Public-private contracting under limited commitment*

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Abstract

A government delegates a build-operate-transfer project to a private firm in a limited-commitment framework. When the contract is signed, parties are uncertain about the operating cost. The firm can increase the likelihood of facing a low cost by exerting some non-contractible effort while building the facility. Once the facility is in place, the firm learns the marginal cost and begins to operate. We characterize the contract which stipulates the efficient allocation. We study the financial structure and duration that secure its enforcement. To this end, we take into account that break-up of the partnership occasions a replacement cost for the government and an expropriation cost for the firm and its lender. Furthermore, both these costs are higher the earlier the contract is terminated. Enforcement is achieved as follows. The firm is instructed to invest some intermediate amount of own and borrowed funds. Under the aegis of a third party that can commit, the government provides guarantees to the lender, conditional on continuation of the partnership. Duration may be shortened, though not to the point where the initial effort of the firm is uncompensated.

Keywords: Public-private contracts; limited enforcement; non-commitment; financial structure; contract duration; conditional guarantees; replacement cost; expropriation

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1 Introduction

Public-private partnerships in infrastructure projects are typically characterized by two features. First, private firms are required to undertake significant investments in the initial stage and, sometimes, at specific contract milestones. Second, projects are highly leveraged.\footnote{In June 2008, The Economist reported that infrastructure spending was mainly funded with corporate bonds issued by private firms running the projects before the economic crisis, and with senior debt after the crisis. According to Sader [34], in developing countries, debt covers three-quarters of the costs of a typical build-operate-transfer infrastructure project. See also Ehrhardt and Irwin [14] and Flyvbjerg \textit{et al}ii [17] on debt finance in large public projects.} These two features of public-private partnerships, in turn, raise two important questions. The first is whether private rather than public investment is desirable. As governments usually face a lower cost of capital than private firms, this question is problematic, and the answer is far from obvious. The second question concerns the role of debt finance. While firms are not always cash-constrained to the point of justifying massive reliance on debt, highly leveraged projects tend to be vulnerable to default. Thus, one may wonder whether there is a strategic scope for debt finance, particularly as it affects the possibility and the outcome of later renegotiation, which is one of the most pervasive difficulties of public-private contracts.\footnote{Although renegotiation occurs mostly in developing countries (Banerjee \textit{et al}ii [3], Estache and Wren-Lewis [16], Guasch [19], Guasch \textit{et al}ii [20] - [21]), it is also widespread in transition economies (Brench \textit{et al}ii [5]) and even in developed countries (Gagnepain \textit{et al}ii [18]).}

The objective of this study is to suggest answers to these questions.

To this end, we rely on a model in which contractual parties have different information about relevant aspects of the project and, in addition, lack the ability to commit to particular actions. Adoption of such a model enables us to consider both information and enforcement issues. Specifically, following Laffont [30] and the related studies of Guasch \textit{et al}ii [20] - [21], we assume that, at the contracting stage, neither the government nor the firm knows the production cost. The firm discovers this as soon as the facility is in place. Furthermore, we allow for the contract to be renegotiated during operation. The model is, however, innovative in a variety of ways. First, the length of the contract is not exogenously given. Second, at the construction stage, the firm decides whether or not to exert some non-contractible effort that makes a low operating cost more likely. This is similar to studies of public-private partnerships in which there are synergies between phases of projects (e.g., Bennett and Iossa [4], Hart [22], Iossa and Martimort [26] - [27], Martimort and Pouyet [33]). Third, instead of focusing alternatively on firm-led or government-led renegotiation, we assume that either party may attempt to renegotiate. Finally, we let private capital be used to finance the project, possibly together with public funds. While in Guasch \textit{et al}ii [20], private capital can only come from bank finance, we also allow the firm to invest its own resources up-front. This representation is consistent with real-world evidence that construction expenses are generally financed with firms’ own funds and bank loans, sometimes complemented by governmental subsidies (Engel \textit{et al}ii [10]).

We first establish a benchmark. Under full commitment, the financial structure of the project does not matter. Information issues are addressed by means of a reward-and-punishment compensation scheme and a sufficiently long contract duration. A contract with these characteristics retains all surplus \textit{ex ante} and yields the efficient output level.
nder limited commitment, by contrast, enforcement of that contract may be difficult. Once
information is revealed, some party may seek to renegotiate. In that case, the financial struc-
ture of the project matters. We show that, with an appropriate mix of funds and contract
duration, incentives to behave opportunistically do not arise and the contract is honoured.

To understand our findings, it is useful to consider what would happen if the parties were
to return to the contracting table. One possibility is that renegotiation fails, the partner-
ship breaks up, and the government replaces the firm with a new operator. Alternatively,
renegotiation succeeds, and the partnership continues under a new contract. Break-up is
not costless for either party. The government incurs a replacement cost, that is, a loss of
reputation/credibility *vis-à-vis* current and prospective partners, customers, and voters. The
firm incurs an expropriation cost, *i.e.*, the initial investment is (partially) foregone. *Rebus
sic stantibus*, both parties are more prone to reach a new agreement than to break up the
relationship because, under a new agreement, they could share what the government saves by
not replacing the firm. Moreover, given the stipulated contract duration, both the replace-
ment and expropriation costs are larger, the earlier the interruption. Hence, the incentive to
sign a new agreement becomes stronger as the residual contractual period lengthens. These
considerations suggest that enforcing the contract is more difficult in two cases: first, when
parties anticipate that, should either party renege, a new deal would be reached; second,
when a long duration is initially agreed upon. Consequently, two steps can be taken to ease
the task: first, ensure parties’ anticipation that any renegotiation attempt would lead to
break-up; second, shorten the contract duration in the first place.

The first step may appear difficult to implement. However, the following procedure
will accomplish this task. The government instructs the firm to take a loan to run the
project. As the firm does not commit to reimburse the lender, the government provides
*conditional* guarantees, which would take effect only if the partnership is preserved. These
guarantees reduce the benefit that the firm could extract under a new deal and increase the
attractiveness to the government of replacing the firm. Thus, it suffices to set guarantees
large enough to ensure that any attempt to renegotiate would lead to break-up. While a
guarantee provision is difficult for a government that fails to commit, this difficulty can
be circumvented by depositing resources with a third party that does have the ability to
commit (*e.g.*, an Investment Insurance Agency). One might object that, while guarantees
are meant to attract external financiers, the latter would refuse to participate, given that
such guarantees would not come into force precisely in the case of break-up, which they
also correctly anticipate. This concern would only be justified if some party were to actually
renege. However, the ultimate goal is to secure contract enforcement. As this goal is achieved,
guarantees will not take effect, in equilibrium.

If the firm is to take out a loan, then the ability to raise funds for the project in question is
a necessary condition. However, this does not imply that a very poor firm could be entrusted
with the project, as long as it has access to credit markets. Nor does it imply that a firm not
facing credit constraints should be instructed to rely heavily on debt. Our analysis delivers
a simple recipe for attaining a suitable mix of private funds. First, the firm’s contribution
should be neither too large nor too small. Thus, another necessary condition is that the firm
is sufficiently wealthy. Second, the loan should not be too large, and it should decrease as the
firm’s contribution increases. Intuitively, the firm will be willing to honour the contract only
if it has enough to recoup from the project. The government will be willing to do the same

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only if there is not much private capital that it could appropriate by ending the contract.

A suitable financial structure can be found, regardless of how the contract duration is chosen to address information issues, provided the firm’s resources are large and, in addition, the replacement cost is large. Under these circumstances, the firm can be required to make any suitable contribution without incentivizing opportunism on the part of the government. However, if the firm is not well endowed and/or the replacement cost is not high, a long-term contract is difficult to enforce. A firm that contributes little has little to recoup, and hence little to lose if the relationship ends. If recovery is diluted over a long time period, at some point, the residual compensation may be driven so low that the firm prefers to quit the project. Moreover, if the replacement cost is always small, or it becomes small as the residual period decreases, then the government has little to lose, at least if break-up does not occur early on. If a long duration is stipulated, it may become attractive, at some point, to incur this cost in order to appropriate the private investment. In such cases, shortening the contract duration does help. However, this strategy is not completely secure. Moral hazard requires that the contract not end too early. Hence, reducing duration may make it impossible to reconcile enforcement with information issues, and the contract cannot be implemented. Taken together, these considerations point to a more general conclusion. The choice of a financial structure that incentivizes parties to abide by their contractual obligations requires stipulation of an appropriate contract length.

1.1 Related literature

Engel et alii [9] argue that private investment in public projects is desirable because disbursing money from the public budget to fund projects occasions administrative and agency costs. Our analysis provides a different motivation for private investment. Requiring a non-committing firm to contribute to the project is a way to induce the firm to honour the contract. de Bettignies and Ross [8] argue that private investment is beneficial in that, unlike public authorities, private firms credibly commit to early termination of socially inefficient projects. We examine situations in which, by contrast, continuation of the project is desirable even when unfavourable conditions are realized.

Lewis and Sappington [31] identify the mix of financing sources that enables the principal to decentralize the efficient allocation through the contract with the agent. However, they consider frameworks of full commitment. Hence, any renegotiation issue is ruled out.

Hart and Moore [23] consider a credit contract for a project, the outcome of which is observable to all parties but not verifiable. Depending upon the observed cash-flow, the firm and the creditor either renegotiate or cancel the agreement. In the latter event, the firm retains the cash-flow, and the creditor liquidates the activity. In our model, the cash-flow (governmental transfers and market revenues) is endogenous and verifiable. However, the firm does not commit to pay the debt and the creditor cannot liquidate assets, which belong to the government and have no alternative potential use. Under these circumstances, a credit contract can be signed not because the creditor is given residual control rights to the assets, as in Hart and Moore [23], but because guarantees are provided to deter default.
1.2 Outline

The remainder of the paper is organized as follows. In section 2, we describe the model, the government-firm contract, and the credit contract. In section 3, we present the full-commitment benchmark and show that the optimal contract yields an efficient allocation. In section 4, we consider limited commitment, introduce conditional guarantees, and describe the renegotiation game. In section 5, we show how the efficient allocation is enforced under limited commitment with a proper choice of contract duration and financial structure. Section 6 concludes. Mathematical details are relegated to an appendix.

2 The model

We consider the contractual relationship between a government G and a private firm F for the realization of a project that includes two tasks: the construction of a facility and the provision of a good (or service) to the collectivity. The project unfolds in two stages. In the first stage, which takes place at date 0, the facility is financed and built (the construction phase). In the second stage, which begins soon thereafter and lasts until \( T > 0 \), the facility is used to provide the good (the operation phase). At date \( T \), the contract ends. The facility is transferred to G, which then manages the activity, possibly through a public firm.

**Technology, production, consumer surplus, demand** At time 0, to build the facility, F bears the sunk cost \( I \) and exerts effort \( a \in \{0,1\} \), incurring a disutility \( \psi(a) \), with \( \psi(1) = \psi > \psi(0) = 0 \). Effort is unobservable to both G and third parties and cannot be contracted upon. At each instant \( \tau \in (0,T) \), F provides \( q \) units of the good, incurring a marginal cost of \( \theta \) and a fixed cost of \( K \). The level of effort exerted affects the distribution of \( \theta \). Specifically, exerting \( a = 1 \) favours the realization of a low marginal cost. As a return on production, F receives a transfer \( t \) from G and collects revenues \( p(q)q \) from the market. This assumption encompasses a variety of real-world situations, ranging from conventional infrastructure provision, where the firm only receives governmental transfers, to traditional concession, where the firm only collects market revenues. Consumption of \( q \) units of the good yields instantaneous gross surplus \( S(q) \), such that \( S'(\cdot) > 0 \), \( S''(\cdot) < 0 \), \( S(0) = 0 \), and the Inada conditions are satisfied. The output produced at some given \( \tau \) is sold on the market at price \( p(q) \equiv S'(q) \), which defines the inverse demand function, and is entirely consumed in that instant. Once the investment is made, both technology and demand remain constant for the duration of the project.

**Information structure** The contract between G and F is signed, the investment \( I \) is made, the effort \( a \) is exerted, and the disutility \( \psi(a) \) is borne, while the value of \( \theta \) is unknown to either party. However, at the contracting stage, it is commonly known that \( \theta \) will be either low (\( \theta_l \)) or high (\( \theta_h \)), with probabilities \( \nu_1 \) and \( 1 - \nu_1 \), respectively, if \( a = 1 \), and with probabilities \( \nu_0 \) and \( 1 - \nu_0 \), respectively, if \( a = 0 \), and such that \( \nu_1 > \nu_0 \). For future reference, we let \( \Delta \theta = \theta_h - \theta_l \) and \( \Delta \nu = \nu_1 - \nu_0 \). Once the facility is in place, F observes \( \theta_l \) and begins to produce.
Project financing  To finance the cost of investment, F injects an amount \( M \in [0, E] \) of its own funds, where \( E \) denotes its resource endowment, and borrows \( C \geq 0 \) in the credit market. G makes an up-front transfer \( t_0 \in \mathbb{R} \) to F such that
\[
M + C + t_0 = I. \tag{1}
\]
The transfer \( t_0 \) is positive when the project is partially financed with public funds, and negative when it is financed only with private funds. In the latter case, the contribution of F includes a fee equal to \( I - (M + C) \) for being awarded the contract.

2.1 Payoffs under complete information

Suppose that not only F but also G knows the effort exerted in construction as well as the marginal cost of production. We present parties’ payoffs in this environment, for some given value of \( \theta \) observed at the outset of the operation phase.

The payoff of F  Let \( d \geq 0 \) be the repayment that F makes to the lender L at each instant \( \tau \in (0, T) \) in return for the amount of money \( C \) received initially. For the given \( \theta \), F obtains the instantaneous operating profit
\[
\pi = t + p(q)q - (\theta q + K) - d. \tag{2}
\]
Further denoting the discount rate as \( r \), the present value at date \( \tau \) of the stream of profits through date \( T \) is given by
\[
\Pi_{\tau} = \int_{\tau}^{T} \pi e^{-r(x-\tau)} dx.
\]
The payoff of F is the net present value of the project:
\[
\tilde{\Pi} = \Pi_0 - (M + \psi(a)). \tag{3}
\]

The payoff of G  The aim of G is to maximize the discounted consumer surplus generated under both private and public management, net of market expenditures and the social cost of transferring resources from taxpayers to the producer. To finance the transfers, G must raise distortionary taxes. Each transferred euro requires collecting \( 1 + \lambda \) euros from taxpayers, where \( \lambda > 0 \) is the shadow cost of public funds (see Dahlby [7], for instance). The imperfections of the tax system do not vary over time, and hence \( \lambda \) remains constant. The discounted return of G over the period \( (\tau, T) \) is formulated as
\[
V_{\tau} = \int_{\tau}^{T} w(q)e^{-r(x-\tau)} dx - (1 + \lambda) (\Pi_{\tau} + D_{\tau}), \tag{4}
\]
with \( w(q) \equiv S(q) + \lambda p(q)q - (1 + \lambda) (\theta q + K) \), and where \( D_{\tau} = \int_{\tau}^{T} e^{-r(x-\tau)} dx \) is the value of the debt of F at date \( \tau \). The credit market is competitive and populated by a large number of lenders, each facing zero outside opportunity, so that \( D_0 = C \). Accordingly, the discounted
No additional investment is required to continue the activity after the conclusion of the contract. Furthermore, the production technology is related to the inner characteristics of the facility, so that, once the facility is in place, the marginal cost of production remains the same, regardless of who runs the activity. Under these circumstances, at date $T$, the discounted optimized return of $G$ from public management is

$$R^*_T = w^*(q_T) dy,$$

where $w^*(q_T)$; and $q^*$ is the output level that maximizes $w(q)$. This level is characterized by the Ramsey-Boiteux condition

$$p(q^*) - \theta = \frac{\lambda}{1 + \lambda |\varepsilon(q^*)|},$$

where $\varepsilon(q) \equiv (dp(q)/dq) q/p(q)$ is the price elasticity of market demand when output is $q$. Given $\theta$, the payoff of $G$ is formulated as

$$W = \tilde{V} + \int_T^\infty w^* e^{-r_y} dy.$$

**Optimized payoffs** Under complete information, $G$ requires that $F$ both exert effort in construction, provided that this is desirable, and produce the output level $q^*$. Moreover, $G$ leaves no surplus to $F$. Thus, $F$ and $G$ obtain, respectively,

$$\Pi^*_0 = M + \psi$$

$$W^* = \int_T^\infty w^* e^{-r_y} dy - (1 + \lambda) (I + \psi).$$

## 2.2 Contracts

Two contracts are signed, one between $G$ and $F$, the other between $F$ and $L$.

**The contract between $G$ and $F$** $G$ makes a take-it-or-leave-it offer to $F$. First, this specifies the financing triplet $(M, C, t_0)$. Second, invoking the Revelation Principle and restricting attention to direct mechanisms under which $F$ releases information, the offer includes the menu of allocations $\{(q_i, t_i); (q_h, t_h)\}$, where $q_i$ is the quantity to be produced and $t_i$ is the transfer to be made at each instant $\tau \in (0, T)$ in the event that the realized cost is $\theta_i$. Assuming that effort is desirable, and given that it affects cost stochastically, conditioning the allocation on the cost realization is a way to address not only the adverse-selection problem but also the moral-hazard problem. Henceforth, the subscript $i$ will be appended to all state-dependent variables. Finally, the offer indicates the date $T$ at which the contract will end.

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As is standard, effort provision is desirable as long as the expected gain from effort exceeds the cost of inducing effort. Specifically, at the Ramsey-Boiteux quantities, this means that $E_i [w^*_i] - \overline{E_i} [w^*_i] > r\psi$, where $E_i$ (resp. $\overline{E_i}$) is the expectation operator over the two states $i$ and $h$, corresponding to $a = 1$ (resp. $a = 0$).
The credit contract  Consistent with the deal made with G, the contract signed between F and L states the amount of money C that F will borrow from L and invest in the project at date 0. Additionally, the contract establishes the repayment \( d_i \) that, in state \( i \in \{ l, h \} \), F will make to L at each instant of the operation phase. This is set to yield neither a surplus nor a loss to L. That is, \( E_i[D_{i,0}] = C \) or, equivalently, \( E_i[d_i] = rC/\left(1 - e^{-rT}\right) \).

3 Full commitment

We begin by considering situations where G and F commit to their reciprocal obligations and, in addition, F commits to its obligations \( \text{vis-à-vis} \) L. We characterize the optimal contract between G and F in this framework, reporting mathematical details in Appendix A. The optimal credit contract is thus determined.

Referring, with a standard change of variables, to the pair of discounted cumulated profits \( \{\Pi_{l,0}, \Pi_{h,0}\} \) rather than to the pair of instantaneous transfers \( \{t_l, t_h\} \), the contractual offer that G makes to F becomes \( \{(M, C, t_0); (q_l, \Pi_{l,0}); (q_h, \Pi_{h,0}); T\} \). To determine the optimal values of the whole set of variables, G solves the following programme:

\[
\begin{align*}
\max \quad & E_i [W_i] \\
\text{s.t.} \quad & \Pi_{l,0} \geq \Pi_{h,0} + \int_0^T \Delta q_h e^{-rx} dx \tag{6a} \\
& \Pi_{h,0} \geq \Pi_{l,0} - \int_0^T \Delta q_l e^{-rx} dx \tag{6b} \\
& \Pi_{l,0} - \Pi_{h,0} \geq \frac{\psi}{\Delta \nu} \tag{6c} \\
& E_i[\Pi_{i,0}] \geq M + \psi \tag{6d} \\
\text{and (1).}
\end{align*}
\]

In the programme, (6a) (resp. (6b)) is the incentive-compatibility constraint whereby type \( \theta_l \) (resp. \( \theta_h \)) is not tempted to choose the quantity-profit pair designed for type \( \theta_h \) (resp. \( \theta_l \)). (6c) is the moral-hazard constraint whereby F is not tempted to shirk during the construction stage. (6d) is the participation constraint, which assumes that the best outside opportunity of F is zero.\(^4\)

The optimal production level is \( q^*_i \) for all \( i \in \{l, h\} \), i.e., the efficient (Ramsey-Boiteux) quantity is included in the contractual offer for each possible cost. Furthermore, the expected profit, evaluated at the optimal quantities, is set to saturate (6d): \( E_i[\Pi^*_{i,0}] = M + \psi \). This implies that no information rent is given up to F \( \text{ex ante} \).

Neglecting (6c) for a moment, the pair of optimal profits satisfying (6a) and (6b) is any

\(^4\)The fact that both F and L have zero outside opportunities means that, risk being equal, equity financing \((M)\) and debt financing \((C)\) have the same rate of return for the project.
\(\Pi_{i,0}^* (z) \equiv M + \psi + (1 - \nu_1) \int_0^T \Delta \theta z e^{-rx} \, dx \) \hfill (7a)
\[\Pi_{h,0}^* (z) \equiv M + \psi - \nu_1 \int_0^T \Delta \theta z e^{-rx} \, dx \] 

determined by picking the "sharing rule" \(z\) within \(Z \equiv [q_h^*, q_l^*]\) for any given duration \(T\). Profits in (7a) and (7b) satisfy (6c) as well, if and only if
\[T \geq T(z) \equiv \frac{1}{r} \ln \frac{\Delta \nu \Delta \theta z}{\Delta \nu \Delta \theta z - r \psi}. \] 

In line with the findings of the literature on public-private partnerships, addressing moral hazard requires that the contract last long enough for the firm to enjoy the benefits of its effort exerted in construction (compare Iossa and Martimort [27]). How tight this requirement is depends upon the sharing rule chosen. As \(T(z)\) is smaller for larger \(z\), (8) is relaxed most when \(z = q_h^*\). Intuitively, by raising \(z\), \(G\) introduces more risk in the distribution of profits and, thus, lessens the temptation to shirk.

In expectation, \(G\) reaps the same net benefit that it would obtain from the project if the realization of \(\theta\) were publicly observed at the outset of the operation phase, i.e.,
\[\mathbb{E}_i [W^*_i] = \int_0^\infty \mathbb{E}_i [w^*_i] e^{-rx} \, dx - (1 + \lambda) (I + \psi). \] 

Importantly, this result is attained regardless of the exact way in which \(M, C,\) and \(t_0\) are mixed to fund the project.

**Proposition 1 (Benchmark)** Assume that \(\exists z \in Z\) for which \(\psi < \Delta \nu \Delta \theta z / r\). Then, under full commitment, the payoff \(\mathbb{E}_i [W^*_i]\) is attained with profits (7a) and (7b), if and only if, for such \(z\), \(T\) is set according to (8). Any triplet \((M, C, t_0)\) satisfying (1) can be chosen.

Henceforth, for the sake of brevity, we let \(\Psi \equiv \{(M, C, t_0); (q_l^*, \Pi_{l,0}^* (z)); (q_h^*, \Pi_{h,0}^* (z)); T\}\) denote the contract that stipulates the efficient (full-commitment) allocation for some \(z \in Z\) and \(T\) fulfilling (8). Under \(\Psi\), discounted profits at date \(\tau\) are given by
\[\Pi_{l,\tau}^* (z) = \left( \frac{(M + \psi) r}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-r(T-\tau)}}{r} \] 
\[\Pi_{h,\tau}^* (z) = \left( \frac{(M + \psi) r}{1 - e^{-rT}} - \nu_1 \Delta \theta z \right) \frac{1 - e^{-r(T-\tau)}}{r}, \] 
in the good and bad states, respectively. Abstracting from the part of profits that differs between states for incentive purposes, (10a) and (10b) illustrate an important feature of the compensation of \(F\). In either state, \(F\) is allowed to recoup its initial investment, including both its monetary contribution \((M)\) and its non-monetary contribution \((\psi)\). Nonetheless, recovery occurs progressively over time, in line with the principle, often invoked in public-private partnerships, that the private partner should be remunerated "as time goes by."
Moreover, under $\Psi$, the discounted return of G in state $i$ at date $\tau$ is given by

$$V^*_i(z) = \omega^*_{i,\tau} - (1 + \lambda) (\Pi^*_i(z) + D_{i,\tau}),$$

where $\omega^*_{i,\tau} \equiv w^*_i \left(1 - e^{-r(T-\tau)}\right) / r$. The surplus that consumption of the optimal quantity yields, net of the production cost in the relevant state, is diminished by the social cost of the profit that G owes to F, plus the debt with which F is burdened.

4 Limited commitment

Consider now a situation where G and F sign $\Psi$, F and L sign the credit contract $\{C; d_l, d_h\}$, but neither G nor F commits to contractual obligations. Using the terminology often adopted in the literature, there is a problem of both non-commitment and limited enforcement (see Estache and Wren-Lewis [16] on these labels). In this environment, enforcement of $\Psi$ may be difficult. Suppose that, after building the facility, F correctly announces $\theta_i$ to G and, accordingly, parties’ payoffs are realized.$^5$ One or the other party may have an incentive to renege on $\Psi$. For example, F may be unhappy with its profit in state $h$, or G may not be willing to reward F in state $l$. In addition, in the absence of reputation concerns in the credit markets, F may cease reimbursement of L. This implies that F would be unable to raise funds in the first place. $\Psi$ yields an efficient allocation. Hence, it would be desirable if it were honoured. Moreover, insofar as a loan is difficult to obtain in the first place, it may be difficult, for budgetary reasons, to undertake the project at all. One thus needs to understand how $\Psi$ can be made self-enforcing in such a way that, even under limited commitment, external financiers can be involved, if desired, and the efficient allocation is implemented for the stipulated duration.

4.1 Conditional guarantees: why and how

To ensure that L is willing to lend money to F, G guarantees the debt of F. Specifically, when contracts are signed, G stipulates that some resources will be transferred to L, should F, at some point, cease to honour its debt obligations. Importantly, guarantees are conditional, i.e., they come into force only if the relationship with F remains in place. The exact amount of resources to be transferred to L depends on whether the relationship continues under the initial or a new contract. Moreover, for each such contract, it depends on the state and date at which the guarantee would take effect. In principle, an entire profile of guarantees could result, one for each state-date pair at which F could stop making payments to L.$^6$

$^5$We report in Appendix C.1 the conditions under which F has no incentive to cheat at the outset of the operation phase, anticipating that a new negotiation will take place at a later stage.

$^6$Guasch et alii [20] assume that the assets of the firm can be used to pledge debt collateral. We do not consider this possibility for the following reason. In the private sector, when debt is not paid back, the creditor undertakes the activity to liquidate or reorganize it. By contrast, in the situation that we represent, the government undertakes the activity, which continues although the initial firm no longer manages it. Hence, in our model, if the relationship between G and F breaks down and F stops making payments to L, the assets of F that are sunk in the project cannot be liquidated to reimburse L. Whether or not L recovers the money depends upon the guarantee that G provides. However, even if there were assets that could be
Reliance on conditional guarantees is consistent with the project finance technique. This technique requires making the project legally and economically self-contained. Ideally, this outcome is attained in two ways. First, a stand-alone firm is created to undertake no other business than running the concerned project and endowed with the sole assets pertaining to it, where such assets are kept separate from the assets of the parent firm(s). Second, lenders are provided no guarantees beyond the right to be paid out of the resources generated by the project, which means that guarantees are foregone, should the firm quit the project (see Yescombe [37] - [38]). In practice, things are often different. While the firm remains responsible for debt as long as it earns profits from the project, the government bails out the project when difficulties arise. At that point, debt responsibilities are passed to taxpayers. This occurred, for instance, with the 2002-03 London Underground maintaining-and-upgrading project (see House of Lords [24] - [25]). To boost banks’ appetite for the project, Transport for London guaranteed 95% of the debt obligations of Metronet, the consortium in charge of the project. As the guarantee was not conditioned on the continuation of the partnership, it came into force when, eventually, Metronet failed and the partnership broke up. To enable Transport for London to meet the guarantee, the Department for Transport had to make a £1.7 billion payment. Overall, taxpayers ended up with a direct loss of between £170 and £410 million, an epilogue that could have been avoided with a conditional guarantee.

One may wonder how G can pledge guarantees in situations where it has no ability to commit. In fact, however, G can use "external" means to tie its hands. One can think of G as depositing resources with a third party, whose task would then be to release money directly to L, should F suspend transfers. Strategies of this kind are frequently adopted, in practice. Governments often mandate Investment Insurance Agencies to act as intermediaries, providing insurance and/or direct cover in the event of payment default by a borrower (or its guarantor) under some loan agreement. Furthermore, in developing countries, the World Bank and other multilateral development banks provide huge guarantees that are less subject to project and country limits than insurance. According to Irwin et alii [29], if properly managed, these guarantees are essential to reinforcing governments’ resolve to abide by their commitments. As will be apparent below, our analysis supports this view.

4.2 The renegotiation game

Our ultimate objective is to establish conditions under which parties have no incentives to renege on $\Psi$. In order to achieve that, it is first necessary to understand what would happen if reneging were to occur. The obvious goal of the reneging party would be to raise its payoff. We thus assume that, once, at some date $\tau \in (0, T)$ and in some commonly-known state $i \in \{l, h\}$, either F or G reneges on $\Psi$, the two parties return to the contracting table.

liquidated without compromising the execution of the project under the new management, only the residual debt, i.e., the part of debt that is not protected by the collateral, would be relevant in our model. Therefore, allowing F to use its assets to pledge collateral would bring no qualitative change to our analysis and results.

Most European governments have set up official Investment Insurance Agencies for the purposes described in the text, and are now party to the "Arrangement on Guidelines for Officially Supported Export Credits," which provides specific rules for project finance, derogating from the usual Consensus Rules to allow, among other things, for longer repayment terms. See Sader [34] on the core role that both bilateral and multilateral Investment Insurance Agencies play in developing countries in providing political risk insurance by pledging guarantees on the debt package in the realization of BOT-type infrastructure projects.
If renegotiation fails, the partnership breaks down and F is replaced by another firm F’. If it succeeds, the partnership continues under a revised contract. The main features of these alternative regimes are illustrated below. Further details and mathematical derivations are reported in Appendix B.

4.2.1 Break-up of the relationship and replacement of F

When renegotiation fails, F is relieved of the activity and no longer receives any compensation. It foregoes the part of the initial contribution that it would have recovered, under the contract, from date $\tau$ to date $T$. F then has no reason to make further payments to L. As the guarantee does not come into force, L foregoes the part of the loan that was supposed to be paid back between date $\tau$ and date $T$. In other words, both F and L incur an expropriation cost, which is larger, the earlier a break-up occurs.

G appropriates the private resources invested in the project that have not been recovered, and continues to benefit from the productive activity undertaken by F’. Specifically, as the marginal cost is still $\theta_i$, G makes the following deal with F’. The latter will produce the quantity $q^*_i$ in return for an instantaneous transfer exactly covering production costs. This yields to G the largest benefit $\omega^*_i$. At the same time, G bears a replacement cost. This reflects the loss of reputation and/or credibility that the government faces for not being authoritative enough to have the contract executed by the project partner and/or for breaking promises vis-à-vis current and prospective investors, customers, and voters.\(^8\) The replacement cost is denoted $R_\delta$, with $\delta = T - \tau$, to capture the circumstance that its size is related to the length of the residual contractual period. Specifically, it is such that $R_\delta > 0 \forall \delta \in (0, T)$, with $\lim_{\delta \to 0} R_\delta = \varepsilon > 0$, and $R_0 = 0$. Furthermore, $R_\delta$ is continuously differentiable on $(0, T)$, and $R'_\delta \equiv (dR/d\delta) > 0 \forall \delta \in (0, T)$.\(^9\)

Overall, appending the superscript $rp$ to indicate the replacement scenario, the payoff of F and the discounted return of G at date $\tau$ are given, respectively, by

$$\Pi^{rp}_{i,\tau} = \int_{\tau}^{T} \pi^{rp}_i e^{-\tau(x-\tau)} dx = 0$$  \hspace{1cm} (12a)$$
$$V^{rp}_{i,\tau} = \omega^*_i - R_\delta.$$  \hspace{1cm} (12b)

\(^8\)See Guasch et alii [20] and Irwin [28]. The latter stresses that government-firm games involving large investments are repeated games in a double sense. Not only is the government concerned with its reputation vis-à-vis the current partner. It also cares about the information that its behaviour and achievements convey to third parties with whom it might interact in the future. See also Martimort [32] regarding the information value that contractual deviations by governments have for third parties, and the negative consequences for governmental credibility. The assumption that, unlike G, F has no reputation concerns reflects the practical circumstance that reputation losses are smaller for private firms, especially if they can diversify activities and locations and/or disguise themselves behind subsidiaries.

\(^9\)The fact that, in our model, the marginal cost is an inner characteristic of the facility makes it unnecessary to run an auction in the first place to select the most efficient firm. If the marginal cost were specific to the production technology, and the project were tendered out, then replacing F would lead to a higher marginal cost. In that situation, the replacement cost would further capture this loss of efficiency.
4.2.2 Renegotiation

We assume that, with probability $\alpha \in [0, 1]$, G makes a take-it-or-leave-it offer to F; with probability $1 - \alpha$, F makes a take-it-or-leave-it offer to G. The party that takes the initiative optimally makes the offer that leaves the partner just indifferent between renegotiation and replacement. For any benefit from renegotiation reaped through a change in the instantaneous transfer, there is nothing to gain from a change in the termination date (see Appendix B.3). Hence, the parties concentrate on quantity-transfer proposals.

F defaults on the loan, letting the guarantee take effect, and parties’ payoffs are determined accordingly. While F receives the same payoff as under replacement, if G makes the offer, it extracts the resources that G would lose in the replacement scenario, net of the guarantee, if F itself makes the offer. In turn, whichever party makes the offer, G obtains the largest benefit $\bar{\omega}_{\tau}^*$ from consumption of the good. This is then diminished by the social cost of the surplus that is given up to F when it makes the offer, plus the guarantee provided to L.

Overall, appending the superscript $rn$ to indicate the renegotiation regime, the payoff of F and the discounted return of G at date $\tau$ are given, respectively, by

$$\Pi_{i, \tau}^{rn} = (1 - \alpha) \left( \frac{R_{\delta}^*}{1 + \lambda} - D_{i, \tau}^{rn} \right)$$

$$V_{i, \tau}^{rn} = \bar{\omega}_{\tau}^* - (1 + \lambda) \left[ (1 - \alpha) \left( \frac{R_{\delta}^*}{1 + \lambda} - D_{i, \tau}^{rn} \right) + D_{i, \tau}^{rn} \right],$$

where, to avoid confusion with the value of the guarantee provided for $\Psi$ (namely, the value of the debt $D_{i, \tau}$), we let $D_{i, \tau}^{rn} = \int_0^T d_{i, \tau}^{rn} e^{-r(x-\tau)} dx$ denote the value at date $\tau$ of the guarantee provided in the event that, in state $i$, the contract is renegotiated at that date and not further renegotiated beyond that date.

4.2.3 Ruling out repeated renegotiation

The expressions in (13a) and (13b) are determined under the implicit assumption that the contract renegotiated at date $\tau$ remains in place until $T$. However, the parties could renegotiate after date $\tau$. We touch on this possibility only briefly, relegating formal details to Appendix B.4. For renegotiation not to occur repeatedly, conditional on state $i$ being correctly announced by F at the outset of the operation phase, it must be the case that, by renegotiating again at $\tau'$, neither F nor G reaps a higher return than under the contract renegotiated at $\tau$. Formally, this occurs for all $\tau \in (0, T)$ and $\tau' \in (\tau, T)$, if and only if

$$D_{i, \tau}^{rn} - \tilde{D}_{i, \tau/\tau'}^{rn} \geq \frac{1}{1 + \lambda} \max \left\{ \left( R_{\delta'} - R_{\delta}^* \right) \frac{1 - e^{-r_{\delta'}}}{1 - e^{-r_{\delta}}} ; \left( R_{\delta} \frac{1 - e^{-r_{\delta'}}}{1 - e^{-r_{\delta}}} - R_{\delta'} \right) \frac{1 - \alpha}{\alpha} \right\},$$

where $\delta' = T - \tau'$ and $\tilde{D}_{i, \tau/\tau'}^{rn} = \int_{\tau'}^T d_{i, \tau}^{rn} e^{-r(x-\tau')} dx$ is the value at $\tau'$ of the guarantee that G provides to L at date $0$, anticipating the possibility of $\Psi$ being renegotiated at $\tau$. By exploiting the cost that replacement at $\tau$ would cause G, starting from that date, F extracts from G an instantaneous benefit equal to $r R_{\delta}/(1 - e^{-r_{\delta}})$. (14) implies that, when that benefit varies with the renegotiation date, the value of the guarantee that takes effect at $\tau'$.
must be sufficiently large relative to the residual value that the guarantee taking effect at \( \tau \) has at \( \tau' \). If it were not so, by renegotiating again at \( \tau' \), there would be more for F to extract from G, should that benefit increase as the renegotiation date approaches \( T \). G would have something to save on the resources conceded to F in the converse case. Remarkably, unless \( r R_R / (1 - e^{-r \delta}) \) is constant, in the absence of guarantees, repeated renegotiation would always be appealing to some party. Throughout, we take guarantees for the renegotiated contract to be set such that (14) holds for all \((\tau, \tau')\) - pairs, so that renegotiation occurs at most once.

4.2.4 The incentives of F and G to renege

Inspection of (12a) to (13b), jointly with (10a) to (11), allows us to identify the exact incentives that contractual parties have to return to the contracting table, conditional on F announcing the true cost at the outset of the operation phase. With these incentives clear in mind, it will be easier to understand how to secure enforcement of \( \Psi \).

Being aware that break-up of the partnership would occasion a replacement cost for the government, F may threaten to abandon the project, causing G to bear that cost, unless the initial deal is revised in a manner favourable to F. This temptation is naturally stronger in state \( h \), in which F receives a lower compensation. By quitting the project, F would in turn lose (a part of) its up-front contribution. Even so, the threat of default may still be effective, provided that replacement is sufficiently costly to G and the initial contribution of F is relatively modest.

On the other hand, being aware that break-up would involve expropriation of the firm, G may threaten to stop compensating F, unless the contract is revised in a manner favourable to G. This incentive is obviously stronger in state \( l \), in which G owes a higher return to F and, yet, rewarding F is no longer optimal, once information has been revealed. Although the activity can continue with a new firm that reaps no surplus, the change of partner would be costless for G. The threat is nonetheless credible, provided that sufficient private capital is involved.

5 Enforcement of \( \Psi \) under limited commitment

We are now ready to establish conditions under which the parties are willing to honour \( \Psi \). This reduces to identifying an appropriate mix of funds \((M, C, t_0)\) and a termination date \((T)\), taking into account that (1) and (8) must hold.

The choice of a financial structure and that of a duration securing enforcement of \( \Psi \) are not disjoint. The duration (more precisely, the residual contractual period) affects the replacement cost, which determines the attractiveness of new negotiations compared to execution of \( \Psi \). At the same time, the duration dictates how rapidly F and L recover the investment as \( \Psi \) is executed and, hence, how much G could appropriate by ending the partnership at a given date. The relationship between the financial structure of the project and the duration of \( \Psi \) is thus evident, yet, far from obvious. It will be clarified in the remainder of the analysis.
5.1 Setting guarantees to facilitate enforcement

As far as private funds are concerned, one can show that enforcement of $\Psi$ involves weaker requirements when replacement, rather than renegotiation, is the relevant epilogue to be prevented. To facilitate enforcement of $\Psi$, one should thus find a way to ensure that, were some party to renege, the relationship would be interrupted, eventually. Our previous analysis suggests that the conditional guarantees that $G$ provides to $L$ when the credit contract is signed can potentially influence the outcome of the renegotiation game, and hence the attractiveness of engaging in renegotiation at all. In fact, comparison of (12a) with (13a) and (12b) with (13b) shows that renegotiation would fail or succeed, depending on the magnitude of $D_{i,\tau}^{rn}$. Thus, properly setting these guarantees accomplishes the task.

In formal terms, let guarantees in the renegotiation scenario be set such that, for each relevant $(i, \tau)$-pair, it is

$$D_{i,\tau}^{rn} \geq \frac{R_i}{1 + \lambda}.$$ (15)

Two consequences follow. First, any benefit that $F$ could extract from $G$ under a new agreement is eliminated; hence, $F$ prefers to abandon the project ($\Pi_{i,\tau}^{TP} \geq \Pi_{i,\tau}^{rn}$). Second, reimbursing $L$ becomes too onerous for $G$; hence, for $G$, it is better to let the debt remain unpaid and end the partnership ($V_{i,\tau}^{TP} \geq V_{i,\tau}^{rn}$). It is thus clear that, provided (15) is satisfied, any renegotiation attempt would indeed fail. Importantly, pledging large guarantees for the renegotiated contract is not an issue, given that they will never actually be paid in equilibrium.

This completes the picture of the "strategic" use of conditional guarantees under limited commitment. By setting the profile $\{D_{i,\tau}^{rn}\}$ according to (14) and (15) at the contracting stage, it is possible to eliminate the prospect of renegotiation, whether repeated or single. Conditional guarantees should thus be pledged with this goal in mind. Once this is done, we can focus on how $F$ and $G$ can be restrained from breaking up the partnership, i.e., we can seek the weakest conditions under which $\Psi$ is enforceable. Hereafter, we follow this approach.

5.2 Removing incentives to break up the partnership

Assume that (14) and (15) hold. We now identify the requirements that own funds and debt must satisfy, for given values of $T$.

5.2.1 Removing the incentives of $F$

For $\Psi$ to be honoured, one must ensure that, conditional on revealing $\theta_i$ at the outset of the operation phase, $F$ is at least as well off in $\Psi$ as it would be if the relationship were to cease at some date $\tau$. In formal terms, the following condition must hold:

$$\Pi_{i,\tau}^*(z) \geq \Pi_{i,\tau}^{TP}.$$ (16)

Recalling that $\Pi_{i,\tau}^{TP} = 0$, (16) reduces to the requirement that $\Pi_{i,\tau}^*(z)$ be non-negative. Additionally, one should ensure that $F$, anticipating that the contract will be renegotiated at a later stage, has no incentive to cheat at the outset of the operation phase. In Appendix C.1,
we show that this incentive arises neither in state \( l \) nor in state \( h \), as long as (16) is satisfied, together with (6a) and (6b), respectively. We can thus neglect this concern and state the following lemma.

**Lemma 1** Assume that, in state \( i \in \{l,h\} \), at some given \( \tau \in (0,T) \), (14) and (15) hold. Then, when \( i = l \), (16) is satisfied. When \( i = h \), it is satisfied if and only if

\[
M \geq \nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} - \psi. \tag{17}
\]

**Proof.** See Appendix C.2. ■

The temptation of \( F \) to quit the project in the bad state is removed by ensuring that \( F \) has a sufficiently large amount of resources to recoup in the partnership. Specifically, \( M \) should be large enough to warrant that, under \( \Psi \), \( F \) incurs no loss, even in state \( h \). As usual, effort provision per se already acts as a commitment device, so that, given \( T \), a rise in \( \psi \) reduces the requirement on \( M \). On the other hand, the committing effect of effort is weaker, the longer is the contract duration. This explains why, conversely, the higher is the value of \( T \), the more stringent is the requirement on \( M \).

### 5.2.2 Removing the incentives of \( G \)

For \( \Psi \) to be honoured, one should also ensure that, conditional on \( F \) revealing \( \theta_i \), \( G \) is at least as well off in \( \Psi \) as it would be if the relationship were stopped at date \( \tau \). Formally:

\[
V_{i,\tau}^*(z) \geq V_{i,\tau}^{rp}. \tag{18}
\]

We can thus state the following lemma.

**Lemma 2** Assume that, in state \( i \in \{l,h\} \), at some given \( \tau \in (0,T) \), (14) and (15) hold. Then, (18) is satisfied for \( i = l \) and \( i = h \), if and only if

\[
M \leq \left( \frac{R_\delta}{1 + \lambda} \frac{r}{1 - e^{-r\delta}} - (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-rT}}{r} - \psi \tag{19}
\]

together with, respectively,

\[
D_{l,\tau} \leq \frac{R_\delta}{1 + \lambda} - \left( \frac{(M + \psi) r}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r} \tag{20}
\]

\[
D_{h,\tau} \leq \frac{R_\delta}{1 + \lambda} - \left( \frac{(M + \psi) r}{1 - e^{-rT}} - \nu_1 \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r}. \tag{21}
\]

**Proof.** See Appendix C.3. ■

Given the duration of the contract, the main determinant of \( G \)'s incentives to end the relationship is the total private contribution that \( G \) would be able to appropriate in so doing. Own investment of the firm and borrowed funds are substitutes, from this perspective. To prevent private expropriation by the public partner, \( F \) should be required to inject an amount of money sufficiently small that, given the disutility of effort, its profit does not exceed the
replacement cost, even in the good state. In addition, sufficiently little debt should be recommended in both states, a requirement that is more stringent for larger contributions of F.

5.3 Enforcing $\Psi$

We can now describe the weakest conditions under which $\Psi$ is sustainable in a limited-commitment framework. This will subsequently enable us to determine how the duration of the contract and the financial structure of the project should be chosen for $\Psi$ to be enforced.

**Proposition 2** $\Psi$ is enforceable if and only if $\exists z \in Z, T \in [T(z), +\infty)$ such that

$$R_\delta \geq (1 + \lambda) \Delta \theta z \frac{1 - e^{-r\delta}}{r}, \forall \delta \in (0, T),$$

and, additionally,

$$(22)$$

$$E \geq \nu_0 \psi$$

$$C > 0.$$  

(23)  

(24)

**Proof.** See Appendix C.4. ■

First, according to (22), for $\Psi$ to be enforceable, the replacement cost must not fall below the residual value of the profit wedge, at date $\tau$, as inflated by the shadow cost of public funds. Suppose the profit is very low in state $h$. Then, F has an incentive to renege. To avoid this, the profit must be increased. But, then, it might be necessary to increase the profit also in state $l$. This is because, as we saw in the full-commitment analysis, information problems require making the profit wedge sufficiently large. However, setting the profit very high in state $l$ would induce G to renege when that state is realized. This problem does not arise as long as $R_\delta$ is big enough, as compared to the profit wedge. Second, from (23), F must be sufficiently wealthy to invest (at least) the minimum amount of its own funds for which it is willing to remain in the contract, even in the bad state, for the shortest admissible length of time $T(z)$. Third, according to (24), F must have access to the credit market and be able to obtain a loan for the concerned project.

Overall, Proposition 2 conveys a strong message regarding private investment in public projects. Private capital must be available in the form of both own funds of the delegated firm and outside financing. Even a very rich firm that could finance the whole project on its own should be instructed to take on some debt. Indeed, own and borrowed funds are both effective, albeit in different ways, in preventing parties from behaving opportunistically. On the one hand, investing own funds reinforces the willingness of the firm to remain in the relationship in order to avoid expropriation. On the other hand, once the firm takes the loan, conditional guarantees can be strategically used to eliminate any prospect of profitable renegotiation. Once this is done, there is no longer an incentive to return to the contracting table. $\Psi$ is then enforceable.
5.3.1 Duration

There are two channels through which the possibility of enforcing $\Psi$ is related to the choice of termination date, namely, the properties of the replacement-cost function and the magnitude of the firm’s endowment. While the former is immediately seen from (22), the latter is perhaps less evident because (23) is independent of $T$. In fact, (23) is only a minimum requirement, as noted above, calibrated on the basis of the shortest admissible duration $T(z)$. F would need to be wealthier than that, if a longer duration is to be stipulated. Recalling that the choice of $T$ is also effective in addressing moral-hazard concerns, the range of admissible durations results from all three of these determinants taken together. The next corollary presents the restrictions that are imposed on $T$; once (23) is satisfied, when $R_\delta$ is steep or flat for all values of $\delta$.

**Corollary 1** Assume that (23) and (24) hold. (i) Suppose that

$$R_\delta' \geq (1 + \lambda) \Delta \theta z e^{-r \delta}, \ \forall \delta \in (0, T), \ \forall T \in [T(z), \infty), \ z \in Z. \quad (25)$$

Then, there exist values of $T$ for which $\Psi$ is enforceable:

$$T \in [T(z), \infty) \text{ when } E \geq \frac{\nu_1 \Delta \theta z}{r} - \psi,$$

where

$$\tilde{T}(z, E) \equiv \frac{1}{r} \ln \frac{\nu_1 \Delta \theta z}{\nu_1 \Delta \theta z - r (E + \psi)} < \infty. \quad (26)$$

(ii) Suppose that

$$R_\delta' < (1 + \lambda) \Delta \theta z e^{-r \delta}, \ \forall \delta \in (0, T), \ \forall T \in [T(z), \infty), \ z \in Z. \quad (27)$$

Then, there exist values of $T$ for which $\Psi$ is enforceable if and only if $R_{T(z)} \geq (1 + \lambda) \psi/\Delta \nu$:

$$T \in \left[ T(z), \min \left\{ \tilde{T}(z, E) ; T(z) \right\} \right] \text{ when } E \in \left[ \frac{\nu_0 \psi}{\Delta \nu}, \nu_1 \frac{\Delta \theta z}{r} - \psi \right),$$

where

$$\tilde{T}(z) \equiv \frac{1}{r} \ln \frac{(1 + \lambda) \Delta \theta z}{(1 + \lambda) \Delta \theta z - r R_{T(z)}} < \infty. \quad (28)$$

**Proof.** See Appendix C.5. ■

While moral hazard imposes a lower bound on the duration of the contract, lack of commitment may impose an upper bound. Thus, it may be difficult to reconcile information and enforcement problems. Let us first consider situations in which no upper bound appears, so that moral hazard is the only relevant concern regarding the choice of $T$ (second part of scenario (i)). These are situations in which the firm’s resources are large and, at the same time,
the replacement cost increases more sharply, relative to the social value of the profit wedge, as the contract duration is extended, given the sharing rule \( z \). This means that replacing \( F \) is more costly to \( G \) than executing \( \Psi \), for all possible residual contractual periods. Under these circumstances, the firm can be required to make any desirable contribution without triggering the partner’s opportunism. Hence, any duration satisfying (8) can be chosen. The story is more complicated, however, when not only information problems but also enforcement problems matter. This occurs when the firm is not particularly well endowed (first part of scenario (i)) and/or when the replacement cost increases less sharply than the profit wedge, so that replacing \( F \) is not necessarily more costly than honouring \( \Psi \) (scenario (ii)). In such situations, enforcement of a long-term contract is difficult. A firm that contributes little has little to recoup and, hence, little to lose if the relationship ends. Thus, if recovery is diluted over a long period of time, at some point, the residual compensation may be driven so low that the firm prefers to quit the project. Moreover, if the replacement cost becomes sufficiently small as the residual period reduces, then the government has little to lose, at least if the relationship is not terminated early on. Thus, if a long duration is stipulated, at some point, it may become rational for the government to incur the replacement cost in exchange for the benefit of appropriating the private investment. Clearly, in these situations, it is useful to stipulate a shorter duration. However, when opportunism is very strong, it becomes necessary to set \( T \) to such a small value that the return from the initial effort would not accrue to the firm for a period long enough to compensate for the disutility of that effort. In that circumstance, there is no possibility for \( \Psi \) to be implemented.

One last observation is in order. Limitations on the choice of \( T \) are tied to the choice of the sharing rule \( z \). Recall that, in (7a) and (7b), \( T \) is inversely related to \( z \), so that the weakest admissible floor is \( T(q^*_l) \). For the same reason, the smaller is \( z \), the longer is the contract that \( F \) can be motivated to honour. Consequently, \( \bar{T}(z, E) \) decreases with \( z \), and the weakest feasible cap is \( \bar{T}(q^*_l, E) \). However, because a change in \( z \) has a stronger impact on \( \bar{T}(z, E) \) than on \( T(z) \), if the firm is poor, it is beneficial to keep \( z \) low in order to have a wider range of available durations. It is more difficult, however, to identify the relationship between \( \bar{T}(z) \) and \( z \). The particular choice of \( z \) not only determines the cost of executing \( \Psi \) for \( G \). It also affects the cost of replacing \( F \), through the effect of the choice of \( z \) on the contract duration, given the characteristics of the replacement-cost function. It is thus complex to target a suitable sharing rule, unless specific cases are considered. Suppose, for instance, that the replacement-cost function is very flat, as is most plausible in scenario (ii). Then, \( \bar{T}(z) \) is also likely to decrease with \( z \), and one can reason as above. Conversely, with \( \bar{T}(z) \) increasing in \( z \), \( z = q^*_l \) would be the strategy that gives rise to the greatest latitude in the choice of \( T \), at least if the firm is not too poor.

5.3.2 Financial structure

We are now left to describe how the mix of private funds should be set for \( \Psi \) to actually be enforced under limited commitment.

**Corollary 2** Assume that (22), (23), and (24) hold. Then, \( \Psi \) is implemented by choosing
$M$ and $C$ such that

$$\nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} - \psi \leq M \leq \left( \frac{R_\delta}{1 + \lambda} \frac{r}{1 - e^{-r\delta}} - (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-rT}}{r} - \psi, \; \forall \delta \in (0, T),$$

together with

$$C \leq \frac{R_T}{1 + \lambda} - (M + \psi).$$

**Proof.** See Appendix C.6. ■

The implications of the corollary are clear and immediate in light of the previous findings. First, $F$ should be required to invest neither too little nor too much. The reason is well-known at this stage: if $M$ is too small, $F$ might prefer not to abandon the activity; if $M$ is too large, $G$ might wish to appropriate the partner’s investment. Second, $F$ should be instructed to take a loan but not be encouraged to rely heavily on external financing. While the presence of debt paves the way for a suitable use of conditional guarantees, too large a value of $C$ would trigger expropriation. Specifically, the higher the amount of own funds chosen in accordance with (29), the lower the admissible amount of borrowed funds. Expropriation is avoided, provided that the maximum benefit that break-up of the partnership would secure for $G$ (namely, $M + C + \psi$) does not exceed the largest replacement cost that $G$ could incur (namely, $R_T/(1 + \lambda)$), a cost realized if replacement occurs at the outset of the operation phase.

Once the $(M, C)$ pair is determined, the up-front transfer $t_0$ is also determined, according to (1). Joint inspection of (29) and (30) enables us to conclude with a word about this aspect of the financial structure. Because the sum $M + C$ is bounded from above, it might be desirable that the cost of the facility not be fully covered by private resources ($M + C < I$) and that $G$ complement the investment with an up-front payment to $F$ ($t_0 > 0$). The project is then undertaken with a mix of private and public funds. However, because the sum $M + C$ is also bounded from below ($M + C > (\nu_1 \Delta \theta z (1 - e^{-rT})/r) - \psi$), it might alternatively be desirable that $F$ invest more (own and borrowed) funds than is strictly required for budgetary reasons. Then, the overall monetary contribution of $F$ would also include a payment to $G$, which can be interpreted as a fee for the award of the contract ($t_0 < 0$). Not surprisingly, the latter scenario is more likely to arise when construction effort is not very costly to the firm.

### 6 Concluding remarks

It is, perhaps, intuitive that, in limited-commitment environments, the financial structure of public projects delegated to private firms can be used as a device to enforce efficient contractual agreements. Where our study makes more genuine progress is in assessing the sources from which funds should be drawn and the ways in which they should be mixed, given the circumstances relevant in a hypothetical renegotiation process.

One important implication of our analysis is that it is essential that the delegated firm take out a loan. Then, the guarantees that the government can provide to secure financiers’ participation can be used to make renegotiation unattractive and, thus, to eliminate incen-
tives for the two parties to behave opportunistically. While the specific value of the guarantees to be provided for the renegotiated contract only determines the out-of-equilibrium payoffs, it is crucial to choose that value properly to achieve the intended objective.

We stressed that guarantees should be "conditional." That is, it should be contractually stipulated that they would take effect only if the government-firm relationship does not break up beforehand. Two observations are in order. First, in several practical instances, loans are guaranteed unconditionally, based on the argument that it would otherwise be impossible to attract outside financiers (e.g., the Metronet case). Our finding that, in equilibrium, the partnership is preserved and the loan is paid back casts doubts on the validity of this argument. The fact that the actual use of guarantees is conditional on continuation of the relationship, rather than the mere act of pledging them, is precisely what makes the contract sustainable, thereby restoring the ability to attract external financiers to the project. Second, under limited commitment, it is undesirable that lenders be actively involved in renegotiation. In our model, if the lender could participate when parties return to the contracting table, it would be willing to accept any repayment proposal to avoid losing the whole residual reimbursement in the event of early break-up of the government-firm relationship. But then out-of-equilibrium guarantees could no longer be used to keep that relationship in place. Hence, for the role of guarantees to be preserved, lenders should remain "passive." Importantly, this is not detrimental to the lenders themselves, as loans are fully recovered.

Another contribution of our study lies in detection of a subtle but essential link between the financial structure of a project and the duration of a contract. This relationship stems from the circumstance that, over time, break-up of the partnership becomes increasingly less costly to contracting parties. We show that, in some cases, it is necessary to shorten the length of the contract to accommodate the absence of commitment. This, however, is at odds with the need, dictated by moral-hazard concerns, to ensure that the contract does not end too early. One may thus wonder whether, rather than stipulating a fixed-term contract, which has the same duration regardless of the marginal cost, achievements could be enhanced by conditioning the duration on the realized state of nature. Contracts with this characteristic, often called flexible-term contracts, have been proposed as tools to warrant that risk-averse firms attain the same return in every possible state and, hence, that they are fully insured (Engel et alii [11] - [12]). In a companion research project, we investigate the benefits that conditioning the contract duration on the realized state has in a framework where not only the firm but also the government may wish to renegotiate, once the state is known, and where the financial structure of the project is viewed as an instrument to secure contract enforcement.

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A Full commitment

A.1 Derivation of (7a) and (7b)

From (6a) and (6b), $\exists \varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ such that

$$\Pi_{l,0} = \Pi_{h,0} + \int_0^T \Delta \theta (q^*_h + \varepsilon_1) e^{-rx} dx$$

$$\Pi_{h,0} = \Pi_{l,0} - \int_0^T \Delta \theta (q^*_l - \varepsilon_2) e^{-rx} dx.$$  

Using the binding constraint (6d), (31) and (32) are rewritten as

$$\Pi_{l,0} = M + \psi + (1 - \nu_1) \int_0^T \Delta \theta (q^*_h + \varepsilon_1) e^{-rx} dx$$

$$\Pi_{h,0} = M + \psi - \nu_1 \int_0^T \Delta \theta (q^*_l + \varepsilon_1) e^{-rx} dx.$$  

Replacing (33) into (32), we further obtain

$$\Pi_{h,0} = M + \psi + (1 - \nu_1) \int_0^T \Delta \theta (q^*_h + \varepsilon_1) e^{-rx} dx - \int_0^T \Delta \theta (q^*_l - \varepsilon_2) e^{-rx} dx.$$  

Moreover, from (34) and (35), we get $q^*_h + \varepsilon_1 = q^*_l - \varepsilon_2$. Setting $z \equiv q^*_h + \varepsilon_1 = q^*_l - \varepsilon_2$, we obtain (7a) and (7b). The conditions $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ are rewritten as $z \in Z \equiv [q^*_h, q^*_l]$.

A.2 Derivation of (8)

From (33), (34) and $z \equiv q^*_h + \varepsilon_1 = q^*_l - \varepsilon_2$, we obtain $\Pi_{l,0} (z) - \Pi_{h,0} (z) = \int_0^T \Delta \theta ze^{-rx} dx$. Using this expression, (6c) is rewritten as (8).

B The renegotiation game

Suppose that, at the outset of the operation phase, F observes $\theta_i$, $i \in \{l, h\}$, and reports it to G. Further suppose that, at date $\tau \in (0,T)$, some party reneges on $\Psi$.

B.1 Replacement

When F is replaced, its instantaneous profit is $\pi_{i,0}^{i,0} = 0$. Thus, the payoff of F at $\tau$ is (12a). At each $x \in (\tau,T)$, F' produces $q^*_{i} = q^*_i$ and receives $\theta_i q^*_i + K - p(q^*_i) q^*_i$ so that, at $\tau$, the payoff of F' is zero and the discounted return of G through date $T$ is (12b).
B.2 Renegotiation

First suppose that G makes the offer to F: at each $x \in (\tau, T)$, F will produce $q^F_i$ and receive $t^F_i$ such that its payoff at $\tau$ is $\Pi_{i,\tau}^G = \Pi_{i,\tau}^F = 0$. This requires setting $t^G_i = \theta_i q^G_i + K - p(q^G_i) q^G_i + d^r_{i,\tau}$ ($t^G_i$ includes the amount $d^r_{i,\tau}$ destined to L; alternatively, F receives $t^F_i - d^r_{i,\tau}$ and L $d^r_{i,\tau}$). Replacing $\Pi_{i,\tau}^G$ and $\Pi_{i,\tau}^F$ into (4) and then maximizing with respect $q^G_i$, we see that G chooses $q^G_i = q^*_i$ so that, under the renegotiated contract, it attains the largest discounted return $V_{i,\tau}^G = w^*_i - (1 + \lambda) D^r_{i,\tau}$ at $\tau$. Next suppose that F makes the offer to G: at each $x \in (\tau, T)$, F will produce $q^F_i$ and receive $t^F_i$ such that, under the renegotiated contract, the discounted return of G at $\tau$ is $V_{i,\tau}^F = V_{i,\tau}^F = \int_\tau^T w^*_i e^{-r(x-\tau)} dx - R_\delta$. This requires setting $t^F_i = (S(q^F_i) - p(q^F_i) q^F_i - w^*_i + r R_\delta/(1-e^{-r(T-\tau)}))/(1 + \lambda)$ together with $q^F_i = q^*_i$. At $\tau$, F attains the largest payoff $\Pi_{i,\tau}^F = R_\delta/(1 + \lambda) - D^r_{i,\tau}$. (13a) and (13b) are computed, respectively, as $\Pi_{i,\tau}^F = \alpha \Pi_{i,\tau}^G + (1 - \alpha) \Pi_{i,\tau}^F$ and $V_{i,\tau}^F = \alpha V_{i,\tau}^G + (1 - \alpha) V_{i,\tau}^F$.

B.3 No incentive to renegotiate $T$

First suppose that G makes the offer and proposes to terminate the contract at some date $T^G > \tau$, $T^G \neq T$. If $T^G > T$, then G proposes the quantity-transfer pair $(q^*_i, t^G_{i,1})$ for all $x \in [\tau, T)$ and the quantity-transfer pair $(q^*_i, t^G_{i,2})$ for all $x \in [T, T^G)$. $t^G_{i,1}$ and $t^G_{i,2}$ are set such that the instantaneous profits are zero ($\pi^G_{i,1} = \pi^G_{i,2} = \pi^F_{i,\tau} = 0$), so that the payoff of F at $\tau$ is zero as well. Using $q_i = q^*_i$ in (2) to rewrite the instantaneous profits $\pi^G_{i,1}$ and $\pi^G_{i,2}$, and denoting $t_i = t^G_{i,1}$ in $\pi^G_{i,\tau}$ and $t_i = t^G_{i,2}$ in $\pi^G_{i,\tau}$, we get $t^G_{i,1} = \theta_i q^*_i + K - p(q^*_i) q^*_i + d^r_{i,\tau}$ for all $x \in [\tau, T)$, and $t^G_{i,2} = \theta_i q^*_i + K - p(q^*_i) q^*_i$ for all $x \in [T, T^G)$. Using $t^G_{i,1}$ and $t^G_{i,2}$ in (4), we derive the discounted return of G at $\tau$:

$$\tilde{V}_{i,\tau}^G = \int_\tau^{T^G} w^*_i e^{-r(x-\tau)} dx - (1 + \lambda) D^r_{i,\tau}. $$

If $T^G < T$, then G proposes the pair $(q^*_i, t^G_{i,1})$ for all $x \in [\tau, T^G]$ so that the payoff of F at $\tau$ is zero. The discounted return of G at $\tau$ is $\tilde{V}_{i,\tau}^G$. Hence, the payoff of G at $\tau$ is given by

$$W_{i,\tau} = \tilde{V}_{i,\tau}^G + \int^{+\infty}_{T^G} w^*_i e^{-r(y-T^G)} dy = \int_\tau^{+\infty} w^*_i e^{-r(x-\tau)} dx - (1 + \lambda) D^r_{i,\tau},$$

which is independent of $T^F$.

Next suppose that F makes the offer and proposes to terminate the contract at some date $T^F > \tau$, $T^F \neq T$. Then, the proposal includes the quantity-transfer pair $(q^*_i, t^F_{i,1})$ for all $x \in [\tau, T^G]$. For the discounted return of G at $\tau$ to be equal to (12b), it must be $t^F_{i,1} = (S(q^*_i) - p(q^*_i) q^*_i - w^*_i + r R_\delta/(1-e^{-r(T^F-\tau)}))/(1 + \lambda)$. The payoff of F at $\tau$ is independent of $T^F$ as it is given by

$$\Pi_{i,\tau}^F = \frac{R_\delta}{1 + \lambda} - D^r_{i,\tau}. $$
B.4 Proof of (14)

Suppose that the cost is \(\theta_i, i \in \{l, h\}\), and that F reveals it to G at the outset of the operation phase. Further suppose that the current period is \(\tau' \in (0, T)\) and that \(\Psi\) was renegotiated at \(\tau < \tau', \tau \in (0, T)\). At \(\tau'\), under the contract renegotiated at \(\tau\) and not further renegotiated beyond that date, F and G, respectively, attain:

\[
\hat{\Pi}_{i,\tau'/\tau}^{rn} = (1-\alpha) \left( \int_{\tau'}^{T} \frac{R_\delta}{1+\lambda} - \frac{r}{1-e^{-r\delta}} e^{-r(x-\tau')} dx - \hat{D}_{i,\tau'/\tau}^{rn} \right)
\]

\[
\hat{V}_{i,\tau'/\tau}^{rn} = \int_{\tau'}^{T} \left( w_i^* - \frac{(1-\alpha) R_\delta}{1-e^{-r\delta}} \right) e^{-r(x-\tau')} dx - \alpha (1+\lambda) \hat{D}_{i,\tau'/\tau}^{rn}.
\]

If the contract is renegotiated at \(\tau'\), then F gets \(\Pi_{i,\tau'/\tau}^{rn}\) and G \(V_{i,\tau'/\tau}^{rn}\). F and G do not wish to renegotiate at \(\tau'\), respectively, if and only if:

\[
\hat{\Pi}_{i,\tau'/\tau}^{rn} \geq \Pi_{i,\tau'/\tau}^{rn} \Leftrightarrow \hat{V}_{i,\tau'/\tau}^{rn} \geq V_{i,\tau'/\tau}^{rn} \tag{36a}
\]

\[
\hat{V}_{i,\tau'/\tau}^{rn} \geq V_{i,\tau'/\tau}^{rn} \Leftrightarrow \hat{D}_{i,\tau'/\tau}^{rn} \geq D_{i,\tau'/\tau}^{rn} \tag{36b}
\]

Using the definitions of \(\hat{\Pi}_{i,\tau'/\tau}^{rn}\) and \(\Pi_{i,\tau'/\tau}^{rn}\), (36a) and (36b) are further equivalent, respectively, to:

\[
D_{i,\tau'/\tau}^{rn} - \hat{D}_{i,\tau'/\tau}^{rn} \geq \left( R_\delta' - R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} \right) \frac{1}{1+\lambda} \tag{37a}
\]

\[
D_{i,\tau'/\tau}^{rn} - \hat{D}_{i,\tau'/\tau}^{rn} \geq - \left( R_\delta' - R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} \right) \frac{1-\alpha}{\alpha (1+\lambda)} \tag{37b}
\]

Combining (37a) with (37b), (14) follows.

C Enforcement of \(\Psi\)

C.1 F is unwilling to misrepresent cost anticipating renegotiation

For \(\Psi\) to be enforceable in state \(i = l, h\), the following conditions must hold:

\[
\Pi_{i,0}^* (z) \geq \int_0^{\tau} (\pi_{i,x}^* + \Delta \theta q_{i}^*) e^{-rx} dx + \max \{0; \Pi_{i,\tau}^{RN} \} \tag{38}
\]

\[
\Pi_{h,0}^* (z) \geq \int_0^{\tau} (\pi_{i,x}^* - \Delta \theta q_{i}^*) e^{-rx} dx + \max \{0; \Pi_{h,\tau}^{RN} \} \tag{39}
\]

\(\Pi_{i,\tau}^{RN}\) denotes the stream of profits that F would obtain in state \(i = l, h\), discounted at time \(\tau\), if it were to misrepresent \(\theta_i\) at the outset of the operation phase and some party were to renege at \(\tau\).

We now show that (38) holds. If F reports \(h\) in state \(l\) and the contract is renegotiated
at \( \tau \in (0, T) \), the instantaneous profit of \( F \) is given by

\[
\pi_{i, \tau}^{RN} = t_{h}^{rn} + p(q_{h}^{*})q_{h}^{\ast} - (\theta q_{h}^{*} + K) - d_{h, \tau}^{rn},
\]

(40)

where \( t_{h}^{rn} = \alpha t_{h}^{F} + (1 - \alpha) t_{h}^{F} \) is the expected transfer that results from renegotiating at \( \tau \), given the report \( h \). Using the formulas of \( t_{G}^{h} \) and \( t_{F}^{h} \) presented in Appendix B, \( t_{h}^{rn} \) becomes

\[
t_{h}^{rn} = \frac{\alpha}{1 + \lambda} \left[ S(q_{h}^{*}) - w_{h}^{*} + \frac{r R_{\delta}}{1 - e^{-\delta}} \right] - \frac{1 + \lambda}{1 + \lambda} p(q_{h}^{*})q_{h}^{*}.
\]

Replacing this expression into (40), we obtain

\[
\Pi_{i, \tau}^{RN} = (1 - \alpha)(r R_{\delta} / (1 + \lambda) (1 - e^{-\delta}) - d_{h, \tau}^{rn}) + \Delta \theta q_{h}^{*}.
\]

This yields

\[
\Pi_{i, \tau}^{RN} = \Pi_{h, \tau}^{rn} + \int_{\tau}^{T} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx,
\]

which we replace into (38) to further get

\[
\Pi_{i, 0}^{*} \geq \Pi_{h, 0}^{*} + \int_{0}^{T} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx
\]

\[+ e^{-rt} \left( \max \left\{ 0; \Pi_{h, \tau}^{rn} + \int_{\tau}^{T} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx \right\} \right) - \left( \Pi_{h, \tau}^{*} + \int_{\tau}^{T} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx \right).
\]

This is implied by (6a) and (16). Hence, (38) does hold.

Symmetrically, one can prove that (39) is implied by (6b) and (16) and is, indeed, satisfied.

C.2 Proof of Lemma 1

In state \( i \in \{l, h\} \), at date \( \tau \in (0, T) \), \( F \) prefers \( \Psi \) to any other regime if and only if:

\[
\Pi_{i, \tau}^{*}(z) \geq \max \left\{ 0; \Pi_{i, \tau}^{rn} \right\}.
\]

(41)

Under (15), (41) reduces to (16). Using (10a) and (10b), (16) becomes

\[
\left( \frac{(M + \psi) r}{1 - e^{-rT}} + (1 - \nu_{l}) \Delta \theta z \right) \frac{1 - e^{-rT}}{T} \geq 0,
\]

(42a)

\[
\left( \frac{(M + \psi) r}{1 - e^{-rT}} - \nu_{l} \Delta \theta z \right) \frac{1 - e^{-rT}}{r} \geq 0,
\]

(42b)

respectively, in state \( l \) and \( h \). (42a) is obviously satisfied. (42b) is rewritten as (17).

C.3 Proof of Lemma 2

In state \( i \in \{l, h\} \), at date \( \tau \in (0, T) \), \( G \) prefers \( \Psi \) to any other regime if and only if:

\[
V_{i, \tau}^{*}(z) \geq \max \left\{ V_{i, \tau}^{rn}, V_{i, \tau}^{rp} \right\}.
\]

(43)

Under (15), (43) reduces to (18). Using (11) together with (12b), (18) becomes

\[
D_{i, \tau} \leq R_{\delta} / (1 + \lambda) - \Pi_{i, \tau}^{*}(z).
\]

Replacing the expression of \( \Pi_{i, \tau}^{*}(z) \) for \( i = l, h \), this yields (20) and
(21), which hold only if
\[
\frac{R_\delta}{1 + \lambda} \geq \left( \frac{(M + \psi)r}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r} \tag{44}
\]
\[
\frac{R_\delta}{1 + \lambda} \geq \left( \frac{(M + \psi)r}{1 - e^{-rT}} - \nu_1 \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r}. \tag{45}
\]

(45) is satisfied if (44) holds. (44) is rewritten as (19).

C.4 Proof of Proposition 2

(6c) is satisfied if and only if \(T(z)\), for any given \(z \in Z\). Take (15) to hold for either type. (17) and (19) hold jointly only if (22) is satisfied. Furthermore, from (17) and \(M \leq E\), we deduce
\[
E \geq \nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} - \psi. \tag{46}
\]

If \(E \geq (\nu_1 \Delta \theta z/r) - \psi\), then (46) is satisfied for all \(T > 0\). Otherwise, it is rewritten as \(T \leq \tilde{T}(z, E), \tilde{T}(z, E)\) being defined as in (26). Then, as \(T \geq \tilde{T}(z)\), it is necessary to have \(\tilde{T}(z) \leq \tilde{T}(z, E)\) or, equivalently, \(E \geq \psi \nu_0/\Delta \nu\). Hence, \(\exists T \geq \tilde{T}(z)\) for which (46) is satisfied if and only if (23) holds.

Suppose that \(C = 0\) and that \(\Psi\) is implemented. Then, \(D_{r_{i,\tau}} = 0\) for all \(i \in \{l, h\}\) and \(\tau \in (0, T)\). Moreover, \(D_{i,\tau} = \tilde{D}_{i,\tau} = 0\) for all \(\tau, \tau' \in (0, T), \tau' > \tau\). Hence, (15) cannot be satisfied. Nor can one have \(D_{i,\tau} - \tilde{D}_{i,\tau} > 0\), as is required for (14) to be met. This contradicts the hypothesis that \(\Psi\) is implemented with \(C = 0\). Hence, \(C\) must be positive.

Overall, \(\exists M, D_{i,\tau}, T\) for which (17), (20) and (21) hold jointly, if and only if (22) and (23) are satisfied. Moreover, \(\exists D_{i,\tau}^*\) for which the out-of-equilibrium conditions (14) and (15) are satisfied, if and only if (24) holds. Recall that (14), (15), (16) (rewritten as (17)), (20), and (21) are the constraints that must be satisfied, in addition to those that must hold under full commitment, for \(\Psi\) to be enforceable under limited commitment. Hence, \(\Psi\) is enforceable if and only if (22), (23), and (24) are satisfied together with (8).

C.5 Proof of Corollary 1

By assumption, \(\lim_{\delta \to 0} R_\delta > 0\) and finite. Hence, (22) holds as \(\delta \to 0\).

First suppose that, for some given \(z\), (25) holds. Hence, as \(\delta\) is raised (i.e., \(\tau\) is decreased and/or \(T\) increased), (22) is relaxed. Then, provided it is satisfied as \(\delta \to 0\), it is for all \(\delta \in (0, T), T \in [\tilde{T}(z), \infty)\).

Next suppose that (27) holds. Then, for any given \(T \geq \tilde{T}(z)\), (22) is tightest as \(\tau \to 0\). Then, replacing \(\tau = 0\), (22) holds if and only if \(T \leq \tilde{T}(z)\). As (8) is necessary for \(\Psi\) to be enforceable, it must be the case that the interval \([\tilde{T}(z), \tilde{T}(z)]\) exists and that \(T \in [\tilde{T}(z), \tilde{T}(z)]\).

The remaining conditions are those presented in the proof of Proposition 2, namely
\[ T \geq \overline{T}(z), \text{ when } E \geq (\nu_1 \Delta \theta z/r) - \psi, \text{ and } T \in \left[ \overline{T}(z), \tilde{T}(z, E) \right], \text{ when } E < (\nu_1 \Delta \theta z/r) - \psi. \]

\section*{C.6 Proof of Corollary 2}

Recall that, under (15), (18) is rewritten as (19) to (21). (17) and (19) are rewritten as (29). Using the definition of \( E_i[D_{i,r}] \) in (20) and (21), we obtain

\[ E_i[D_{i,r}] \leq \frac{R_{\delta}}{1 + \lambda} - (M + \psi) \frac{1 - e^{-r_{\delta}}}{1 - e^{-r_{\bar{T}}}}. \]  

(47)

Recalling that \( E_i[D_{i,0}] = C \), (47) together with \( C > 0 \) reduces to (30).