and $(P_q)_z = \{0.90, 0.95\}$, Clément predicted discharge occurring at the upstream end of the network during the peak period is calculated using the formula Q (i, j, z), accounting for the flexibility parameters.

$$\mathrm{Q}_{i,j,z} = \sum_x \frac{A * \mathrm{q}_s}{r_j} + \mathrm{U}(\mathrm{Pq}_z)\sqrt{\sum_x \frac{1}{R_x}\left(eh_i r_j - \left(\frac{A * q_s}{r_j}\right)^2\right)} \qquad \textbf{(3)}$$

The Combined Optimization and Performance Analysis Model (COPAM) (Lamaddalena & Sagardoy, 2000) was used in this study, with the first module estimating the system's upstream discharge and the second optimizing distribution pipe diameters for performance and cost over a range of flows using Labye's Iterative Discontinuous Method (LIDM), extended for several flow regimes (ELIDM) (Kadi, Beaucaire, & Cléroux, 1990; Lamaddalena, Khadra, & Tlili, 2012; Lamaddalena & Sagardoy, 2000; Sawassi et al., 2021). The final solution of the second model should satisfy all the examined discharge configurations derived from the first one. At this level, an iteration-based process of the abovementioned design hypothesis is performed, based on allocating different values of elasticity in each repetition, in which a single run of iteration generates a final cost.

2.2 The Study background: Tunisian context

Between 2006 and 2017, several agricultural investments in the sector of the modernization of irrigated schemes took place in Tunisia's lower Madjerda valley (LMV) (Molle & Sanchis-Ibor, 2019). PISEAU II is one of the modernization projects financed by the World Bank (WB), the French Development Agency (AFD), and the German Bank of Cooperation (KFW), which covers seven large-scale irrigation systems in the Manouba region, Tunisia. The project covers a total area of 4,071 hectares (10,060 Acres), including the systems: Bir Aouini, Mehrine East, Mehrine West, Sidi Neji, Mansoura East, Mansoura West, and Habibia. Even though the modernization of these irrigation systems has entailed significant financial resources, the systems continue to face the challenge of inadequate operation. Users continuously report a high frequency of unexpected drops in the flow and/or pressure that affect the quality of the rendered services; very low water availability at hydrants with the required frequency, discharge, pressure, and duration; and long durations of inadequate flow rate and/or pressure, especially during the peak period. Qualitative assessments were conducted via walk-throughs into the irrigation districts and sectors, observations, and interviews with farmers that were undertaken in a collaboration between the International Center for Advanced Agronomic Studies (CIHEAM-Bari, Italy) and the Tunisian Ministry of Agriculture through the General Directorate of Rural Engineering and Water Exploitation (DGGREE). The quantitative assessment showed the restricted selection of flexibility parameters. With an average elasticity of e_h =1.2, the system was designed to operate 20 hours per day and had a 90% probability that peak discharge would not be overcome during the driest period of the year, so it seems that, from the beginning, the system was designed to operate under its maximum capacity. This rigid selection turns out to be the primary reason behind the system's failure, endangering livelihoods based primarily on agricultural production. The inability to deliver water to farmers adequately (insufficient pressure and discharge) is related directly to future uncertainties noted previously that were not well-considered, resulting in the non-flexible design (Sawassi et al., 2021). Designers, on the other hand, have justified the system's failure by noting that the simplistic approach minimized the overall costs of the new irrigation system.

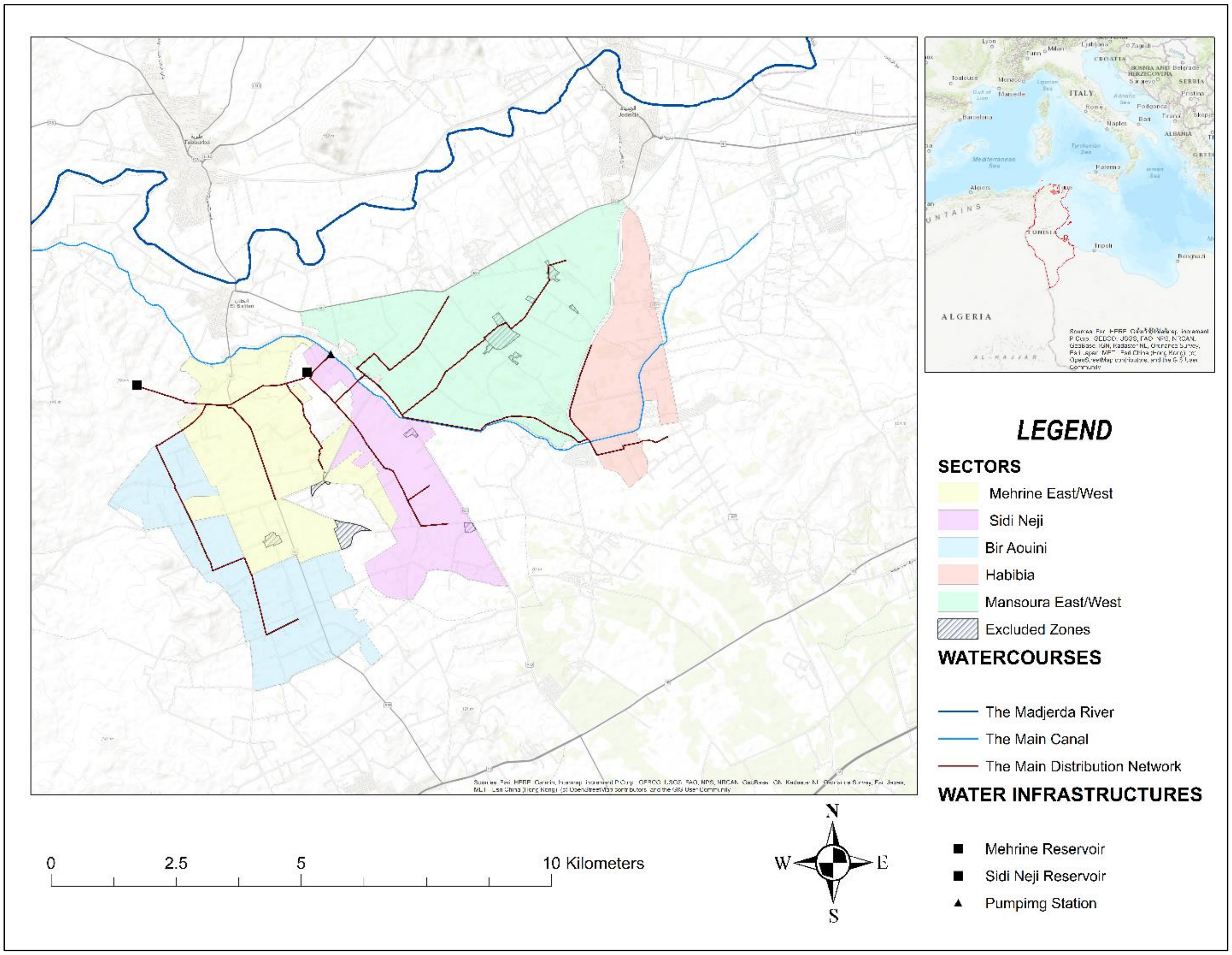


Figure 1. The study area: PISEAU II project in Manouba, Tunisia

2.3 The Dominance-based Rough Set Approach (DRSA)

The challenge facing water designers is to strike a certain balance between current user demands and project budget, user constraints and future demand, as well as future uncertainties and the upgrading investments. As a result, several factors are examined in order to reach a trade-off between these options. This research helps water planners to identify the correlation between design flexibility that guarantees a good service to users/ratepayers with the least amount of associated expenses. Having all the uncertainties and the criteria to account for, we opted to use the Multiple Criteria Decision Aiding (MCDA) method developed by (Greco, Matarazzo, & Slowinski, 2001, 2002) as an extension of the Rough Set Theory (Pawlak, 1982; Pawlak & Pawlak, 1991).

This study aims to implement a strategy for identifying the best range of design elasticity to be taken by decision-makers accounting for a cost-driven perspective using the MCDA, in particular the Dominance-based Rough Set Approach (DRSA).

When designing an irrigation system, the decision-maker should first establish a set of criteria to consider, including performance, adaptability, cost-effectiveness, user satisfaction, and others. The decision-makers should next determine their most significant criteria and assign it a certain preference. The DRSA approach takes into account these decision-maker preferences, as well the typical inconsistency of decision problems (Kadziński, Greco, & Słowiński, 2014). The approach substitutes the indiscernible relation with a dominance relation in a rough approximation of decision classes, making it possible to discover the inconsistencies concerning the dominance principle (Sawicki & Żak, 2014). In other words, DRSA builds the conceptual groundwork for discovering decision rules that have a syntax concordant with the dominance principle between different criteria (Boggia et al., 2014; Greco et al., 2001). In this context, the MCDA approach is used to assist decision-makers in identifying an effective range of flexibility adjusted for cost-effectiveness. This classification model permits the acquisition of posterior information about the most relevant criteria that delineate the objects in the form of "*if..., then ...*" resulting in a set of decisional rules as outputs. These "*if..., then ...*" rules are simply translating the decision possibilities that irrigation system designers are facing, "*if (the design is more flexible by this amount), then (we expect this range of cost increment),*" which are easy to interpret and decide on.

In the present study, the DRSA approach is denoted as the 'Classification model' (Figure 2) and is applied according to the following procedure (Błaszczyński, Greco, Matarazzo, Słowiński, & Szeląg, 2013; Boggia et al., 2014; Greco et al., 2016, 2001; Roma et al., 2020).

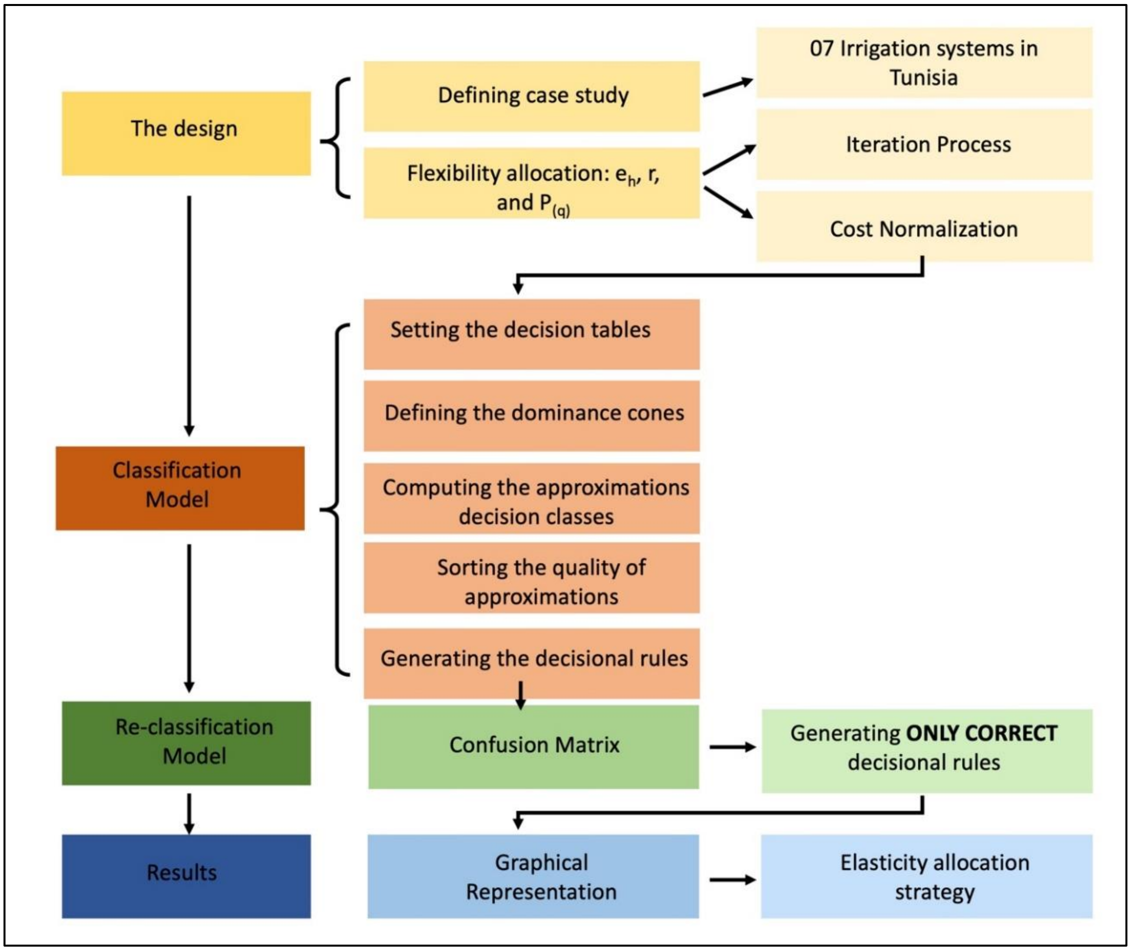


Figure 2. Schematic representation of the methodology

2.3.1 Setting the decision tables

The DRSA application starts from a data matrix code in which the rows include the objects and indicate the values of criteria for each corresponding object, while the columns include the criteria and indicate the values of each corresponding criterion for the objects. This data matrix is a 4-tuple decision table $S = \langle U, Q, V, f \rangle$, where U is a finite set of objects: Q is a finite set of criteria: $V = \bigcup_{q \in Q} V_q$, where V_q is the domain of the criterion q; and $f: U \times Q \rightarrow V$ is an information function such that $f(x,q) \in V_q$ for every $(x,q) \in U \times Q$. Hence, set Q is divided into condition criteria (set $C \neq \emptyset$) and the decision criterion (class) d. Note that $f(x,q)$ is an evaluation of object x on criterion $q \in C$, while $f(x,d)$ is the class assignment (decision value) of the object.

The domain of a criterion $q \in Q$ is assumed to be completely pre-ordered by an outranking relation $\geq q$; $x \geq_q y$, meaning that x is at least as good as (outranks) y with respect to the criterion q. Without loss of generality, it is assumed that the domain of q is a subset of reals, $V_q \subseteq \mathbb{R}$, and that the outranking relation is a simple order between real numbers such that the following relation holds: $x \geq_q y \Leftrightarrow f(x,q) \geq f(y,q)$. This relation is straightforward for gain-type criterion (i.e., "the more …, the better …"), whereas this relation can be satisfied by negating the values from V_q for cost-type criterion (i.e., "the less …, the better …").

Our datasets were built-in decision tables representing the irrigation systems subject of this study. The data matrix code contains three sets of objects attributes (A) ($\in U$), preferences (P) ($\in V$), and decisions (S) ($\in S$). Therefore, for any given combination in the condition attributes (A), which represents the design assumptions A = {e_h, r, $P_{(q)}$}, a unique decision attribute, which is the cost increment of the project, is allocated (S) = {low, medium, high, very high}. Besides, the decision-maker preference (P) is assigned to each condition attributes as gain-type or cost-type criterion, which helps to link the condition criteria to the decision criteria through this preference information.

In this context, the elasticity at the hydrant (e_h) is assigned a "gain-type" preference because the higher the value of elasticity (e_h), the higher the price of the project. The coefficient of utilisation of the network (r) is assigned a "cost-type" preference because the lower the coefficient of utilization of the network, the higher the estimated price of the project; and the operation quality parameter ($P_{(q)}$) is assigned a "gain preference" since the higher the value of $P_{(q)}$, the higher the price of the project. The input information related to each system was elaborated using the software "jMAF" developed by the Laboratory of Intelligent Decision Support Systems (Poznań University of Technology, Poland) (Błaszczyński et al., 2011, 2013).

The domain of the decision criterion V_d consists of n elements. Hence, without loss of generality, it is assumed that $V_d = T$, with $T = \{1, \dots, n\}$. V_d induces a partition of U into n classes $Cl = \{Cl_t, t \in T\}$, where $Cl_t = \{x \in U: f(x,d) = t\}$.

Each object $x \in U$ is assigned to one and only one class. The classes are preference-ordered according to an increasing order of class indices, (i.e., for all $r, s \in T$ such that $r \geq s$, the objects from Cl_r are strictly preferred to the objects from Cl_s). For this reason, we can consider the upward and downward unions of classes, defined respectively, as:

$$\boldsymbol{Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s \quad Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s \ , t \in T} \qquad \textbf{(4)}$$

2.3.2 Defining the dominance cones

The approximation of upward and downward unions of decision classes is represented by granules of knowledge that are generated by the criteria. These granules are also defined as dominance cones in the criteria values space. In particular, x dominates y with respect to $P \subseteq C$, denoted by xD_py, if x is better than y on every criterion from $P, x \geq_q y, \forall_q \in P$.

For each $P \subseteq C$, the dominance relation D_p is reflexive and transitive, namely, it is a partial pre-order. Given $P \subseteq C$ and $x \in U$, the granules of knowledge used for approximation in DRSA are as follows: a set of objects dominating x, called *P*-dominating set and given by $D_P^+(x) = \{y \in U: yD_px\}$; and a set of objects dominated by x, called *P*-dominated set and given by $D_P^-(x) = \{y \in U: xD_py\}$.

2.3.3 Computing the approximations of ordered decision classes

The *P*-lower and the *P*-upper approximation of $Cl_t^{\geqslant}, t \in T$ with respect to $P \subseteq C$, denoted as $\underline{P}(Cl_t^{\geqslant})$ and $\overline{P}(Cl_t^{\geqslant})$, are defined as:

$$\boldsymbol{\underline{P}(Cl_t^{\geqslant}) = \{x \in U: D_P^+(x) \subseteq Cl_t^{\geqslant}\}} \quad \textbf{(5)}$$

$$\boldsymbol{\overline{P}(Cl_t^{\geqslant}) = \{x \in U: D_P^-(x) \cap Cl_t^{\geqslant} \neq \emptyset\}} \quad \textbf{(6)}$$

In the same way, the *P*-lower and the *P*-upper approximation of $Cl_t^{\leqslant}, t \in T$ with respect to $P \subseteq C$, denoted as $\underline{P}(Cl_t^{\leqslant})$ and $\overline{P}(Cl_t^{\leqslant})$, are defined as:

$$\boldsymbol{\underline{P}(Cl_t^{\leqslant}) = \{x \in U: D_P^-(x) \subseteq Cl_t^{\leqslant}\}} \quad \textbf{(7)}$$

$$\boldsymbol{\overline{P}(Cl_t^{\leqslant}) = \{x \in U: D_P^+(x) \cap Cl_t^{\leqslant} \neq \emptyset\}} \quad \textbf{(8)}$$

The lower approximations group contains the objects that belong certainly to a class union $Cl_t^{\geqslant}$ (respectively $Cl_t^{\leqslant}$). Indeed, object $x \in U$ belongs to the lower approximation $\underline{P}(Cl_t^{\geqslant})$ (respectively $\underline{P}(Cl_t^{\leqslant})$) if no other object in U contradicts the claim that every object $x \in U$ which *P*-dominates x, also belong to the class union $Cl_t^{\geqslant}$ (respectively $Cl_t^{\leqslant}$). The upper approximations group contains the objects that could belong to $Cl_t^{\geqslant}$ (respectively $Cl_t^{\leqslant}$), since object $x \in U$ belongs to the upper approximation $\overline{P}(Cl_t^{\geqslant})$ (respectively $\overline{P}(Cl_t^{\leqslant})$), if there is another object $y \in U$ *P*-dominated by x from a class union $Cl_t^{\geqslant}$ (respectively $Cl_t^{\leqslant}$). The *P*-lower and *P*-upper approximations satisfy the following properties for all $t \in T$ and any $P \subseteq C$:

$$\boldsymbol{\underline{P}(Cl_t^{\leqslant}) = \{x \in U: D_P^-(x) \subseteq Cl_t^{\leqslant}\}} \quad \textbf{(9)}$$

$$\boldsymbol{\underline{P}(Cl_t^{\leqslant}) \subseteq Cl_t^{\leqslant} \subseteq \overline{P}(Cl_t^{\leqslant})} \quad \textbf{(10)}$$

Consequently, the *P*-boundaries (i.e., *P*-doubtful regions) of $Cl_t^{\geqslant}$ and $Cl_t^{\leqslant}$ are defined as:

$$\boldsymbol{Bn_P(Cl_t^{\geqslant}) = \overline{P}(Cl_t^{\geqslant}) - \underline{P}(Cl_t^{\geqslant})} \quad \textbf{(11)}$$

$$\boldsymbol{Bn_P(Cl_t^{\leqslant}) = \overline{P}(Cl_t^{\leqslant}) - \underline{P}(Cl_t^{\leqslant})} \quad \textbf{(12)}$$

2.3.4 Sorting the quality of approximations and the reduction of attributes

For every $P \subseteq F$, the quality of approximation of the ordinal classification Cl by a set of attributes P is defined as the ratio of the number of objects consistent with the dominance principle and the number of all the objects in U. Therefore, the ratio:

$$\boldsymbol{\gamma_P(Cl)} = \frac{\left|\boldsymbol{U} - \left(\left(\cup_{\boldsymbol{t \in T}} \boldsymbol{Bn_P}\left(\boldsymbol{Cl_t^{\geq}}\right)\right) \cup \left(\cup_{\boldsymbol{t \in T}} \boldsymbol{Bn_P}\left(\boldsymbol{Cl_t^{\leq}}\right)\right)\right)\right|}{|\boldsymbol{U}|} \quad \textbf{(13)}$$

defines the quality of approximation of the partition Cl into classes using the set of criteria P. This ratio expresses the relation between all the correctly classified objects and all the objects in the decision table. Furthermore, every minimal sub-set $P \subseteq C$ such that $\gamma_P(Cl) = \gamma_C(Cl)$ is called a reduct of C and is denoted by RED_{Cl}. A decision table may have more than one reduct. The intersection of all reducts is known as $CORE_{Cl}$. The criteria in this core cannot be removed without affecting the quality of approximation; thus, there are three categories of criteria: (i) indispensable criteria included in the core; (ii) exchangeable criteria included in some reducts but not in the core; and (iii) redundant criteria, neither indispensable nor exchangeable, which are not included in any reduct.

2.3.5 Generating the decisional rules

Based on the approximations obtained by the dominance relations, it is possible to describe the preferential information included in the table in terms of decision rules. These rules are expressed in the form "*If… then…,*" which indicates a dependency between condition criteria and decision criteria.

The Windy Gap Firming Project is a collaboration between 12 Northeastern Colorado water providers to address water shortages and improve the reliability of water supplies. The 12 participants represent different stakeholders with different preferences to consider: Nine municipalities, two water districts, and one power provider. To address this shortage, designers have used multiple approaches accounting for multiple criteria, alternatives, and preferences, including conservation plans, alternative transfer methods with farms, water reuse, and providing additional supplies (Municipal Subdistrict Northern Colorado Water Conservancy, 2018). The reason why procedures for generating decision rules should always follow an inductive learning principle.

In the present study, we used only the "certain rules" that represent the certain knowledge extracted from the data, which have the following form:

$if\ f(x, q_1) \geq r_1\ and\ f(x, q_2) \geq r_2\ and \dots f\left(x, q_p\right) \geq r_p\ then\ x \in Cl_t^{\geq}$

$if\ f(x, q_1) \leq r_1\ and\ f(x, q_2) \leq r_2\ and \dots f\left(x, q_p\right) \leq r_p\ then\ x \in Cl_t^{\leq}$

Possible rules have a similar syntax, but the consequent part of the rule has a different form:

$if\ f(x, q_1) \geq r_1\ and\ f(x, q_2) \geq r_2\ and \dots f\left(x, q_p\right) \geq r_p\ then\ x\ could\ belong\ to\ Cl_t^{\geq}$

$$if\ f(x,q_1) \le r_1\ and\ f(x,q_2) \le r_2\ and\ \dots f(x,q_p) \le r_p\ then\ x\ could\ belong\ to\ Cl_t^{\le}$$

On the other hand, the approximate rules have the following syntax:

$$if\ f(x,q_1) \ge r_1\ and\ f(x,q_2) \ge r_2\ and\ \dots f(x,q_k) \ge r_k\ and\ f(x,q_{k+1}) \le r_{k+1}$$

$$and\ f(x,q_{k+2}) \le r_{k+2}\ and\ \dots f(x,q_p) \le r_p\ then\ x \in Cl_s \cup Cl_{s+1} \cup Cl_t$$

A set of decision rules is complete when it covers all objects from the decision table in such a way that consistent objects are re-classified to their original classes and inconsistent objects are classified into clusters of classes referring to this inconsistency. Moreover, a set of decision rules is called minimal if is complete and non-redundant so that the exclusion of any rule from this set makes it non-complete.

Several algorithms can be used for the induction of decision rules. Among these algorithms, the "Dominance-based Learning from Examples Module" (DOMLEM) (Błaszczyński, Słowiński, & Szelg, 2011) enables the generation of a minimal set of rules.

3. Results

3.1 Iteration and Cost normalization

The parameters adopted from the design assessment of the seven modernized irrigation systems were considered as "existing sets," and their associated costs were referred to as "baselines." Any incremental cost resulting from any sequential change of the condition alternatives (e_h, r, and $P_{(q)}$), in respect to the "baseline," was recorded as a percentage. Then, the increment was normalized into four bins (low, moderate, high, and very high). Accordingly, the increment interval is calculated as the increment range divided by the number of bins, as illustrated in Table 2. The normalization allocates a 0% increment to all the irrigation sectors while in the baseline. For the low category, an average interval from 0% to 13% is assigned to (Bir Aouini, Mehrine West, and Mansoura West sectors and from 0% to 6% to the Mehrine East, Sidi Neji, Mansoura East, and Habibia sectors. The medium bin ranges from 12% to 19% for the Bir Aouini, Mehrine West, and Mansoura West sectors and from 6% to 12% for the Mehrine East, Sidi Neji, Mansoura East, and Habibia sectors. The level high ranges from 19% to 33% for the Bir Aouini, Mehrine West, and Mansoura West sectors and from 12% to 24% for the Mehrine East, Sidi Neji, Mansoura East, and Habibia sectors. Finally, any cost increments exceeding 24% for the Mehrine East, Sidi Neji, Mansoura East, and Habibia sectors are categorized as very high, as are the Bir Aouini, Mehrine West, and Mansoura West sectors if the cost exceeds 33%. The reasons behind this difference in cost increment ranges are, the total area served, the difference in elevations, and the number of users (hydrants) in each sector.

Table 1. Ranges of cost increment (in percentage) corresponding to each sector

Classes	**Bir Aouini**	**Mehrine East**	**Mehrine West**	**Sidi Neji**	**Mansoura West**	**Mansoura East**	**Habibia**
"base line"	0%	0%	0%	0%	0%	0%	0%
low	[0%;12%]	[0%; 06%]	[0%;13%]	[0%;08%]	[0%; 14%]	[0%; 04%]	[0%;05%]
medium	[12%;19%]	[06% ;10%]	[13%;18%]	[08% ;16%]	[14%23%]	[04%;08%]	[05%;12%]
high	[19%;33%]	[10%;23%]	[18%;29%]	[10%;24%]	[23%;42%]	[08%;16%]	[12%;25%]
very high	[33%;39%]	[23%;27%]	[29%;35%]	[24%;32%]	[42%;51%]	[16%;25%]	[25%;32%]

3.2 Calculation of the unions of approximations

At this level, it is important to understand a priori which type of rules are needed. First, three types of rules are available: certain, possible, and approximate. In particular, certain rules are generated from lower approximations of unions of classes and possible rules are generated from upper approximations of unions of classes, while approximate rules are generated from boundary regions. However, in this work, we decided to only use **certain** rules. Furthermore, any significant difference between the lower and upper approximation unions of classes may open doors to the uncertainty of incorrect or ambiguous results.

The calculation results of the standard unions of approximations corresponding to all of the sectors are plotted in Table 3. A strong matching in the number of objects supporting the rules from the upper and the lower approximations is illustrated, which shows the high accuracy of the resulting rules.

Table 2. Standard union of classes of the 07 irrigation systems

Union name	**Accuracy**	**Lower approx.**	**Upper approx.**	**Boundary**	**Cardinality**
At least low	1,00	20	20	0	20
At most low	1,00	2	2	0	2
At least medium	0,90	18	20	2	19
At most medium	0,82	9	11	2	10
At least high	1,00	17	17	0	17
At most high	0,85	17	20	3	19
At least very high	0,67	4	6	2	5

3.3 DRSA Application: The Classification Model

One of the first steps of data analysis using the multiple-criteria analysis is to set the decision tables, which is based on understanding the relationship between the decision variables and preference criteria (the more, the better or the less, the better). In this context, the elasticity at the hydrant (eh) is assigned a “gain-type” preference because the higher the value of elasticity (eh), the higher the price of the project. The coefficient of utilisation of the network (r) is assigned a “cost-type” preference because the lower the coefficient of utilization of the network, the higher the estimated price of the project; and the operation quality parameter P(q) is assigned a “gain preference” since the higher the value of P(q), the higher the price of the project. The second step is the calculation of dominance cones (P-dominating sets and P-dominated sets), the DRSA unions, and their approximations. Then, the model assigns the final dominance relation between objects: an object “a” is non-dominated in set U (Pareto-optimal), if and only if there is no other object “b” in set U such that “b” is not worse than “a” on all considered criteria, and strictly better on at least one criterion. Finally, the relationship between the parameters of elasticity and their associated cost is classified by applying the DOMLEM algorithm (Błaszczyński et al., 2011) based on the DRSA approach. The model extracts certain decisional rules by identifying the parameters that are most influencing the cost. The rules are sorted into two categories that characterize the classes of the decision (low, medium, high, and very high). For instance, the “at least rules” indicate the smallest that is possible or likely (e.g., $If\ (e_h \geq 3)$, then the result could be at least medium), while “at most rules” indicate the maximum that is possible or likely (e.g., $If\ (e_h \leq 5)$, then the result could be at most high).

The approach was applied on seven different irrigation systems, and the results revealed 26 decisional rules that are illustrated in Table 10 according to their decision class. All the rules shown in Table 10 are dependable to each other, therefore, any interpretation of a single rule does not reflect the decision class borders.

The first rule proposes a wide range of possibilities with, “if the elasticity $(e_h) \geq 2.5$, then the cost may be at least low.” The range is made narrow with the next rule announcing that, “if the elasticity $(e_h) \geq 3$, then the cost may be at least medium.”

Rules 6 and 7 claim two different and true conditions within the same decision class of high. The second declares that if only the elasticity $(e_h) \geq 4$, then the cost increment may be at least high, and the third says that if $(e_h) \geq 4\ and\ (P_{(q)}) \geq 95\%$, then the cost increment may be at least high. Similar outcomes generated with rules 8 and 9 always in the same class “high” and with rules 15 and 16 in the class “very high” clearly show that the operation quality parameter $P_{(q)}$ has only a minor effect on the cost increment and the decision class.

No “at most” rules were generated for the very high class since it is the ultimate class decided in this framework.

Table 3. Generated decision rules that correspond to all the systems

	At least rules	At most rules
LOW	1. *If* $(e_h \geq 2.5)$	1. *If* $(e_h \leq 2.5), (r \geq 22)$, *and* $(U_{(pq)} \geq 95\%)$
MEDIUM	2. *If* $(e_h \geq 3)$ 3. *If* $(r \leq 18)$ 4. *If* $(U_{(pq)} \geq 95\%)$	2. *If* $(e_h \leq 3)$ *and* $(r \geq 18)$ 3. *If* $(e_h \leq 3), (r \geq 22)$, *and* $(U_{(pq)} \leq 90\%)$ 4. *If* $(e_h \leq 4)$ *and* $(r \geq 18)$ 5. *If* $(e_h \leq 4), (r \geq 18)$, *and* $(U_{(pq)} \leq 90\%)$ 6. *If* $(e_h \leq 5), (r \geq 22)$, *and* $(U_{(pq)} \leq 90\%)$
HIGH	5. *If* $(e_h \geq 3)$ *and* $(r \leq 16)$ 6. *If* $(e_h \geq 4)$ 7. *If* $(e_h \geq 4)$ *and* $(U_{(pq)} \geq 95\%)$ 8. *If* $(e_h \geq 5)$ 9. *If* $(e_h \geq 5)$ *and* $(U_{(pq)} \geq 95\%)$ 10. *If* $(e_h \geq 5)$ *and* $(r \leq 18)$ 11. *If* $(r \leq 16)$	7. *If* $(e_h \leq 3)$ 8. *If* $(e_h \leq 4)$ 9. *If* $(e_h \leq 5)$ 10. *If* $(r \geq 18)$
VERY HIGH	12. *If* $(e_h \geq 4)$ *and* $(r \leq 16)$ 13. *If* $(e_h \geq 5)$ *and* $(r \leq 16)$ 14. *If* $(e_h \geq 6)$ *and* $(r \leq 16)$ 15. *If* $(e_h \geq 6)$ 16. *If* $(e_h \geq 6)$ *and* $(U_{(pq)} \geq 95\%)$	**NO Rules**

3.4 The Reclassification models

Inaccuracies while generating the rules may yield misleading and unbalanced results. Many large-scale projects have realized that checking the efficiency classification model is an important task because it allows for a more detailed analysis than a mere proportion of accuracy. For instance, with the Windy Gap Firming Project in Colorado, thirteen water providers entered the formal federal permitting process for the Windy Gap Firming Project; The Subdistrict published an alternatives report detailing 170 methods to potentially accomplish project goals, which were then narrowed down to seven (Municipal Subdistrict Northern Colorado Water Conservancy, 2018).

The reclassification task is performed using the confusion matrix. Table 4 illustrates all the generated rules in a matrix form. The row of the matrix indicates the decision classes in the target dataset, and the column represents the decision classes by the classification algorithm. The diagonal elements represent the generated elements correctly classified for each class. The diagonal sums indicate the total number of correct elements. Therefore, any off-diagonal positive number within the matrix square arrangement signifies an error that may be incorrect or ambiguous classification.

The matrix displays a tally of 19 correct results out of 26 elements. One element belongs to the class "Low," five to the class "medium," nine to the class "high," and four correct rules belong to the class "very high." The results show that the confusion matrix includes a low number of ambiguous or incorrect cases: only seven rules are misclassified, which represents the high consistency of the results.

Table 4. The confusion matrix (Reclassification of the rules) for all the system

CONFUSION MATRIX				
CLASS	**Low**	**Medium**	**High**	**Very high**
Low	1	1	0	0
Medium	1	5	2	0
High	0	2	9	0
Very high	0	0	1	4

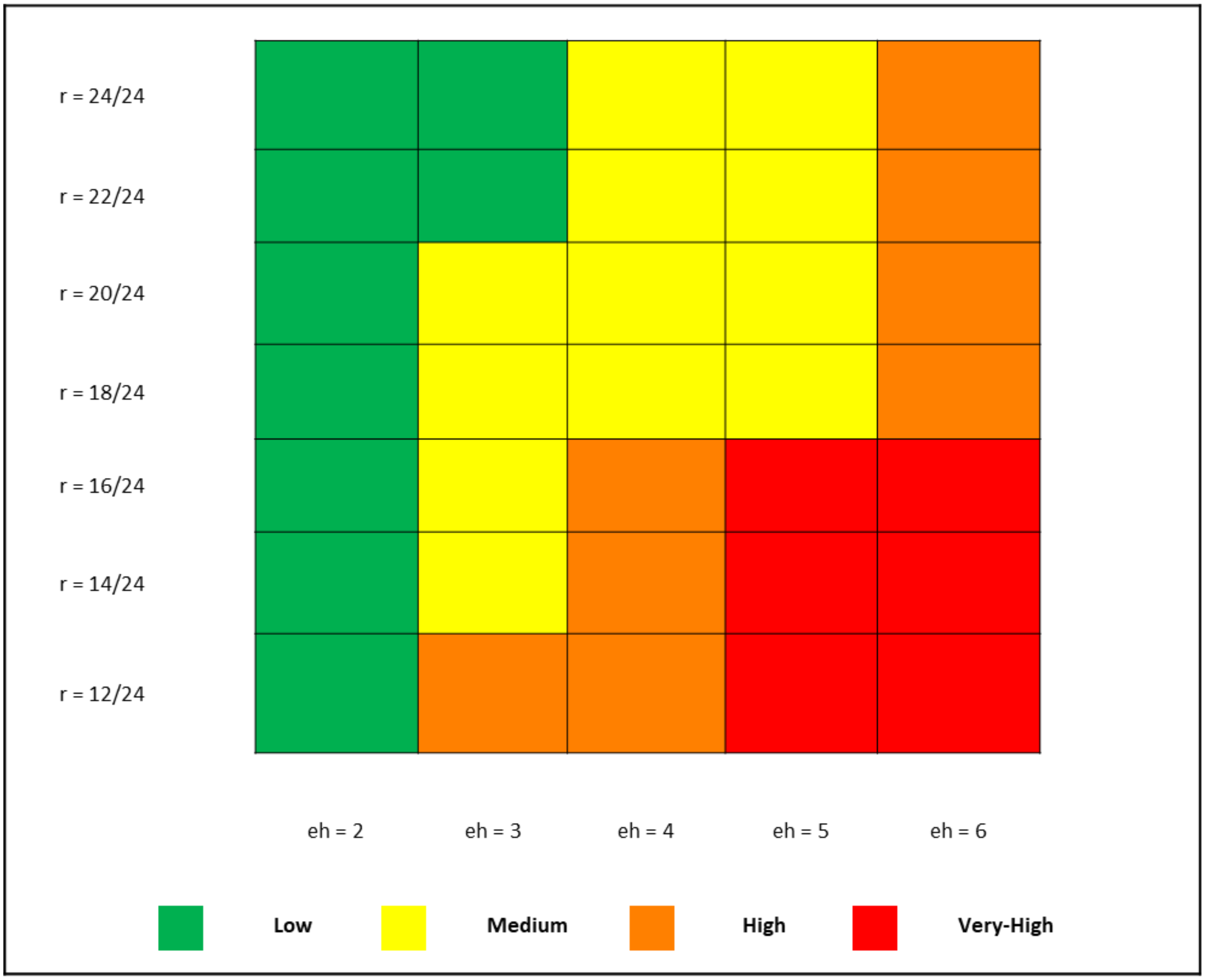
Ambiguous cases represent the inconsistency of a rule that can neither be reassigned to its original decision class nor reclassified, while incorrect cases may occur due to significant differences between lower and upper approximations of the unions of decision classes (i.e., rules supported by few objects from lower approximations). In this context, both ambiguous and incorrect rules are being declined from our analysis to avoid weakening the decision rules. Table 5 explains the reclassification results, in which one ambiguous rule seemed to include both “low” and “medium” decisions in its regulations. The case is recorded in the Mehrine East system. Within the class “medium,” three ambiguous rules are responding to “low,” “medium,” and “high” decisions at the same time. These rules are all recorded in the Mehrine East system. Two rules within the “high” class, recorded in Mehtine East and Mansoura East systems, are also classified as ambiguous as they are responding to the “medium” and “high” decisions synchronically. Finally, one incorrect rule is registered in the very high decision, recorded in Sidi Neji, Mansoura East, and Habibia systems.

Table 5. The reclassification results

RE-CLASSIFICATION RESULTS				
Class	**Correct**	**Incorrect (reclassified)**	**Ambiguous**	**Total**
LOW	01	00	01	02
MEDIUM	05	00	03	08
HIGH	09	00	02	11
VERY HIGH	04	01	00	05
			Total	**27**

3.5 Graphical representation of the rules

Graphical representation refers to the use of graphs to visually display, analyse, clarify, and interpret numerical data, functions, and other qualitative structures. Figure 3 displays the exported rules corresponding to each system in a worksheet plot. An *X–Y* data object is a two-dimensional array of data.



	eh = 2	eh = 3	eh = 4	eh = 5	eh = 6
r = 24/24	🟩	🟩	🟨	🟨	🟧
r = 22/24	🟩	🟩	🟨	🟨	🟧
r = 20/24	🟩	🟨	🟨	🟨	🟧
r = 18/24	🟩	🟨	🟨	🟨	🟧
r = 16/24	🟩	🟨	🟧	🟥	🟥
r = 14/24	🟩	🟨	🟧	🟥	🟥
r = 12/24	🟩	🟧	🟧	🟥	🟥

🟩 **Low** 🟨 **Medium** 🟧 **High** 🟥 **Very-High**

Figure 3. Graphical representation of the rules

The type of graphical representation used in this context is a heatmap. Heatmaps display information in a two-dimensional grid of cells, and each cell represents a grouping of data. The cell colour indicates the relative value, which is the project's cost increment in this study. Four colours are considered in this framework, green, yellow, orange, and red, representing the classes low, medium, high, and very high, respectively. The X versus Y graph stores in its two columns the generated rules confirmed to be correct, where the *X*-column represents the elasticity (e_h) and the *Y*-column represents the coefficient of utilization (r). The operation quality parameter (P_q) is not considered in the graphical representation as it was confirmed to have a minor effect on the cost. The approach is providing fundamentals in identifying the suitable strategy of elasticity allocation according to the budget. Therefore, selecting a single X and Y data outcome sorts a graph cell that displays the associated cost according to the related flexibilities.

The graphical representation shows the sensitivity of the cost to the elasticity at the hydrant (e_h) more than the coefficient of utilization of the network. For any assigned coefficient of utilization r, an elasticity (e_h) less than 3 would result in a low-cost increase. When the elasticity is equal to 3, the related cost increases as the network coefficient changes. The medium-cost increase appears with an elasticity of 4 and 5 only when the network's coefficient of utilization is equal to or greater than 18/24. However, the very high-cost increment class occurs with an elasticity of 5 to 6 when the network's coefficient of utilization is equal to or less than 16/24.

4. Discussion

The findings of this research are supposed to help water planners identify the correlation between design flexibility parameters that guarantees a good service to users/ratepayers with the least amount of associated expenses. First, the elasticity at the hydrant ($\boldsymbol{e_h}$) defines the flexibility assigned at hydrant level (farmer). This flexibility means that the nominal discharge of the hydrants ($\boldsymbol{d}$) has to be selected much higher than the duty ($\boldsymbol{D}$) allowing farmers to irrigate for a duration lower than 24 hours. This condition implies that higher the elasticity assigned at hydrant, bigger the pipe sizes selected, and eventually higher the cost of system. The network's coefficient of utilization defines the flexibility at the system level in an irrigation system operating on-demand, should have a value equal to one because in reality the system is operating 24 hours per day. The study showed that a system with a lower network coefficient of utilization costs more than a system with a higher coefficient of utilization. From a design standpoint, this is confirmed because we build a system to satisfy the same downstream demand in fewer operational hours, so the pipe size will be bigger and the price is higher.

The present study provides a simplistic mathematical model based on the multiple-criteria decision analysis, which explains how the flexibility in the design can influence the project's economic outcomes in the field of irrigation systems. Other technics were similarly used to address this issue, such as scenario-based planning, multistage scenario trees, multistage decision analysis, and Bayesian techniques. For instance, scenario-based planning that has gained a lot of acceptance in the water resources community through preparing a range of possible futures provides flexibility and adds robustness to the system so it can respond to uncertain events at reasonable costs while maintaining community confidence in their utilities. Kang and Lansey, 2013 applied the scenario-based multiobjective optimization model to regional-scale water and wastewater infrastructure design in the southeast of Tucson-Arizona, providing flexibility within a system to allow it to adapt to a range of uncertain events at a reasonable cost. The study involved seven scenarios representing potential system growth variations along the planning time horizon, and results showed that the flexible planning process reduces the regret costs by 30% (Kang & Lansey, 2013). Many authors used scenario-based planning to tackle the cost-effectiveness of flexible design as an approach to adapt to future uncertainties, such as (Hadjikakou et al., 2019; Herman, Reed, Zeff, & Characklis, 2015; Kang & Lansey, 2014). On the other hand, Erfani (2018) used the multistage decision analysis to inform about least-cost capacity expansion scheduling through multistage stochastic mathematical programming applied to London's water system. The results show that ignoring adaptive planning costs 15.4% of the total Net Present Value and considering flexible decision making has a value of 6% of the total Net Present Value of London's water supply system (Erfani, Pachos, & Harou, 2018). Fletcher (2019) developed a new planning framework that assesses the potential to learn about flexible approaches applied to a real problem in the water supply system in Mombasa, Kenya using a Bayesian statistical model. The approach showed that the flexible dam design cost per m3 of additional capacity added is assumed to be 50% greater than that of the original capacity. (Fletcher et al., 2019). Fletcher (2017) used probability-based risk quantification, which allows the evaluation and comparison of flexible design versus rigid design in a way that takes into account the diverse and multifaceted uncertainties water planners face. The study was applied to a desalination plant in Melbourne, Australia, and results show that in 10% of simulations, rigid design leads to regret of greater than $10 US billion compared with a flexible desalination plant. (Fletcher et al., 2017).

The methodology presented in this study provides a considerable understanding of the different levels of flexibility in the cost and the design process in the field of irrigation systems, rather than concluding that a final solution is better than the others. This is why, it seemed important in this work to illustrate the main limitations of the multiple criteria decision analysis, as well as opportunities to improve and extend the approach of flexibility in the design of irrigation water system planning.

First, the present research has only considered one planning objective, which is to find a compromise between the flexibility and the final cost of the project. In reality, planners and stakeholders may have other objectives and preferences as well, such as energy intensity, maintaining ecosystem services, equitable distribution, and access, water quality, etc. One notable objective that is not addressed in this work is the reliability of the irrigation water systems, despite the fact that it is an important objective in evaluating the cost-effectiveness of flexible investments. Second, the types and goals of flexibility are addressed in this Tunisian case study. Other applications in the literature may consider flexible planning as the timing of capacity additions, or even the volume of capacity additions to a system or an infrastructure. In this work, we provide an approach to assess the value of flexibility as a strategy for achieving reliable planning that satisfies users and planners and proves that it is a worthwhile goal. However, the application of this method across a wider range of many different water systems (supply systems, desalination plans (Fletcher et, al, 2019), hydro-power systems, etc.) or maybe other planning alternatives could enable the identification of properties of water systems that enable the high value of flexibility. Third, the model has a quite specific representation of the decision rules “If-Then”. It might be non-standard and does not support a hierarchical structure of the decision criteria, but it rather corresponds directly to how human experts typically make decisions. Fourth, the monotonicity problem: this constraint occurs when the value sets of attributes and the predefined classes are preference ordered, and there exist monotonic relationships between condition and decision attributes. Fifth, the choice problem related to the multiple objective optimizations, occurs when the set of objects is infinite and defined by the constraints of a mathematical program. Sixth, the discrete choice problem occurs when the set of objects is finite and reasonably small enough to be listed.

Future work and options to extend the framework to incorporate social and institutional issues more thoroughly. This work has represented decisions from the standpoint of a single planner, concentrating on realistic options that the planner has the authority to implement. In reality, many various actors interact with water resource systems, and the planner's actions might impact the behavior of other actors and vice versa. Moreover, the methodologies and ideas used in this dissertation could be applied to various disciplines of study. Many other engineering systems fields, such as energy or transportation, deal with infrastructure planning decisions in the face of many decision criteria and changing uncertainties. The paradigm given here could be applied to a broad variety of systems. In particular, it could be applied to other infrastructure systems whose performance is impacted by future uncertainties to assess the value of flexible approaches to managing risk.

5. Conclusions and recommendations

Success in solving today's water management challenges for future generations requires a data-informed plan and water planners with the ability to consider the fundamental concepts, algorithms, techniques, and info-tools that could be applied to this data to assist with or inform particular problem solving, as systematic thinking fosters success in data-driven decision making. In this context, the pattern extraction process concerning the flexibility afforded to the design of a large-scale irrigation system and its associated cost is investigated with well-defined stages from a give-and-take perspective. Understanding this process and its stages helps water managers structure problem solving, consider uncertainty, and create well-informed suitable designs that meet users' needs. To this aim, seven large scale pressurized irrigation systems in the Manouba region in Tunisia, operating under on-demand service and designed according to the Clément formula, are considered as the context of the application of the current study.

In this analysis, we present a simplified model that allows us to easily understand the way in which the elasticity at the hydrant interacts with the coefficient of utilization from different networks to influence economic outcomes for project realization in the field of irrigation systems.

First, an iteration-based process of the design hypothesis was performed, which generated a final cost of each system associated with a single run of iteration. Twenty-one iterations were recorded relative to each of the seven systems, assigning a different combination of elasticity in every iteration made.

Second, a classification model permits the acquisition of posterior information about the most relevant combination delineates the outcome in the form of "*if..., then ...*" resulting, therefore, in a set of 26 decisional rules. The methodology applied helps to extract certain decisional rules by identifying the parameters that most influence the cost of a project. The rules are sorted into two categories, "at least" and "at most" rules, that characterize the classes of the decision (low, medium, high, and very high). For instance, "at least rules" indicate the smallest conceivable or probable cost that may occur when altering the elasticity, whereas "at most rules" indicate the greatest cost that may occur when changing the elasticity. These two categories set the intervals (min and max) of the decision classes. The results showed that some rules would present two different and true conditions within the same decision class. For example, the second rule declares that if only the elasticity $e_h \geq 4$, then the cost increment may be at least high, and the third rule says that if $e_h \geq 4 \; and \; P_{(q)} \geq 95\%$, then the cost increment may be at least high as well. Similarly outcomes with other examples indicate that the operation quality parameter $P_{(q)}$ has only a minor effect on the cost increment and the decision class. Afterward, these rules were reclassified using the confusion matrix model, and results were plotted in a heatmap graphical representation. The reclassification resulted in 19 correct elements, indicating the good efficiency and high consistency of the classification model based on a low number of ambiguous or incorrect cases (only seven rules).

The results revealed the sensitivity of cost to the elasticity at the hydrant e_h more than the coefficient of utilization of the network. Any elasticity (e_h) less than 3 associated with any given (r), would result in a low cost increment. The cost increases as the coefficient (r) decreases for any given value of (e_h). An elasticity from 4 to 5 associated with a network's coefficient (r) equal or greater than 18/24 leads to a medium cost increment. Finally, any (e_h) from 5 to 6 associated with an (r) equal or less than 16/24 would result in a very high cost increment

This methodology provides a considerable understanding of the different levels of flexibility in the cost and the design process. Rather than concluding that a final solution is better than the others, trade-offs likely exist between achieving mathematical optimality and presenting options to decision-makers. Yet, this approach provides an interactive representation that helps in identifying the appropriate range of flexibility, justified with the expense criterion. Future efforts may also expand the outcome presented here to an interactive-dynamic setting.

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