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Abstract

There have been studied regularities affecting the magnitude of the dynamic component of the plough resistance for cylindrical mouldboards, and its peculiarities for cylindroical mouldboards depending on the direction of arrival of the soil layer on the ploughshare. The dynamic component of the draught resistance of the plough, which is directly proportional to the square of the ploughing velocity, can be estimated by the value of a dimensionless coefficient, conditionally named the "wrapping angle of the layer". This coefficient can be found for the mouldboard surface if the path of the movement of the mid-point of the section of the soil layer is known. By example of a cylindroical surface of the mouldboard it is shown how the dynamic component of the resistance varies depending on the design parameters of the mouldboard and the direction of arrival of the soil layer onto the ploughshare.

Keywords	layer of soil; mouldboard of the plough; geodesic line
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Highlights

- Motion of a soil layer along mouldboards and dynamic component of draught resistance.
- Dynamic component of plough draught resistance and "wrapping angle of layer" factor.
- Mouldboard design and arrival angle of soil layer onto ploughshare affect resistance.
- The proposed approach considers only the geometry of the surface of mouldboards.

A theoretical study of the limit path of the movement of a layer of soil along the plough mouldboard

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Abstract

There have been studied regularities affecting the magnitude of the dynamic component of the plough resistance for cylindrical mouldboards, and its peculiarities for cylindroical mouldboards depending on the direction of arrival of the soil layer on the ploughshare. The dynamic component of the draught resistance of the plough, which is directly proportional to the square of the ploughing velocity, can be estimated by the value of a dimensionless coefficient, conditionally named the "wrapping angle of the layer". This coefficient can be found for the mouldboard surface if the path of the movement of the mid-point of the section of the soil layer is known. By example of a cylindroical surface of the mouldboard it is shown how the dynamic component of the resistance varies depending on the design parameters of the mouldboard and the direction of arrival of the soil layer onto the ploughshare.

Keywords: layer of soil, mouldboard of the plough, path of the movement, geodesic line.

1. Introduction

Soil tillage requires great amount of energy in agricultural production and in last decades a topic of interest of many researchers has been the evaluation of tillage draught (Karmakar and Kushwaha, 2006; Tong and Moayad, 2006; Bentaher et al., 2008; Shmulevich, 2010). Taking into account that the tillage forces are mainly a function of soil mechanical properties, working parameters of the tool (e.g. depth and speed) and tool geometry, simulation of soil-tool interaction for farming operations has also been analysed for the design and optimization of tillage implements (Zhang et al., 2018; Abo-Elnor, et al. 2004; Zhu et al., 2017; Bentaher, et al., 2013; Chi and Kushwaha, 1990).

In the well-known rational formula of the draught resistance force of the plough, obtained by V.Goryachkin, there are three components, one of which is directly proportional to the square of the velocity of relative movement of the layer along the mouldboard. It is the so-called dynamic component (Goryachkin, 1998). These components are in complex dependence on the properties of the soil, the speed of ploughing, the shape of the mouldboard surface. The author of this formula himself noted: "... in the future, in a more detailed study, each of the three terms of the formula may have to be developed and replaced by more complex functions." Therefore, the search for new dependencies of these components of the draught resistance of the plough upon its design parameters is an actual scientific task.

L.Gyachev considered in detail the forces that arise during the interaction of the soil layer with the mouldboard, and compiled a differential equation for the path of the movement of the mid-point of the soil layer along the mouldboard (Gyachev, 1981). In addition, he took into account the dimensions and specific gravity of the soil layer, its elasticity, the ploughing velocity, and linked these parameters with the normal and geodesic curvature, and the length of the path. From the compiled differential equation he obtained a result according to which, when the rigidity of the soil layer and velocity of its movement along the mouldboard increases, the geodesic curvature of the path approaches zero, i.e. the soil layer tends to move along the geodesic line of the surface, which is the upper limit path. Thus the search for the ultimate path of the layer movement is reduced to finding the geodesic line of the mouldboard surface, which, in its turn, makes it possible to consider interaction of the layer with the mouldboard.

A.Vilde, who developed the theory by V.Goryachkin, investigated the draught resistance of helical mouldboard surfaces of the plough under the conditions of Latvia (Vilde, 2004, Rucins et al., 2007, Vilde, 2008). Although a number of contemporary researchers have continued research in the plough surfaces in order to improve the quality of soil cultivation in particular soil and climatic zones, and to reduce energy requirement, these tasks still remain topical. By means of the created test-bench, profilograms (shape lines) were obtained for the share-mouldboard surfaces of some bodies mainly used on the farms of Latvia, as well as their parameters and suitability for the Latvian conditions were determined (Vilde, 2012).

It is well known that geodesic lines on a surface are the shortest paths between two points on the surface (Volkov, 2002). Just as it is possible to draw a bundle of straight lines in all directions, it is possible to draw a bundle of geodesic lines from a point on the surface, among which there can be straight lines (forming surfaces if the surface is linear) and curves (planar and spatial). P.Vasilenko pointed out that in the case of the movement of a material point by inertia, in order to determine the path of this movement, one can use the solution of the differential equation of the geodesic line instead of solving the differential equation of the movement of a point (Vasilenko, 1980). Geodesic lines on the surface are described by second-order differential equations, for which numerical methods must be applied. Modern means of the computer technology provide a possibility not only to find them but also to visualise (Bulgakov, 2010).

However, it is possible to do the opposite - to set the desired limit path of the movement of the soil layer and, with its help, to find the surface itself. Designing the plough mouldboard from unfolding surfaces along a pre-set limit path of the layer movement without a force interaction is discussed in (Bulgakov, 2011).

The aim of this work was a theoretical study of the limit path of the movement of a soil layer along the linear surfaces of the mouldboards and its impact upon the dynamic component of the draught resistance of the plough.

2. Materials and Methods

In the analytical study, there were used methods of agricultural mechanics, higher mathematics and differential geometry, as well as programming and numerical calculations on the PC were used.

If the soil is hard, coherent, intertwined with plant roots, then its movement along the mouldboard differs from the movement of a loose (incoherent) soil. Let us consider interaction of such a soil with a part of a cylindrical surface, which we refer to a three-dimensional coordinate system xyz (Figure 1). In this case, the lower generatrix of this surface makes angle γ with axis y. The tangent plane to the cylinder along the lower generatrix is inclined to the horizontal plane at angle ε. In the way considered a part of this cylindrical surface can be regarded as a plough mouldboard. Then the path of the movement of the elastic layer of the soil along the plough mouldboard can be simulated with some approximation in the form of a movement of a narrow, flexible, elastic band if it is somehow moved along the mouldboard in a pre-set direction at angle γ. The direction of movement of this band is indicated by an arrow (Fig. 1 a).

If the band adheres to the surface, then its position will determine the geodesic line. The actual path of the soil layer will differ from the one described in the case with a flexible elastic band because, under the action of its weight, it will deflect downwards along the mouldboard.

Let us consider theoretical preconditions for the formation of a path by example of a cylindrical surface since the differential equations for finding geodesic lines on it are greatly simplified, and the geodesic lines of such a surface themselves are easy to perceive – they all intersect the rectilinear generatrices at a constant angle γ, and after unfolding they turn into straight lines.

Next we will simulate the movement of the soil layer, distinguishing in it an infinitely small element (elementary parallelepiped) having geometric dimensions a, b and ds (Fig. 1 b).

In a general case of movement this infinitesimal element is taken as a material particle. We suppose that this material particle is moving along the spatial curve with acceleration, one component of which being directed along tangent $\overline{\tau}$, and the second – in the direction of the main normal $\overline{n}$ of the curve at the point where this particle is located. If the material particle is moving at a constant velocity along the curve (and we accept just this case since velocity V of the movement of the soil layer along the mouldboard is equal to the velocity of the plough movement aggregated with the tractor, and it is constant by its magnitude), then the first component will be zero, and the value of the second component is determined from expression V^2k, where k is the curvature of the path at the point of location of the material particle. If the curve (i.e. the path of the movement) is located on the surface, then the material particle will interact with it creating pressure with a certain force F. The pressure force, like the surface reaction, is always directed along normal $\overline{n}$ towards the surface. On the whole, this force of pressure is created by components of the weight of the material particle, the centrifugal force, the force that bends the layer of the soil in case it is sufficiently elastic, and other less significant forces. Besides, if the weight of a material particle does not depend on the curvature of the path, then the curvature of the path has a significant impact upon the last two components of the indicated pressure force.

Let us consider in greater detail this material particle moving along the inner surface of a cylinder (Fig. 2). Let this particle (the element of the soil layer) be at point A of the path of movement. Through point A we will draw vectors of a natural trihedron of the path - the tangent line $\overline{\tau}$ and the principal normal $\overline{n}$. The curvature vector k will be directed along the principal normal $\overline{n}$ of the path. The centrifugal force $\overline{F}_c$ acting upon the material particle and defined by such an expression:

$$F_c = mV^2k, \tag{1}$$

is directed in the opposite direction.

At point A of the path we will draw plane μ, tangent to the cylinder. The perpendicular, drawn to this plane μ at point A is a normal $\bar{N}$ to the surface of the cylinder.

The direction of the centrifugal force $\bar{F}_c$ does not coincide with the direction of the normal $\bar{N}$ to the surface; therefore the vector of its action must be decomposed into components along the normal $\bar{n}$ to the surface and onto the tangent plane μ. This is equivalent to the decomposition of curvature k along the same directions, component k_n on the normal $\bar{N}$ to the surface being named a normal curvature, and component k_g on the tangent plane μ to the surface – a geodesic curvature.

Angle ε between the normal $\bar{N}$ to the surface and the principal normal $\bar{n}$ of the path through which decomposition is carried out is determined by the methods of differential geometry, and it depends on the shape of the curve on the surface. In a general case it is variable and depends on the location of the point on the curve, as well as curvature k, i.e.:

$$\varepsilon = \varepsilon(s), \tag{2}$$

and

$$k = k(s), \tag{3}$$

where s – arc length of the arc of the curve, m.

For the geodesic line, angle ε is zero at all points, i.e., the principal normal $\bar{n}$ of the curve coincides with the normal $\bar{N}$ to the surface.

Let us suppose further that elasticity of the soil layer is sufficiently small, i.e., it practically does not exert resistance to the bend of the layer. Then two forces act upon the element of the soil layer of the size $a \times b \times ds$ (Fig. 1 b), which we will further consider as elementary: the elementary force of weight, equal to:

$$dP = (a \times b \times ds)\eta \cdot g = a \cdot b \cdot \eta \cdot g \cdot ds, \tag{4}$$

where η – the soil density, kg·cm^{-3}; g – acceleration of gravity, m·s^{-2};

and the elementary centrifugal force, which will be equal to:

$$dF_c = mV^2k = a \cdot b \cdot \eta \cdot V^2 \cdot k \cdot ds\,. \tag{5}$$

Both these forces must be decomposed along the directions, as shown in Fig. 2. The component of the elementary centrifugal force dF_c projected on the normal $\bar{N}$ to the surface, creates pressure of the soil layer element on the mouldboard. Its value is determined from this expression:

$$dF_{cn} = a \cdot b \cdot \eta \cdot V^2 \cdot k_n \cdot ds\,. \tag{6}$$

This force is balanced by the reaction of the cylinder surface.

The second component, equal to:

$$dF_{cg} = a \cdot b \cdot \eta \cdot V^2 \cdot k_g \cdot ds\,, \tag{7}$$

which is projected onto the tangent plane, is balanced by the component of the weight force of the layer element (in case the force of weight is projected onto the tangent plane):

$$mg_c = a \cdot b \cdot \eta \cdot g \cdot ds_c\,, \tag{8}$$

where ds_c – the projection of the length of the elementary parallelepiped onto the tangent plane μ.

The soil layer, as a rule, does not exert great resistance to bending, and the component of the part of the weight mg_c deflects it in the tangent plane from the rectilinear direction. Nevertheless, when increasing the ploughing velocity V, the component of the centrifugal force dF_{cg} increases directly in proportion to the square of velocity, i.e., with a slight increase in velocity, the component of the centrifugal force dF_{cg} increases very significantly.

Since the value of the component force of weight mg_c, which balances it, does not depend on velocity V, the increasing component of the centrifugal force dF_{cg} tries to straighten the path of the movement of the soil layer element, bringing it closer to the geodesic line. With infinite increasing in velocity V, the geodesic line will be the path of the movement of the soil layer. Consequently, the geodesic line can be considered as the limit path of the movement of the soil layer, which can be real for it at a high ploughing velocity or an absolutely elastic soil layer.

Let us further consider the action of forces when the soil layer is elastic and exerts a certain resistance to its bending. In this case the soil layer bends in two directions: in the tangent and the normal planes of the path. However, in these planes its resistance to bending will be different and it will depend on the hardness of the soil layer. The rigidity of the layer depends on the geometric dimensions of the section of the soil layer and is determined, in its turn, by the product E of the elastic modulus of the soil by the inertia moment I of the section of the soil layer. For normal and tangential planes, the rigidity (EI) of the soil layer will accordingly have the following values:

$$EI_n = \frac{Eab^3}{12}, \tag{9}$$

and

$$EI_g = \frac{Ea^3b}{12}. \tag{10}$$

Since $a > b$, then rigidity EI_g and, accordingly, the resistance to the bend of the soil layer will be greater in the tangent plane than in the normal plane.

So another factor that approximates the path of the movement to the geodesic line of the mouldboard is increase in the width of the strip in comparison with its height, i.e., the depth of ploughing.

In order to find the forces necessary for bending the soil layer in both planes, we will use the well-known provisions of the theory of material resistance according to which curvature k of the elastic axis of the rod is directly proportional to the applied moment M and inversely proportional to the rigidity of the rod, i.e.:

$$k = \frac{M}{E \cdot I}. \tag{11}$$

After finding moment M and its differentiating along the length of the elastic axis (in this case along the length of path S), we will obtain a force that bends the layer of the soil. Another differentiation will give a distributed force per unit of the length of the path.

Thus the force for bending F_b of the layer along its length within the limits of the mouldboard is determined from such expressions:

– in a normal plane: $F_{bn} = \dfrac{dM_n}{ds} = E\dfrac{ab^3}{12}\dfrac{dk_n}{ds}$, (12)

– in a tangent plane: $F_{bg} = \dfrac{dM_g}{ds} = E\dfrac{a^3 b}{12}\dfrac{dk_g}{ds}$. (13)

The soil layer creates pressure upon the mouldboard with a force determined by expression (12), causing a friction force fF_{bn}, where f is the coefficient of friction of the soil layer along the mouldboard. However, it also depends on the elasticity of the soil layer and may be absent in the case when the soil layer does not exert resistance to bending. At the same time, the normal component of the centrifugal force is always present and does not depend on the properties of the soil (bearing in mind that its density is constant). Since the elementary force with which the element of the soil layer presses upon the mouldboard is determined from expression (6), then it is necessary to integrate this expression along the length of arc S of the path. Assuming that the dimensions of the section of the soil layer are constant, the ploughing velocity and the soil density are constant, force F_{cn} will be determined from such an expression:

$$F_{cn} = a \cdot b \cdot \eta \cdot V^2 \int k_n ds . \quad (14)$$

Consequently, the force of pressure, caused by the action of the centrifugal force, which is always present in the case of a curvilinear path, can be reduced by decreasing the integral entering into expression (14). Yet this characteristic curve on the surface is purely geometric and does not depend on the properties of the soil. It will be determined if the path of the soil layer movement along the mouldboard is determined. However, if we assume that the path of the soil layer approaches the geodesic line or moves along it, then the problem of finding this integral becomes quite definite.

In this case angle ε (Fig. 2 a) becomes equal to zero and $k_n = k$, i.e. decomposition of the curvature vector does not occur, and the entire centrifugal force acts along the normal to the surface of the mouldboard. This integral has a geometric sense, which follows from the definition of the curvature k of the curve. Since:

$$k = \frac{d\theta}{ds}, \qquad (15)$$

then:

$$\theta = \int k ds, \qquad (16)$$

where θ – an angle at which the tangent to the curve turns as it moves along it from the initial to the final value of the arc.

Professor L.Gyachev named it the "wrapping angle of the mouldboard". In this case this angle is measured in radians but can be translated into degrees. It will be possible to show this angle on the mouldboard of the plough, taking into account certain geometric peculiarities. Thus it can be seen on the cylindrical surface in case the angle of entry of the layer is = 90º.

As already mentioned, the geodesic lines on the cylindrical surfaces intersect the rectilinear generatrices at a constant angle γ and, on its unfolding, they turn into straight lines. If this angle is straight, then the geodesic line is an orthogonal section of the cylinder, i.e., it is flat. In the case when $\gamma = 0$, the geodesic line is the rectilinear generatrix of the surface. At all the other values of angle γ, the geodesic lines are the spatial curves. If they are all projected onto a plane perpendicular to the generatrices of the cylinder, then their projection will be a flat curve – an orthogonal section of the cylinder. There is a relationship between the curvature k of the curve and the curvature k_s of its projection (i.e., the cross section of the cylinder), as well as between the lengths of their arcs s and s_s, which are determined by such well-known expressions:

$$k = k_s \cdot \sin^2 \gamma, \qquad (17)$$

and

$$ds = \frac{ds_s}{\sin\gamma}. \tag{18}$$

Using expressions (17) and (18) and substituting them into expression (16), we can write the dependence of angle θ in this way:

$$\theta = \int k ds = \sin\gamma \int k_s ds_s . \tag{19}$$

When analysing dependence (19), one can draw an important conclusion that the greatest value of angle θ will be at γ = 90º, i.e., for a flat curve of an orthogonal section of the cylinder. For other values of angle γ, it decreases in direct proportion to its sine.

For better understanding of the essence of this angle we will discuss a concrete example of geodesic lines on a cylindrical surface. Let the orthogonal section of the cylinder be an arc of the parabola, located in a vertical plane and having parametric equations, which ultimately determine its appearance:

$$\begin{aligned} x &= -v, \\ z &= av^2, \end{aligned} \tag{20}$$

where a – a constant value, i.e., the parameter of the parabola shape parameter; v – an independent variable.

Such an approach previously found sufficiently wide application in designing the plough mouldboards when a parabola is adopted as the curve for the cross section of the cylinder, which was set by several parameters: the starting point on the blade of the ploughshare, drawn through its tangent, the angle of inclination of the rear side of the ploughshare to the bottom of the furrow, and the end point with the tangent drawn to it. Despite the fact that the circumference of the cylinder may be a circle (or any other curve), the cross section of the cylinder in the form of a parabola, however, has undoubted advantages connected with the control of the shape of the curve by changing the angles of inclination of the tangents at the initial and final points, and changing distances between the points. In addition, it was more convenient to draw, using graphical methods without the application of equations. In any case, the presence of a curve of the cross section of the mouldboard is necessary as a guiding curve of the surface.

The parametric equations of the cylinder can be written as follows:

$$X = -v,$$
$$Y = -u, \tag{21}$$
$$Z = av^2,$$

where u – the second independent variable (in this case it is the length of the rectilinear generatrix).

The cross-section of a surface with height h and a length of generatrices, equal to five linear units, is shown in Fig. 3 a. In order to build a line on the surface, we have to join together two independent variables u (the length of a straight-line generatrix) and v with a certain dependency. For the curve intersecting all generatrices at a constant angle γ, i.e., the geodesic curve, this dependency has the form:

$$u = \mathrm{ctg}\gamma \int \sqrt{x'^2 + z'^2}\, dv = \mathrm{ctg}\gamma \int \sqrt{1 + 4a^2v^2}\, dv = \frac{2av\sqrt{1+4a^2v^2} + \mathrm{Arc}\sinh(2av)}{4a \cdot \tan\gamma}, \tag{22}$$

where $\mathrm{Arc}\sinh$ – the hyperbolic arcsine.

Consequently, on the cylindrical surface (21) the parametric equations of the geodesic line acquire the following form:

$$\begin{aligned} x_g &= -v, \\ y_g &= -\frac{2a \cdot v\sqrt{1+4a^2v^2} + \mathrm{Arc}\sinh(2av)}{4a \cdot \mathrm{tg}\gamma}, \\ z_g &= av^2. \end{aligned} \tag{23}$$

In Fig. 3 a, on the cylindrical surface (21) geodesic lines are constructed according to equations (23) for different angles of arrival of the layer onto the surface, including angle $\gamma = 42^\circ$ accepted for the ploughs. Let us find the wrapping angle θ of the layer by the section of the surface of the mouldboard for each of these lines. We find curvature k and the differential of arc ds through the first and second derivatives according to the known formulas (we present the final results of differentiation):

$$k = \frac{\sqrt{\begin{vmatrix} y'_g & z'_g \\ y''_g & z''_g \end{vmatrix}^2 + \begin{vmatrix} z'_g & x'_g \\ z''_g & x''_g \end{vmatrix}^2 + \begin{vmatrix} x'_g & y'_g \\ x''_g & y''_g \end{vmatrix}^2}}{\left(x_g'^2 + y_g'^2 + z_g'^2\right)^{3/2}} = \frac{2a\sin^2\gamma}{\left(1+4a^2v^2\right)^{3/2}}. \tag{24}$$

$$ds = \sqrt{x_g'^2 + y_g'^2 + z_g'^2}\,dv = \frac{\sqrt{1+4a^2v^2}}{\sin\gamma}dv. \tag{25}$$

Consequently, the wrapping angle θ of the layer will be equal to:

$$\theta = \int k\,ds = 2a\sin\gamma\int\frac{dv}{1+4a^2v^2} = \sin\gamma\,Arctg\left(2av\right). \tag{26}$$

By formula (26) one can find angle θ for any of the geodesic lines shown in Fig. 3 a. Since for all of them changing parameter v occurs within the same boundaries (from v_1 the moment when the layer enters the lower generatrix to v_2 when it leaves the upper generatrix), the values of angle θ will differ with each other, depending on $\sin\gamma$, i.e., we get a result, found earlier. Hence it follows that, when the layer is raised to the same height h (Fig. 3), the greatest force of pressure and, accordingly, the draught resistance will be at γ = 90º, i.e., when the layer is lifted by the shortest path perpendicular to the generatrices. At γ = 30º, the draught resistance is reduced two times, although the path of the layer increases. This is explained by the fact that the curvature of the path decreases more intensely. Nevertheless, if we follow this way by reducing the draught resistance, we have to increase the length of the mouldboard significantly. Obviously, the value of angle θ does not depend on the point of arrival of the layer onto the lower generatrix at a preset angle γ.

Now let us discuss the physical essence of angle θ. It can be best seen (and at the same time it corresponds to its name) at γ = 90º. In this case the tangent to the path of its movement from the lower to the upper point turns by angle φ (Fig. 3 b). Accordingly, the normal to the surface turns by the same angle, i.e., this is the angle between the lower and upper normals when the layer is elevated to a particular height h. For this case $\theta = \varphi$. For other paths at $\gamma \neq 90^{\circ}$ angle θ is determined by multiplying angle φ by $\sin\gamma$, i.e., it will be smaller despite the same height h of elevation. This can be explained by the fact that angle θ arises by turning not the normal in relation to the surface along the path but the tangent to the path, and only at $\gamma = 90^{\circ}$ these angles are equal.

If for the cylindrical surface the relationships of the formation of angle θ have been clarified and described by relatively simple dependences, for more complex (non-unfolding) surfaces its finding becomes more complicated. Let us consider the peculiarities of the formation of angle θ for cultural cylindrical mouldboards. Cylindroidal surfaces are linear non-unfolding ones, and, in order to find the geodesic lines on them, it is necessary to solve the second-order differential equations (Gjachev, 1981). The cylindroidal surface of the cultural mouldboard is formed by a flat guiding curve, which is a parabola and which is located in a vertical plane, perpendicular to the ploughshare blade, at a distance of 2/3 from its origin. It is defined by two points O and Q, and tangents in them, drawn at angles ε_1 and ε_2 to axis x (Fig. 4, a). Point O will be located at the origin of the coordinates, then the coordinates of the second point Q will be: $x = L$; $z = H$. Height H is assumed to be equal to 55 cm, but offset L, angles ε_1 and ε_2 can vary within the preset boundaries. By means of the preset flat guiding curve it is possible to construct a surface of the mouldboard both of a cylindrical and a cylindroidal shape. If the surface is cylindrical, then all its rectilinear generatrices are parallel to each other, parallel to the furrow bottom forming angle γ = 42° with the furrow wall. If the surface is cylindrical, then all its rectilinear generatrices are also parallel to the bottom of the furrow, but with the furrow wall they form a variable angle γ depending on height h from the bottom of the furrow. Prof. N.Shchuchkin gives graphs of variable angles $\gamma = \gamma(h)$, which he has built as a result of a massive study of the best cultural mouldboards and experimental plough bodies (Shchuchkin, 1982). A graph of this dependence is presented in Fig. 4 b, and it can be described by an algebraic curve of the fourth order:

$$\gamma = c_1 h^4 + c_2 h^3 + c_3 h^2 + c_4 h + c_5 \ , \tag{27}$$

where c_1, c_2, c_3, c_4, c_5 – coefficients found from the condition that the graph is passing through characteristic points (γ = 42º at h = 0, γ = 40º at h = 10 cm, γ = 47º at h = 55 cm).

If the parametric equations of the guiding parabola have the form (Bulgakov, 2011):

$$\begin{aligned} x &= \left(x_Q - 2x_R\right)v^2 + 2x_R v, \\ h &= \left(z_Q - 2z_R\right)v^2 + 2z_R v, \end{aligned} \tag{28}$$

where v – a variable assuming a value within the limits v = 0...1;

x_Q, x_R, z_Q, z_R – constant values, found from the preset design parameters ε_1, ε_2, L, H, then the parametric equations of the cylindrical surface of the mouldboard are written as follows:

$$\begin{aligned} X &= x\cos\gamma_0 - u\sin\gamma, \\ Y &= x\sin\gamma_0 + u\cos\gamma, \\ Z &= h, \end{aligned} \tag{29}$$

where u – the second independent surface variable specifying the length of the rectilinear generatrix;

$\gamma = \gamma\left(h\right)$, $x = x(v)$, $h = h(v)$ – dependencies presented in (27) and (28);

γ_0 = 42º – the installation angle of the ploughshare blade to the wall of the furrow.

If a certain relationship is established between the independent variables u and v in the form $u = u(v)$, then a line will be assigned on surface (29). Let this line be geodesic for which the dependence $u = u(v)$ will be sought. In this case the equations of the geodesic line are written as follows:

$$\begin{aligned} x_g &= x\cos\gamma_0 - u\sin\gamma, \\ y_g &= x\sin\gamma_0 + u\cos\gamma, \\ z_g &= h\ . \end{aligned} \tag{30}$$

In order for the line (30) to be geodesic, equality (3) must be fulfilled:

$$\begin{vmatrix} N_x & N_y & N_z \\ x'_g & y'_g & z'_g \\ x''_g & y''_g & z''_g \end{vmatrix} = 0, \tag{31}$$

where N_x, N_y, N_z – coordinates of the unitary vector of the normal to the surface; $x'_g, y'_g, z'_g, x''_g, y'', z''_g$ – the first and the second derivatives of equations (30) with respect to the variable v.

Identification of the unitary vector of the normal to the surface is shown in work (Vasilenko, 1980). For the surface (29) we propose a finished result of expressions for the vector along the geodesic line:

$$\begin{aligned} N_x &= -z'_g \cos\gamma, \\ N_y &= -z'_g \sin\gamma, \\ N_z &= x'_g \cos(\gamma - \gamma_0) - u\gamma'. \end{aligned} \tag{32}$$

The derivatives of equations (30) with respect to variable v will be:

$$\begin{aligned} x'_g &= x' \cos\gamma_0 - u' \sin\gamma - u\gamma' \cos\gamma, \\ y'_g &= x' \sin\gamma_0 + u' \cos\gamma - u\gamma' \sin\gamma, \\ z'_g &= h', \end{aligned} \tag{33}$$

$$\begin{aligned} x''_g &= x'' \cos\gamma_0 + \left(u\gamma'^2 - u''\right)\sin\gamma - \left(u\gamma'' + 2u'\gamma'\right)\cos\gamma, \\ y''_g &= x'' \sin\gamma_0 - \left(u\gamma'^2 - u''\right)\cos\gamma - \left(u\gamma'' + 2u'\gamma'\right)\sin\gamma, \\ z''_g &= h''. \end{aligned} \tag{34}$$

Since $\gamma = \gamma(h)$ according to (27), but $h = h(v)$ according to (28), the first and the second derivatives of angle γ will be written in the following way:

$$\gamma' = \frac{d\gamma}{dh}\frac{dh}{dv}, \tag{35}$$

$$\gamma'' = \left(\frac{d\gamma}{dh}\frac{dh}{dv}\right)' = \frac{d^2\gamma}{dh^2}\left(\frac{dh}{dv}\right)^2 + \frac{d\gamma}{dh}\frac{d^2h}{dv^2}\ . \tag{36}$$

Substitution of expressions (32) and (33) by (35, 36) into (31) leads to a differential equation of the following form:

$$u'' = \frac{A - B + C + D + E}{F}, \tag{37}$$

where the components that are functions of variable v have the following expressions:

$$\begin{aligned}
A &= \gamma'^2 u\left(2\gamma'^2 u^2 + 4u'^2 + 2z_g'^2 + x_g'^2\right) + u'\left(2\gamma'\gamma'' u^2 + 2z_g' z_g'' + x_g' x_g''\right),\\
B &= \left[u x_g'\left(2\gamma'^3 u + \gamma'' u'\right) + \gamma' u'\left(2u' x_g' + u x_g''\right)\right]\cos\left(\gamma - \gamma_0\right),\\
C &= x_g'\left(\gamma'^2 x_g' u + x_g'' u'\right)\cos\left[2\left(\gamma - \gamma_0\right)\right],\\
D &= 2\left[\gamma' u\left(\gamma' u x_g'' - 2\gamma' u' x_g' - \gamma'' u x_g'\right) + z_g'\left(x_g'' z_g' - x_g' z_g''\right)\right]\sin\left(\gamma - \gamma_0\right),\\
E &= x_g'\left(\gamma'' u x' + 2\gamma' u' x_g' - \gamma' u x_g''\right)\sin\left[2\left(\gamma - \gamma_0\right)\right],\\
F &= 2\gamma'^2 u^2 + 2z_g'^2 + x_g'^2 - 4\gamma' u x_g' \cos\left(\gamma - \gamma_0\right) + x_g'^2 \cos\left[2\left(\gamma - \gamma_0\right)\right].
\end{aligned} \tag{38}$$

3. Results and discussion

The obtained differential equation (37), taking into account equations (38), was solved by numerical methods on the PC. The integration constants were selected in such a way that the geodesic line started at a preset point of the blade and crossed it at angle γ_o = 42º, accepted for the ploughs. The obtained dependence $u = u(v)$ was inserted into equations (30), according to which a geodesic line was built on the surface of the mouldboard (29). We limited the surface of the mouldboard to the frontal contour the automated construction of which was described in detail in (Bulgakov, 2011). Unlike a cylindrical surface, the values of angle θ for the geodesic lines of the same height emerging from different points of the mouldboard differ, yet with a slight divergence (Fig. 5). Obviously, this divergence will be the greater, the more the cylindroidal surface will differ from the cylindrical surface, i.e., from the way of regularity (27), which establishes the distribution law of the rectilinear generatrices by height. If $\gamma = \gamma_o - const$, the cylindroidal surface turns into a cylindrical one, and the value of angle θ for the geodesic lines along the length of the mouldboard becomes equal.

In Fig. 5 for the accepted angles ε_1 = 30º, ε_2 = 95º, there are geodesic lines constructed for different values of offset L with the geodesic lines emerging from three points on the mouldboard.

The numerical values of angle θ for each line are indicated by an arrow. Comparing the respective numerical values of angle θ for the geodesic lines with different offset values L, we can conclude that this indicator increases when value L increases.

In Fig. 6 for the accepted angles ε_1 = 30º, ε_2 = 95º, offset L = 30 cm geodetic lines from the middle point of the blade have been constructed at different arrival angles of the layer arrival onto the mouldboard, the geodesic line for the accepted value γ_o = 42º being presented by a dashed line.

At the arrival angles close to 90º, the value of angle θ is the largest, and the arrival angle decreasing, it also decreases, but not so as on a cylindrical surface where this regularity is described analytically.

Let us find out how angle θ depends on angles ε_1 and ε_2 (Fig. 4 a) for the corresponding shape of the guiding curve. Fig. 7 a presents three mouldboards with different offsets and geodesic lines emerging from the middle of the blade, depicted so that the blade is projected into a point. Apparently, increase in value θ is due to the increase in the path of the layer when it is raised to the same degree. In Fig. 7 b geodesic lines from the middle point of the blade are constructed by numerical integration, indicating angle θ for different values of angles ε_1 and the accepted values L = 30 cm, ε_2 = 95º, and in Fig. 7 c – for different values of angles ε_2 and the accepted values L = 30 cm, ε_1 = 30º. It is evident from the obtained results that, angle ε_1 increasing, the value of angle θ decreases, but, when angle ε_2 increases, on the contrary, it also increases.

4. Conclusions

1. The dynamic component of the draught resistance of the plough, the value of which is directly proportional to the square of the ploughing velocity, can be estimated by the value of a dimensionless coefficient, conditionally named the wrapping angle of the layer. This parameter for the mouldboard surface can be found if the path of the movement of the mid-point of the layer section is known. If the geodesic line is taken as the curve of the layer movement as a

limiting path, the problem is reduced to finding the corresponding value of the wrapping angle.

2. For cylindrical surfaces, the geodesic line intersects all the generatrices at a constant angle, and the wrapping angle depends on the value of this angle and the length of the line on the mouldboard.
3. For cylindroidal surfaces, the angle of the section is variable; therefore it is not possible to establish certain analytical regularities to determine the geodesic lines with the predicted value of the wrapping angle. However, by numerical methods it is possible to determine the impact of one or another design parameter of the mouldboard upon the value of the wrapping angle of the layer, which is directly related to the value of the dynamic component of the draught resistance.
4. The proposed approach is not the determining (most important) for designing mouldboards, since it takes into account only the geometry of the surface without considering other agrotechnical requirements. For instance, the dynamic component of the resistance will completely disappear if the surface turns into a flat wedge; yet in this case no turn of the layer will take place. Therefore, a search for the ways how to reduce the wrapping angle of the layer carried out by mouldboard should be combined with the quality requirements of ploughing.

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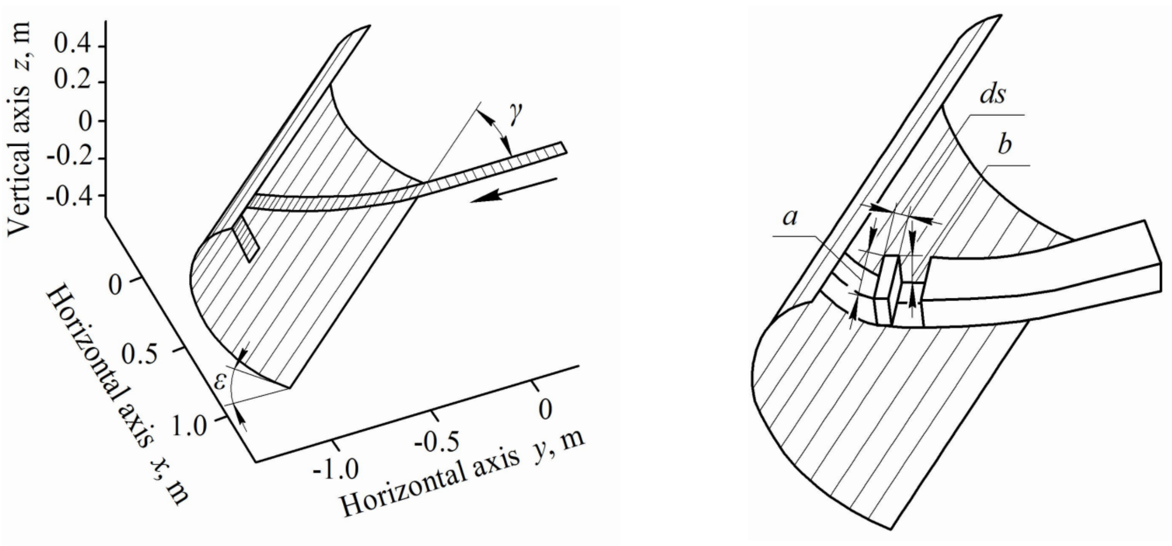


a) b)

Fig. 1. On determination of the direction of movement of the soil layer along the cylindrical surface: a) simulation of the movement of the layer along the inner surface by means of a flexible elastic band that crosses all generating surfaces at a constant angle γ ; b) geometric dimensions of the layer and its infinitesimal element in the form of a parallelepiped

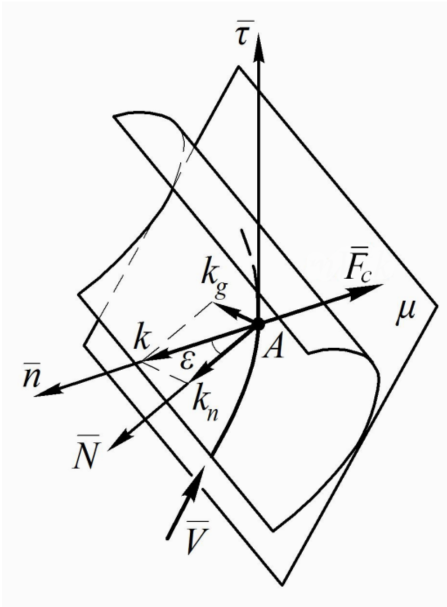
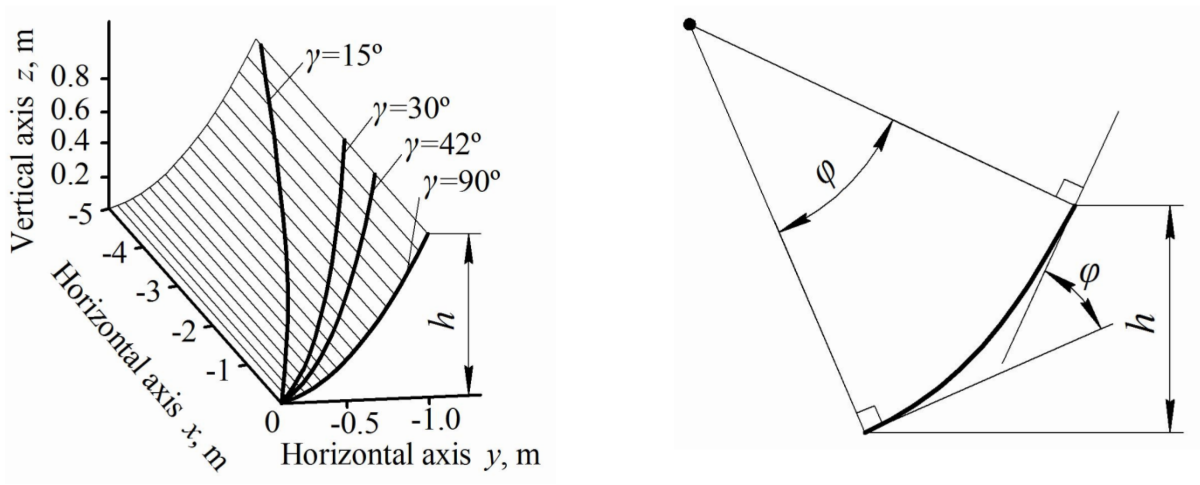


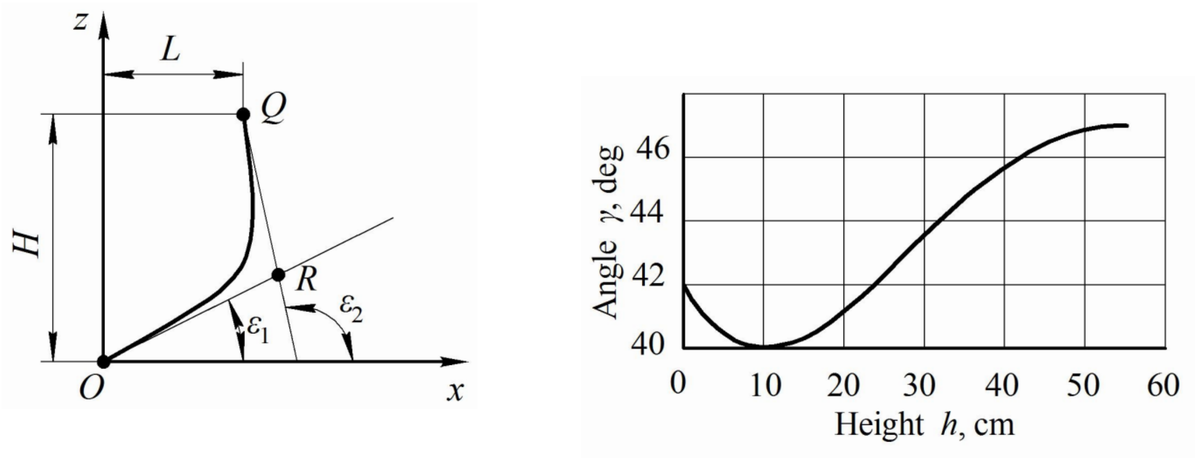
Fig. 2. Decomposition of the curvature vector and the forces acting upon the element of the layer at point A of the path of its movement at velocity V



a)

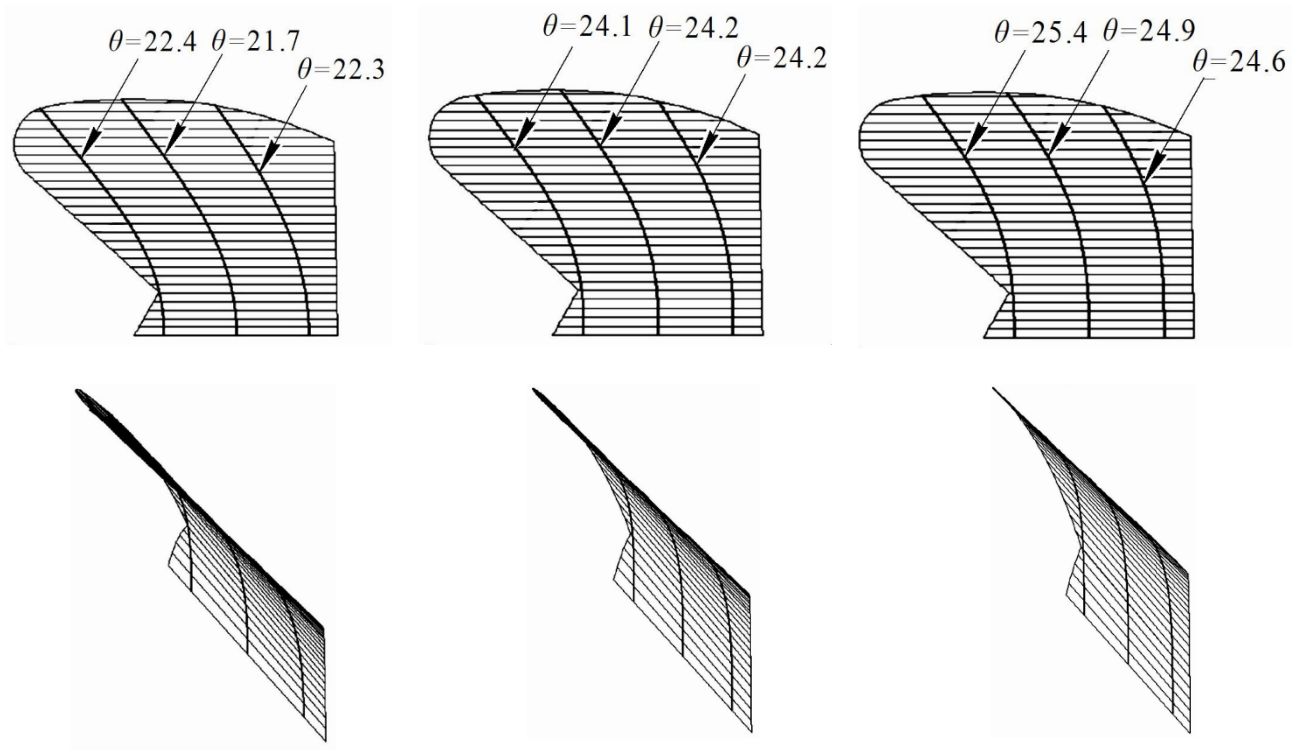
b)

Fig. 3. A cylindrical surface defined by expression (22), and its projection onto a plane, perpendicular to the rectilinear generatrices: a) geodesic lines on the surface indicating the value of angle γ ; b) an orthogonal section of the surface with tangents and normals at the ends of the curve



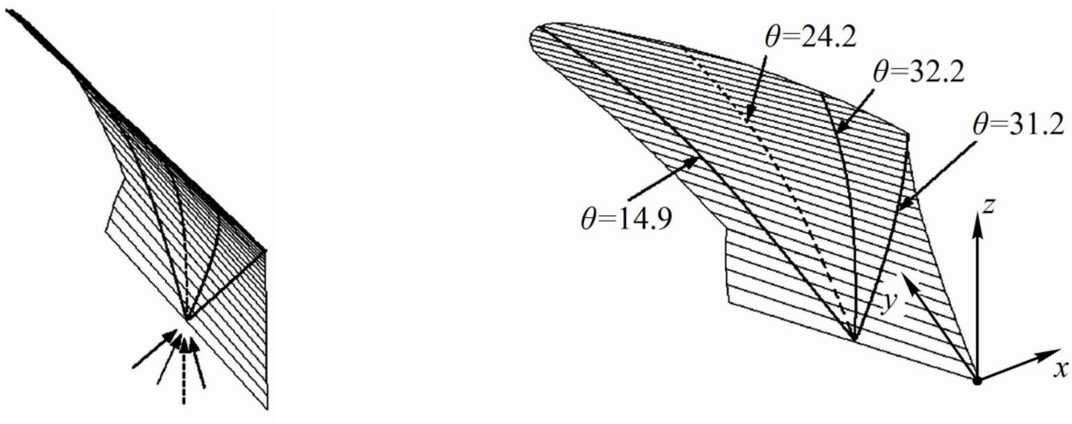
a) b)

Fig. 4. On the definition of a particular surface of a cultural mouldboard: a) the guiding curve (the parabola in a vertical plane, perpendicular to the ploughshare); b) a graph of the variable inclination angle $\gamma = \gamma(h)$ of the rectilinear surface generatrix to the furrow wall



a) b) c)

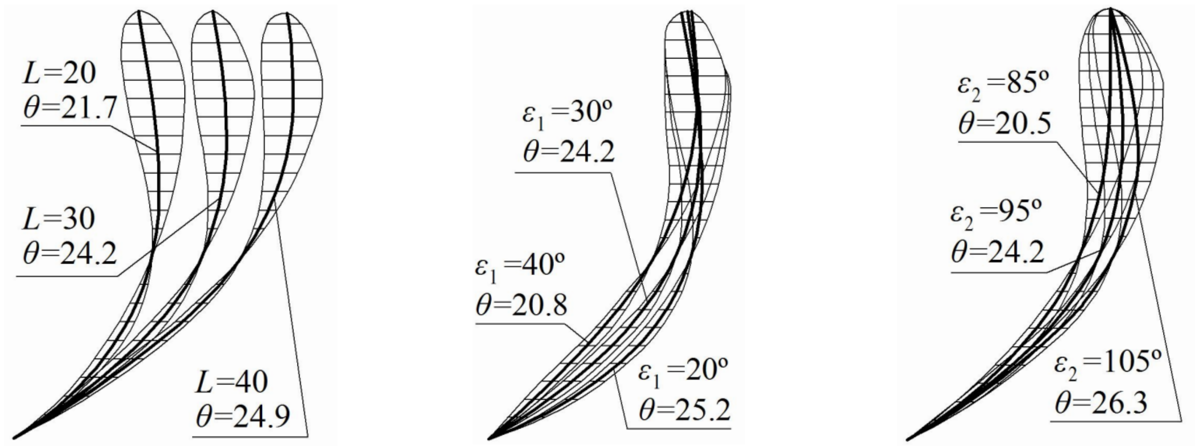
Fig. 5. Cultural mouldboards of ploughs (in two projections) with geodesic lines for the preset design parameters ε_1 = 30º, ε_2 = 95º and various offset values (see Fig. 4 a): a) L = 20 cm; b) L = 30 cm; c) L = 40 cm



a)

b)

Fig. 6. a) Direction of arrival of the soil layer; b) the value of the wrapping angle of the corresponding path for a cultural mouldboard



a) b) c)

Fig. 7. Geodesic lines emerging from the middle of the blade, and corresponding values of angle θ for various forms of the guiding parabola of a cultural (cylindroidal) mouldboard: a) a different offset value L for the accepted angles ε_1 = 30º, ε_2 = 95º; b) a different value of angle ε_1 at the accepted values ε_2 = 95º, L = 30 cm; c) a different value of angle ε_2 at the accepted values ε_1 = 30º, L = 30 cm